

ΑΣΚΗΣΗ – 17

Να υπολογίσετε το ολοκλήρωμα  $\int_1^2 \frac{1}{x} dx = \ln 2$  :

- i. χρησιμοποιώντας τον τραπεζοειδή κανόνα (ξεκινώντας με  $n = 2$  και διπλασιάζοντας κάθε φορά το  $n$ )
- ii. χρησιμοποιώντας στη συνέχεια τον κανόνα του Romberg και φθάνοντας μέχρι τον όρο  $R_5^4$  (η 1<sup>η</sup> στήλη του πίνακα Romberg είναι  $R_1^0$  μέχρι  $R_5^0$ ).

[ΑΠΑΝΤΗΣΗ :  $R_5^4 = 0.693147181\dots$ ]

```
function[y]=func(x)
y          = 1/x;
end
```

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**script (R17.m)**

```
close all; clear; clc;
a = 1;
b = 2;
I = Romberg(@ (x) func(x), a, b);
fprintf('I\t=\t%0.15f\n\n', I);
fprintf('RombergDisp ..... \n');
RombergDisp(@ (x) func(x), a, b, 4);
```

```

function[I]=Romberg(func,a,b,tol,kmax)
% function[I]=Romberg(func,a,b,tol,kmax)
% Romberg integrates function 'func' of one variable and
nonsingular
% from 'a' to 'b' with tolerance 'tol' and maximum order of 'kmax'.
% 0<tol<1 & kmax>0
% I=Romberg(@func,a,b,tol,kmax)
% I=Romberg(@func,a,b,tol),default kmax=15
% I=Romberg(@func,a,b),default tol=1e-10, kmax=15
% e.g.
% I=Romberg(@sin,0,pi)
% I=2.0000000000000000
switch nargin
    case 3
        tol      = 1e-10;
        kmax     = 15;
    case 4
        kmax     = 15;
end
kmax      = abs(kmax);
R          = zeros(1,kmax+1);
err        = 1;
Ip         = 0;
R(1)       = ((b-a)/2)*(func(a)+func(b));
k          = 1;
while(err>tol*abs(Ip))
    R(k+1)  = trapm(func,a,b,k+1,R(k));
    for j=k:-1:1
        p   = 4^(k-j+1);
        R(j) = (p*R(j+1)-R(j))/(p-1);
    end
    err     = abs(R(1)-Ip);
    Ip      = R(1);
    k       = k+1;
    if k==kmax
        warning('Maximum trials exceeded with no convergence!');
        break;
    end
end
I          = R(1);
function[I]=trapm(func,a,b,k,Ip)
I          = 0;
H          = b-a;
for i=1:2^(k-2)
    I       = I+func(a+(2*i-1)*H/(2^(k-1)));
end
I          = 0.5*Ip+(H/(2^(k-1)))*I;
end
end

```

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```

function []=RombergDisp(func,a,b,K)
% function []=RombergDisp(func,a,b,k)
% Romberg integrates function "func" of one variable and
nonsingular
% from "a" to "b" with order of "K" and displays all
results.
% k>0
% I = RombergDisp(@func,a,b,k)
% e.g.
% RombergDisp(@sin,0,pi,2)
% 0.0000000000000000
% 1.570796326794897 2.094395102393195
% 1.896118897937040 2.004559754984421 1.998570731823836
tol = 1e-15;
kmax = abs(K);
R = zeros(1,kmax+1);
err = 1;
Ip = 0;
R(1) = (b-a)/2*(func(a)+func(b));
fprintf('%0.15f\n',R(1));
k = 1;
while(err>tol*abs(Ip))
    R(k+1) = trapm(func,a,b,k+1,R(k));
    fprintf('%0.15f\t',R(k+1));
    for j=k:-1:1
        p = 4^(k-j+1);
        R(j) = (p*R(j+1)-R(j))/(p-1);
        fprintf('%0.15f\t',R(j));
    end
    fprintf('\n');
    err = abs(R(1)-Ip);
    Ip = R(1);
    k = k+1;
    if k==K+1
        break;
    end
end
function [I]=trapm(func,a,b,k,Ip)
I = 0;
H = b-a;
for i=1:2^(k-2)
    I = I+func(a+(2*i-1)*H/(2^(k-1)));
end
I = 0.5*Ip+(H/(2^(k-1)))*I;
end
end

```