ΕΠΑΝΑΛΗΠΤΙΚΕΣ ΑΣΚΗΣΕΙΣ, ΕΑΡΙΝΟ 2024

ΑΣΚΗΣΗ – 8

Χρησιμοποιώντας σειρές Taylor, να βρείτε το όριο

α) $\lim_{x\to 0}\frac{\sin x^2}{1-\cos 2x}$ β) $\lim_{x\to 0}\frac{\sin x - x}{x^3}$

$$\sin x^2 = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \dots$$

The Maclaurin series for $\cos x$ is

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots,$$

so the series for $\cos 2x$ is

$$1 - \frac{4x^2}{2!} + \frac{16x^4}{4!} - \dots.$$

Therefore,

$$\lim_{x\to 0}\frac{\sin x^2}{1-\cos 2x} = \lim_{x\to 0}\frac{x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \dots}{1-\left(1-\frac{4x^2}{2!}+\frac{16x^4}{4!}-\dots\right)} = \lim_{x\to 0}\frac{x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \dots}{\frac{4x^2}{2!} - \frac{16x^4}{4!} + \dots}.$$

Dividing numerator and denominator by x^2 yields

$$\lim_{x\to 0}\frac{1 - \frac{x^4}{3!} + \frac{x^8}{5!} - \dots}{\frac{4}{2!} - \frac{16x^2}{4!} + \dots} = \frac{1}{\frac{4}{2!}} = \frac{2}{4} = \frac{1}{2}.$$

$$\lim_{x\to 0}\frac{\sin x - x}{x^3} = \lim_{x\to 0}\frac{\left(x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \cdots\right) - x}{x^3}$$

$$= \lim_{x\to 0}\frac{-\frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \cdots}{x^3}$$

$$= \lim_{x\to 0} -\frac{1}{3!} + \frac{1}{5!}x^2 - \frac{1}{7!}x^4 + \cdots = -\frac{1}{3!} = -\frac{1}{6}$$