

ΦΥΛΛΑΔΙΟ 6 ΑΣΚΗΣΗ 3, ΕΑΡΙΝΟ 2024

Να βρείτε τις τιμές $y(0.1)$ και $y(0.2)$ από την ακόλουθη διαφορική εξίσωση

$$\frac{dy}{dx} = x^2 + y^2 \text{ με } y(0) = 1$$

Solution. Let $h = 0.05$, $x_0 = 0$, $y_0 = 1$.

Then

$$x_1 = x_0 + h = 0.05$$

$$y_1 = y(0.05) = y_0 + hf(x_0, y_0) = 1 + 0.05 \times (0 + 1) = 1.05$$

$$x_2 = x_1 + h = 0.1$$

$$y_2 = y(0.1) = y_1 + hf(x_1, y_1) = 1.05 + 0.05 \times (0.1^2 + 1.05^2) = 1.105625$$

$$x_3 = x_2 + h = 0.15$$

$$y_3 = y(0.15) = y_2 + hf(x_2, y_2) = 1.105625 + 0.05 \times (0.1^2 + 1.105625^2) \\ = 1.167245$$

$$x_4 = x_3 + h = 0.2$$

$$y_4 = y(0.2) = y_3 + hf(x_3, y_3) = 1.167245 + 0.05 \times (0.15^2 + 1.167245^2) \\ = 1.236493.$$

Hence $y(0.1) = 1.105625$, $y(0.2) = 1.236493$.

Algorithm 8.1 (Euler's method). This algorithm finds the solution of the equation $y' = f(x, y)$ with $y(x_0) = y_0$ over the interval $[x_0, x_n]$, by Euler's method

$$y_{i+1} = y_i + hf(x_i, y_i), \quad i = 0, 1, 2, \dots, n-1.$$

Algorithm Euler

Input function $f(x, y)$

Read x_0, y_0, x_n, h // x_0, y_0 are the initial values and x_n is the last value of x //
// where the process will terminate; h is the step size //

for $x = x_0$ to x_n step h do

$$y = y_0 + h * f(x, y_0);$$

Print x, y ;

$$y_0 = y;$$

endfor;

end Euler

```

/* Program Euler
   Solution of a differential equation of the form  $y'=f(x,y)$ ,
    $y(x_0)=y_0$  by Euler's method. */
#include<stdio.h>
#include<math.h>
void main()
{
    float x0,y0,xn,h,x,y;

    float f(float x, float y);
    printf("Enter the initial (x0) and final (xn) values of x ");
    scanf("%f %f",&x0,&xn);
    printf("Enter initial value of y ");
    scanf("%f",&y0);
    printf("Enter step length h ");
    scanf("%f",&h);
    printf(" x-value   y-value\n");
    for(x=x0;x<xn;x+=h)
    {
        y=y0+h*f(x,y0);
        printf("%f   %f \n",x+h,y);
        y0=y;
    }
} /* main */
/* definition of the function f(x,y) */
float f(float x, float y)
{
    return(x*x+x*y+2);
}

```

A sample of input/output:

```

Enter the initial (x0) and final (xn) values of x
0 .2
Enter initial value of y 1
Enter step length h .05
 x-value   y-value
0.050000  1.100000
0.100000  1.202875
0.150000  1.309389
0.200000  1.420335

```