ΦΥΛΛΑΔΙΟ 5 , ΑΣΚΗΣΗ 16, 2023

$$\frac{dV}{dt} = F^{\text{in}}(t)$$

$$\int_0^V dV = V = \int_0^t F(t)dt = \int_0^{t=5} 1 - e^{-t}dt$$

h	$I_1, O(h^2)$	$I_2, O(h^4)$	$I_3, O(h^6)$
$h = 5$	$I_1(5) = 2.4832$		
$h = 2.5$	$I_1(2.5) = 3.5364$	$I_2(2.5) = 3.8874$	
$h = 1.25$	$I_1(1.25) = 3.8807$	$I_2(1.25) = 3.9955$	$I_3(1.25) = 4.0027$

- $I_1(5) = \frac{5}{2}[f(0) + f(5)] = 2.4832$
- $I_1(2.5) = \frac{5}{2\times2}[f(0) + 2f(2.5) + f(5)] = 3.5364$
- $I_1(1.25) = \frac{5}{4\times2}[f(0) + 2f(1.25) + 2f(2.5) + 2f(3.75) + f(5)] = 3.8807$
- $I_2(2.5) = \frac{4I_1(2.5) - I_1(5)}{4-1} = \frac{4\times3.5364 - 2.4832}{3} = 3.8875$ using the messy formula with $j = 1$ and $k = 1$
- $I_2(1.25) = \frac{4I_1(1.25) - I_1(2.5)}{4-1} = \frac{4\times3.8807 - 3.5364}{3} = 3.9955$ with $j = 1$ and $k = 2$
- $I_3(1.25) = \frac{16I_2(1.25) - I_2(2.5)}{16-1} = \frac{16\times3.9955 - 3.8875}{15} = 4.0027$ with $j = 2$ and $k = 2$

The final estimate for the integral is the last column in the last row, so $I_3(1.25) = 4.003$

The true value of the integral is easy to find: $\int_0^{t=5} 1 - e^{-t}dt = t + e^{-t}|_0^5 = 4 + e^{-5} = 4.0067379$; using this we can see the Romberg method had lower error.