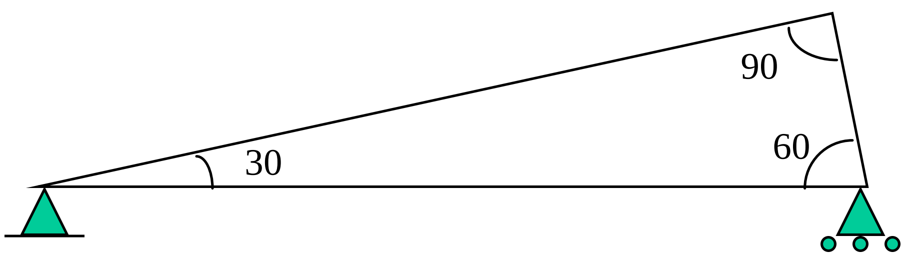
ΑΡΙΘΜΗΤΙΚΗ ΑΝΑΛΥΣΗ

Αριθμητική Επίλυση

Συστημάτων Γραμμικών Εξισώσεων

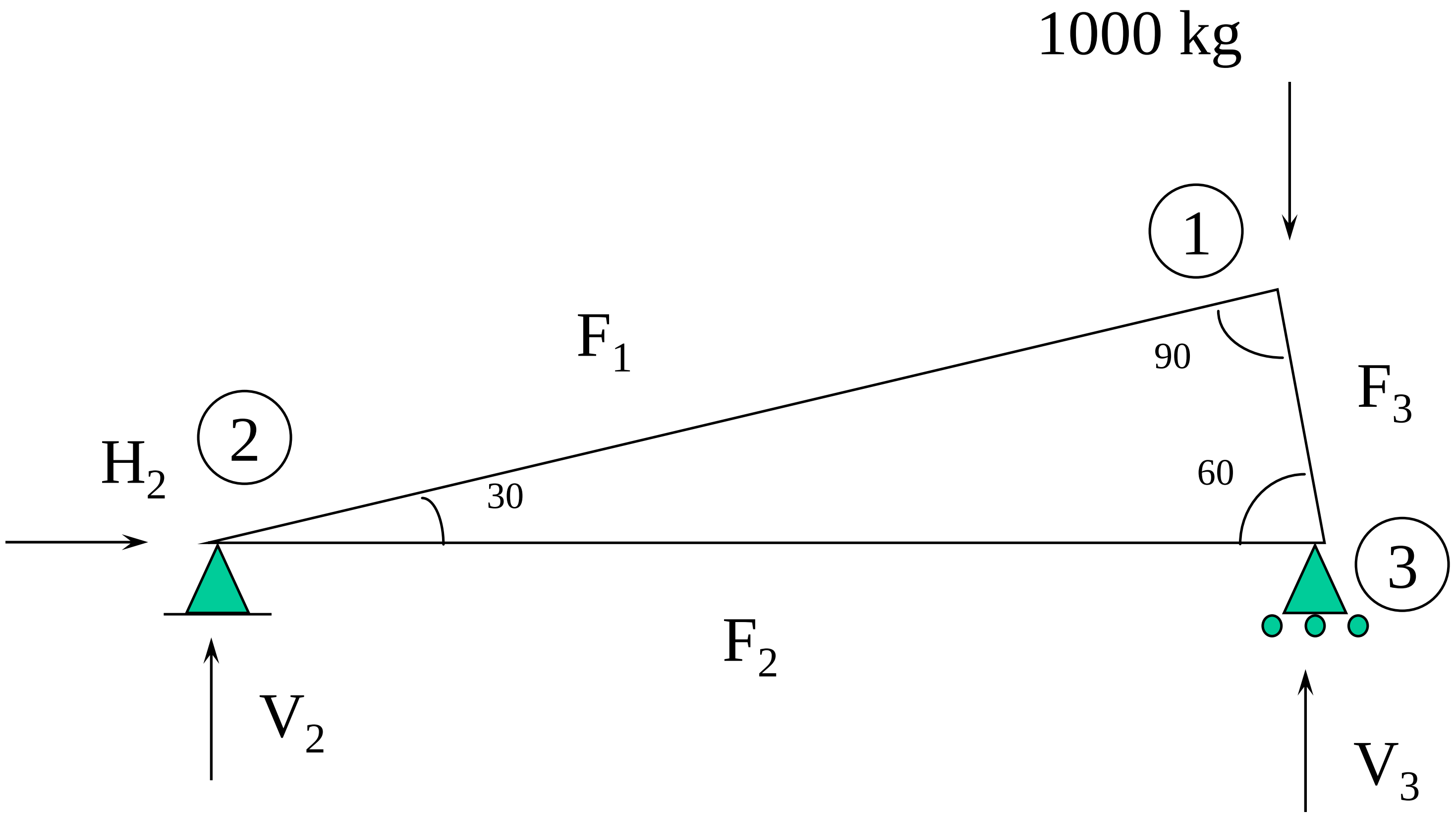
Introduction : Practical application

- Consider a problem in structural engineering
- Find the forces and reactions associated with a statically determinant truss



