ΦΥΛΛΑΔΙΟ 6 ΑΣΚΗΣΗ 4 , ΕΑΡΙΝΟ 2024

Να προσδιορίσετε την τιμή του y όταν $x = 0.1$ και $x = 0.2$ δοθέντος ότι $y(0) = 1$ και $y' = x^2 - y$. Να χρησιμοποιήσετε την τροποποιημένη μέθοδο Euler.

Solution. Let $h = 0.1, x_0 = 0, y_0 = 1, x_1 = 0.1, x_2 = 0.2$ and $f(x, y) = x^2 - y$.

$$\begin{aligned}
y_1^{(0)} &= y_0 + hf(x_0, y_0) = 1 + 0.1f(0, 1) = 1 + 0.1 \times (0 - 1) = 0.9000. \\
y_1^{(1)} &= y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(0)})] \\
&= 1 + \frac{0.1}{2}[(0^2 - 1) + (0.1^2 - 0.9)] = 0.9055. \\
y_1^{(2)} &= y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(1)})] \\
&= 1 + \frac{0.1}{2}[(0^2 - 1) + (0.1^2 - 0.9055)] = 0.9052. \\
y_1^{(3)} &= y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(2)})] \\
&= 1 + \frac{0.1}{2}[(0^2 - 1) + (0.1^2 - 0.9052)] = 0.9052.
\end{aligned}$$

Therefore, $y_1 = y(0.1) = 0.9052$.

$$\begin{aligned}
y_2^{(0)} &= y_1 + hf(x_1, y_1) = 0.9052 + 0.1f(0.1, 0.9052) \\
&= 0.9052 + 0.1 \times (0.1^2 - 0.9052) = 0.8157. \\
y_2^{(1)} &= y_1 + \frac{h}{2}[f(x_1, y_1) + f(x_2, y_2^{(0)})] \\
&= 0.9052 + \frac{0.1}{2}[(0.1^2 - 0.9052) + (0.2^2 - 0.8157)] = 0.8217. \\
y_2^{(2)} &= y_1 + \frac{h}{2}[f(x_1, y_1) + f(x_2, y_2^{(1)})] \\
&= 0.9052 + \frac{0.1}{2}[(0.1^2 - 0.9052) + (0.2^2 - 0.8217)] = 0.8214. \\
y_2^{(3)} &= y_1 + \frac{h}{2}[f(x_1, y_1) + f(x_2, y_2^{(2)})] \\
&= 0.9052 + \frac{0.1}{2}[(0.1^2 - 0.9052) + (0.2^2 - 0.8214)] = 0.8214.
\end{aligned}$$

Hence, $y_2 = y(0.2) = 0.8214$.

Algorithm 8.2 (Modified Euler's method). This algorithm solves the initial value problem $y' = f(x, y)$ with $y(x_0) = y_0$ over the interval $[x_0, x_n]$ with step size h. The formulae are given by

$$
\begin{aligned}
y_{i+1}^{(0)} &= y_i + hf(x_i, y_i) \\
y_{i+1}^{(k)} &= y_i + \frac{h}{2}[f(x_i, y_i) + f(x_{i+1}, y_{i+1}^{(k-1)})], \text{ for } k = 1, 2, \ldots
\end{aligned}
$$

Algorithm Modified_Euler
Input function $f(x, y)$;
Read x_0, x_n, y_0, h; //initial and final values of x, initial value of y and step size h.//
Read ε; //ε is the error tolerance.//
Set $y = y_0$;
for $x = x_0$ to x_n step h do
    Compute $f_1 = f(x, y)$;
    Compute $y_c = y + h * f_1$; //evaluated from Euler's method//
    do
        Set $y_p = y_c$;
        Compute $y_c = y + \frac{h}{2}[f_1 + f(x + h, y_p)]$ //modified Euler's method//
    while ($|y_p - y_c| > \varepsilon$) //check for accuracy//
    Reset $y = y_c$;
    Print x, y;
endfor;
end Modified_Euler

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/* Program Modified Euler
   Solution of a differential equation of the form y'=f(x,y),
   y(x0)=y0 by Modified Euler's method. */
#include<stdio.h>
#include<math.h>

void main()
{
 float x0,y0,xn,h,x,y;/*x0, xn the initial and final values of x*/
   /* y0 initial value of y, h is the step length */
 float eps=1e-5; /* the error tolerance */
 float yc,yp,f1;
 float f(float x, float y);
 printf("Enter the initial (x0) and final (xn) values of x ");
 scanf("%f %f",&x0,&xn);
 printf("Enter initial value of y ");
 scanf("%f",&y0);
 printf("Enter step length h ");
 scanf("%f",&h);
 printf(" x-value    y-value\n");
 y=y0;
 for(x=x0;x<xn;x+=h)
    {
      f1=f(x,y);
      yc=y+h*f1; /* evaluated by Euler's method */
      do
      {
        yp=yc;
        yc=y+h*(f1+f(x+h,yp))/2; /*modified Euler's method*/
      }while(fabs(yp-yc)>eps);
      y=yc;
      printf("%f   %f\n",x+h,y);
    }
} /* main */
/* definition of the function f(x,y) */
float f(float x, float y)
 {
   return(x*x-2*y+1);
 }
```