ΦΥΛΛΑΔΙΟ 6 ΑΣΚΗΣΗ 6, ΕΑΡΙΝΟ 2024

Να προσδιορίσετε την τιμή $y(0.1)$ της εξίσωσης $y' = x^2 + y^2$ με $x = 0, y = 1$ χρησιμοποιώντας τη μέθοδο Runge-Kutta 4ης τάξης.

Solution. Here $h = 0.1, x_0 = 0, y_0 = 1, f(x, y) = x^2 + y^2$.

$$\begin{aligned}
k_1 &= hf(x_0, y_0) = 0.1 \times (0^2 + 1^2) = 0.1000.\\
k_2 &= hf(x_0 + h/2, y_0 + k_1/2) = 0.1 \times f(0.05, 1.05)\\
&= 0.1 \times (0.05^2 + 1.05^2) = 0.1105.\\
k_3 &= hf(x_0 + h/2, y_0 + k_2/2) = 0.1 \times f(0.05, 1.0553)\\
&= 0.1 \times (0.05^2 + 1.0553^2) = 0.1116.\\
k_4 &= hf(x_0 + h, y_0 + k_3) = 0.1 \times f(0.1, 1.1116)\\
&= 0.1 \times (0.1^2 + 1.1116^2) = 0.1246.
\end{aligned}$$

Therefore,

$$\begin{aligned}
y_1 &= y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)\\
&= 1 + \frac{1}{6}(0.1000 + 2 \times 0.1105 + 2 \times 0.1116 + 0.1246)\\
&= 1.1115.
\end{aligned}$$

Algorithm 8.3 (Fourth-order Runge-Kutta method). This algorithm finds the solution of the differential equation $y' = f(x, y)$ with $y(x_0) = y_0$ using fourth-order Runge-Kutta method, i.e., using the formula

$$y_{i+1} = y_i + \frac{1}{6}[k_1 + 2(k_2 + k_3) + k_4]$$

within the interval $[x_1, x_n]$ at step h.

Algorithm RK4
Input function $f(x, y)$;
Read x_0, x_n, y_0, h; //initial and final value of x, initial value of y and step size.//
Set $y = y_0$;
for $x = x_0$ to x_n step h do
    Compute $k_1 = h * f(x, y)$;
    Compute $k_2 = h * f(x + h/2, y + k_1/2)$;
    Compute $k_3 = h * f(x + h/2, y + k_2/2)$;
    Compute $k_4 = h * f(x + h, y + k_3)$;
    Compute $y = y + [k_1 + 2(k_2 + k_3) + k_4]/6$;
    Print x, y;
endfor;
end RK4

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/* Program Fourth Order Runge-Kutta
   Solution of a differential equation of the form y'=f(x,y),
   y(x0)=y0 by fourth order Runge-Kutta method. */
#include<stdio.h>
#include<math.h>
void main()
{
 float x0,y0,xn,h,x,y,k1,k2,k3,k4;
 float f(float x, float y);
 printf("Enter the initial values of x and y ");
 scanf("%f %f",&x0,&y0);
 printf("Enter last value of x ");
 scanf("%f",&xn);
 printf("Enter step length h ");
 scanf("%f",&h);
 y=y0;
 printf(" x-value    y-value\n");

 for(x=x0;x<xn;x+=h)
    {
      k1=h*f(x,y);
      k2=h*f(x+h/2,y+k1/2);
      k3=h*f(x+h/2,y+k2/2);
      k4=h*f(x+h,y+k3);
      y=y+(k1+2*(k2+k3)+k4)/6;
      printf("%f   %f\n",x+h,y);

    }
} /* main */
/* definition of the function f(x,y) */
float f(float x, float y)
 {
   return(x*x-y*y+y);
 }
```