ΦΥΛΛΑΔΙΟ 6 ΑΣΚΗΣΗ 8, ΕΑΡΙΝΟ 2024

Να λυθεί το ακόλουθο ζεύγος διαφορικών εξισώσεων

$$\frac{dy}{dx}=\frac{x+y}{z},\quad \frac{dz}{dx}=xy+z \qquad \text{με αρχικές συνθήκες } x_0=0.5,\ y_0=1.5,\ z_0=1 \text{ για } x=0.6$$

Solution. Let $h=0.1$ and calculate the values of $k_1,k_2,k_3,k_4;\ l_1,l_2,l_3,l_4$.
Here $f(x,y)=\dfrac{x+y}{z},\quad g(x,y)=xy+z$.

$k_1=hf(x_0,y_0,z_0)=0.1\times\dfrac{0.5+1.5}{1}=0.2$

$l_1=hg(x_0,y_0,z_0)=0.1\times(0.5\times1.5+1)=0.175$

$k_2=hf(x_0+h/2,y_0+k_1/2,z_0+l_1/2)=0.1\times\dfrac{0.55+1.6}{1.0875}=0.197701$

$l_2=hg(x_0+h/2,y_0+k_1/2,z_0+l_1/2)=0.1\times(0.55\times1.6+1.0875)$
$=0.19675$

$k_3=hf(x_0+h/2,y_0+k_2/2,z_0+l_2/2)=0.1\times\dfrac{0.55+1.59885}{1.098375}=0.195639$

$l_3=hg(x_0+h/2,y_0+k_2/2,z_0+l_2/2)=0.1\times(0.55\times1.59885+1.098375)$
$=0.197774$

$k_4=hf(x_0+h,y_0+k_3,z_0+l_3)=0.1\times\dfrac{0.6+1.695639}{1.197774}=0.191659$

$l_4=hg(x_0+h,y_0+k_3,z_0+l_3)=0.1\times(0.6\times1.695639+1.197774)$
$=0.221516.$

Hence,

$$y(0.6)=y_1=y_0+\frac{1}{6}[k_1+2(k_2+k_3)+k_4]$$
$$=1.5+\frac{1}{6}[0.2+2(0.197701+0.195639)+0.191659]=1.696390.$$
$$z(0.6)=z_1=z_0+\frac{1}{6}[l_1+2(l_2+l_3)+l_4]$$
$$=1.0+\frac{1}{6}[0.175+2(0.19675+0.197774)+0.221516]=1.197594.$$

Algorithm 8.4 (Runge-Kutta method for a pair of equations). A pair of first order differential equation of the form $y' = f(x, y, z), z' = g(x, y, z)$ with initial conditions $x = x_0, y(x_0) = y_0$ and $z(x_0) = z_0$ can be solved by this algorithm using fourth-order Runge-Kutta method. The formulae are

$$y_{i+1} = y_i + \frac{1}{6}[k_1 + 2(k_2 + k_3) + k_4],$$
$$z_{i+1} = z_i + \frac{1}{6}[l_1 + 2(l_2 + l_3) + l_4]$$

where

$k_1 = hf(x_i, y_i, z_i)$
$l_1 = hg(x_i, y_i, z_i)$
$k_2 = hf(x_i + h/2, y_i + k_1/2, z_i + l_1/2)$
$l_2 = hg(x_i + h/2, y_i + k_1/2, z_i + l_1/2)$
$k_3 = hf(x_i + h/2, y_i + k_2/2, z_i + l_2/2)$
$l_3 = hg(x_i + h/2, y_i + k_2/2, z_i + l_2/2)$
$k_4 = hf(x_i + h, y_i + k_3, z_i + l_3)$
$l_4 = hg(x_i + h, y_i + k_3, z_i + l_3)$.

Algorithm RK4_Pair
Input functions $f(x, y)$ and $g(x, y)$;
Read x_0, y_0, z_0, h, x_n; //initial values of x, y, z; step size and final value of x.//
Set $y = y_0$;
Set $z = z_0$;
for $x = x_0$ to x_n step h do
    Compute the following
    $k_1 = hf(x, y, z)$;
    $l_1 = hg(x, y, z)$;
    $k_2 = hf(x + h/2, y + k_1/2, z + l_1/2)$;
    $l_2 = hg(x + h/2, y + k_1/2, z + l_1/2)$;
    $k_3 = hf(x + h/2, y + k_2/2, z + l_2/2)$;
    $l_3 = hg(x + h/2, y + k_2/2, z + l_2/2)$;
    $k_4 = hf(x + h, y + k_3, z + l_3)$;
    $l_4 = hg(x + h, y + k_3, z + l_3)$;

    $y = y + [k_1 + 2(k_2 + k_3) + k_4]/6$;
    $z = z + [l_1 + 2(l_2 + l_3) + l_4]/6$;
    Print x, y, z;
endfor;
end RK4_Pair