hinge: transmits both vertical and horizontal forces at the surface

roller: transmits vertical forces

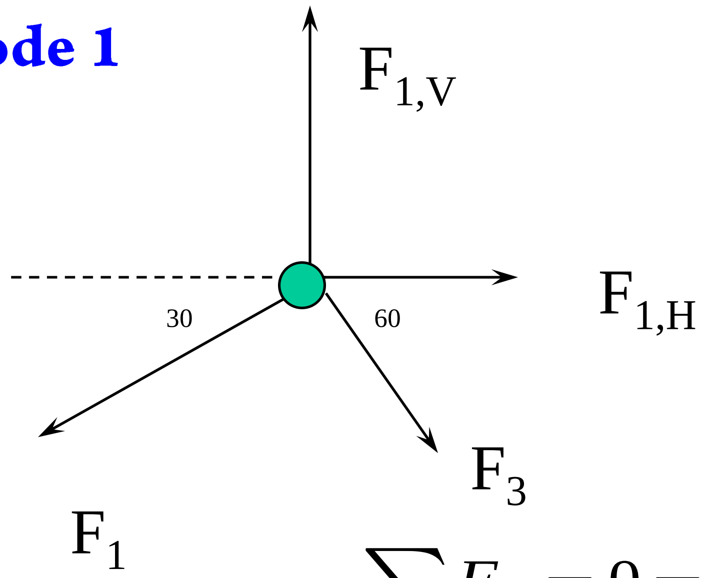


FREE BODY DIAGRAM

$$\sum F_H = 0$$

$$\sum F_v = 0$$

Node 1



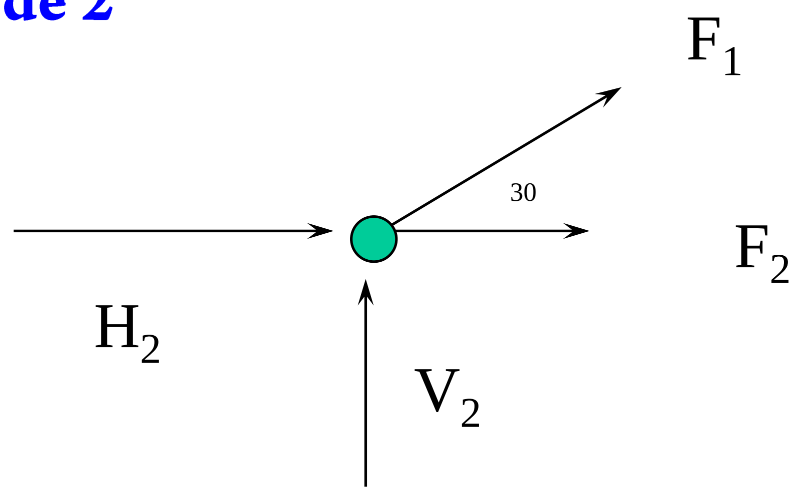
$$\sum F_H = 0 = -F_1 \cos 30^\circ + F_3 \cos 60^\circ + F_{1,H}$$

