ΦΥΛΛΑΔΙΟ 6 ΑΣΚΗΣΗ 1 , ΕΑΡΙΝΟ 2024

Χρησιμοποιώντας σειρές Taylor να λύσετε την εξίσωση

$$\frac{dy}{dx} = x - y, \quad y(0) = 1$$

για $x = 0.1$

Solution. The Taylor's series around $x = 0$ for $y(x)$ is

$$y(x) = y_0 + xy_0' + \frac{x^2}{2!}y_0'' + \frac{x^3}{3!}y_0''' + \frac{x^4}{4!}y_0^{iv} + \cdots .$$

Differentiating y' repeatedly with respect to x and substituting $x = 0$, we obtain

$$\begin{aligned} y'(x) &= x - y, \; y_0' = -1 \\ y''(x) &= 1 - y', \; y_0'' = 1 - y_0' = 2 \\ y'''(x) &= -y'', \; y_0''' = -2 \\ y^{iv}(x) &= -y''', \; y_0^{iv} = 2 \end{aligned}$$

and so on.
The Taylor's series becomes

$$\begin{aligned} y(x) &= 1 - x + \frac{x^2}{2!} \cdot 2 + \frac{x^3}{3!} \cdot (-2) + \frac{x^4}{4!} \cdot 2 + \cdots \\ &= 1 - x + x^2 - \frac{x^3}{3} + \frac{x^4}{12} - \cdots . \end{aligned}$$

This is the Taylor's series solution of the given differential equation at any point x. Now,

$$y(0.1) = 1 - (0.1) + (0.1)^2 - \frac{(0.1)^3}{3} + \frac{(0.1)^4}{12} - \cdots = 0.909675.$$