

**ΦΥΛΛΑΔΙΟ 6 ΑΣΚΗΣΗ 5, Εαρινό 2024**

Να προσδιορίσετε τις τιμές  $y(0.1)$  και  $y(0.2)$  της εξίσωσης  $y' = y^2 - x^2$  με  $y(0) = 2$  χρησιμοποιώντας τη μέθοδο Runge-Kutta 2ης τάξης.

**Solution.** Here  $h = 0.1, x_0 = 0, y_0 = 2, f(x, y) = y^2 - x^2$ .  
Then

$$\begin{aligned}k_1 &= hf(x_0, y_0) = 0.1(2^2 - 0^2) = 0.4000. \\k_2 &= hf(x_0 + h, y_0 + k_1) = 0.1 \times f(0 + 0.1, 2 + 0.4000) \\&= 0.1 \times (2.4^2 - 0.1^2) = 0.5750.\end{aligned}$$

Therefore,  $y_1 = y_0 + \frac{1}{2}(k_1 + k_2) = 2 + \frac{1}{2}(0.4000 + 0.5750) = 2.4875$ , i.e.,  $y(0.1) = 2.4875$ .  
To determine  $y_2 = y(0.2)$ , let  $x_1 = 0.1$  and  $y_1 = 2.4875$ .

$$\begin{aligned}k_1 &= hf(x_1, y_1) = 0.1 \times f(0.1, 2.4875) = 0.1 \times (2.4875^2 - 0.1^2) \\&= 0.6178. \\k_2 &= hf(x_1 + h, y_1 + k_1) = 0.1 \times f(0.2, 2.4875 + 0.6178) \\&= 0.1 \times f(0.2, 3.1053) = 0.1 \times (3.1053^2 - 0.2^2) = 0.9603.\end{aligned}$$

Therefore,  $y_2 = y_1 + \frac{1}{2}(k_1 + k_2) = 2.4875 + \frac{1}{2}(0.6178 + 0.9603) = 3.2766$ .  
Hence,  $y(0.2) = 3.2766$ .