

ΦΥΛΛΑΔΙΟ 6 ΑΣΚΗΣΗ 7, ΕΑΡΙΝΟ 2024

Δίνεται η διαφορική εξίσωση 2^{ης} τάξης $y'' + xy' + y^2 = \sin(x)$ με αρχικές συνθήκες $y(0) = 1$, $y'(0) = 1$. Να χρησιμοποιήσετε τη μέθοδο Euler για να βρείτε τις τιμές των y και y'' για $x = 0.5$. Να χρησιμοποιήσετε βήμα 0.1.

On substituting $y' = z$ in the given second-order ODE, the following system of first order ODEs is obtained

$$\begin{aligned}\frac{dy}{dx} &= y' = z \\ \frac{dz}{dx} &= z' = \sin(x) - xz - y^2 \\ y(0) &= 1, z(0) = 1\end{aligned}$$

For this system, we have

initial conditions $x_0 = 0$, $y(x_0) = y_0 = 1$, $z(x_0) = z_0 = 1$,
functions $f(x, y, z) = z$ and $g(x, y, z) = \sin x - xz - y^2$
step size $h = 0.1$.

On applying Euler formula as in previous examples, we get

$$\begin{aligned}y_1 &= y(x_1) = y(0.1) = y_0 + h f(x_0, y_0, z_0) \\ &= y_0 + h(z_0) = 1 + 0.1(1) = 1.1 \\ z_1 &= z(x_1) = z(0.1) = z_0 + h g(x_0, y_0, z_0) \\ &= z_0 + h(\sin x_0 - x_0 z_0 - y_0^2) = 1 + 0.1(0 - 0 - 1) = 0.9\end{aligned}$$

Similarly, other iterations are as follows

$$y_2 = 1.190000 \quad z_2 = 0.779983$$

$$y_3 = 1.267998 \quad z_3 = 0.642641$$

$$y_4 = 1.332262 \quad z_4 = 0.492131$$

$$y_5 = 1.381476 \quad z_5 = 0.333896$$

Note that $y(0.5) = y_5 = 1.381476$ and $y'(0.5) = z(0.5) = z_5 = 0.333896$.