

ΦΥΛΛΑΔΙΟ 5, ΑΣΚΗΣΗ 4, 2024

x	1.0	1.2	1.4	1.6	1.8
$f(x)$	1.54	1.81	2.15	2.58	3.11

Solution:

Simpson's 1/3 rule:

$$\int_a^b f(x)dx = \frac{h}{3} (f_0 + 4f_1 + 2f_2 + 4f_3 + 2f_4 + \dots + 4f_{n-1} + f_n) + O(h^4)$$

$$h = 0.4, \quad I_1 = 0.4/3(1.54 + 4 \times 2.15 + 3.11) = 1.7667$$

$$h = 0.2, \quad I_2 = 0.2/3(1.54 + 4 \times 1.81 + 2 \times 2.15 + 4 \times 2.58 + 3.11) = 1.7673$$

$$\text{Romberg integral } I_3 = I_2 + \frac{I_2 - I_1}{2^4 - 1} = 1.767340$$

(Note that the order of error in Simpson's 1/3 rule is 4.)