$$\sum F_V = 0 = -F_1 \sin 30^\circ - F_3 \sin 60^\circ + F_{1,V}$$

$$-F_1 \cos 30^\circ + F_3 \cos 60^\circ = 0$$

$$-F_1 \sin 30^\circ - F_3 \sin 60^\circ = -1000$$

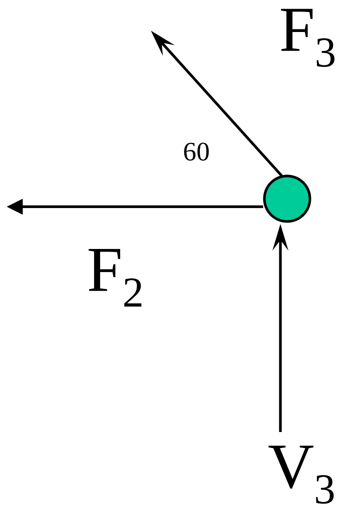
Node 2



$$\sum F_H = 0 = H_2 + F_2 + F_1 \cos 30^\circ$$

$$\sum F_V = 0 = V_2 + F_1 \sin 30^\circ$$

Node 3



$$\sum F_H = 0 = -F_3 \cos 60^\circ - F_2$$

$$\sum F_V = 0 = F_3 \sin 60^\circ + V_3$$

$$-F_1 \cos 30^\circ + F_3 \cos 60^\circ = 0$$

$$-F_1 \sin 30^\circ - F_3 \sin 60^\circ = -1000$$

$$H_2 + F_2 + F_1 \cos 30^\circ = 0$$

$$V_2 + F_1 \sin 30^\circ = 0$$

$$-F_3 \cos 60^\circ - F_2 = 0$$

$$F_3 \sin 60^\circ + V_3 = 0$$

SIX Equations

SIX Unknowns

Do some book keeping

	F_1	F_2	F_3	H_2	V_2	V_3	
1	-cos30	0	cos60	0	0	0	0
2	-sin30	0	-sin60	0	0	0	-1000
3	cos30	1	0	1	0	0	0
4	sin30	0	0	0	1	0	0
5	0	-1	-cos60	0	0	0	0
6	0	0	sin60	0	0	1	0

This is the basis for your matrices and the equation [A]{x}={b}

$$\begin{bmatrix} -0.866 & 0 & 0.5 & 0 & 0 & 0 \\ -0.5 & 0 & -0.866 & 0 & 0 & 0 \\ 0.866 & 1 & 0 & 1 & 0 & 0 \\ 0.5 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.866 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ H_2 \\ V_2 \\ V_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -1000 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

System of Linear Equations

- Now we will deal with the case of determining the values of $x_1, x_2, \ldots\ldots x_n$, that simultaneously satisfy a set of equations
  - $a_{11}x_1 + a_{12}x_2 + \ldots\ldots a_{1n}x_n = b_1$
  - $a_{21}x_1 + a_{22}x_2 + \ldots\ldots a_{2n}x_n = b_2$
  - ……….
  - $a_{n1}x_1 + a_{n2}x_2 + \ldots\ldots a_{nn}x_n = b_n$

Cramer's Rule

- Linear System of Equations

$$
\begin{array}{ccccccc}
a_{11}x_1 & a_{12}x_2 & \cdot & \cdot & a_{1n}x_n & = & b_1 \\
a_{21}x_1 & a_{22}x_2 & \cdot & \cdot & a_{2n}x_n & = & b_2 \\
\cdot & \cdot & \cdot & \cdot & \cdot & = & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & = & \cdot \\
a_{n1}x_1 & \cdot & \cdot & \cdot & a_{nn}x_n & = & b_n
\end{array}
$$

Example, n=2

$$
\begin{bmatrix} 0.01 & -1.0 \\ 1.0 & 0.01 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 1.0 \\ 1.0 \end{Bmatrix}
$$

$$
\begin{aligned}
x_1 &= \frac{1.0 \cdot 0.01 - 1.0 \cdot (-1.0)}{0.01 \cdot 0.01 - 1.0 \cdot (-1.0)} = 1.0099 \\
x_2 &= \frac{1.0 \cdot 0.01 - 1.0 \cdot 1.0}{0.01 \cdot 0.01 - 1.0 \cdot (-1.0)} = -0.9899
\end{aligned}
$$

Cramer's Rule, n=2

$$
\begin{aligned}
D &= a_{11}a_{22} - a_{21}a_{12} \\
D_1 &= b_1 a_{22} - b_2 a_{12} \\
D_2 &= b_2 a_{11} - b_1 a_{21}
\end{aligned}
$$

$$
x_1 = \frac{D_1}{D} = \frac{b_1 a_{22} - b_2 a_{12}}{a_{11}a_{22} - a_{21}a_{12}}
$$

$$
x_2 = \frac{D_2}{D} = \frac{b_2 a_{11} - b_1 a_{21}}{a_{11}a_{22} - a_{21}a_{12}}
$$

Cramer's rule inconvenient for n>3

Solution Techniques

- **Direct solution methods**
  - Finds a solution in a finite number of operations by transforming the system into an equivalent system that is 'easier' to solve.
  - Diagonal, upper or lower triangular systems are easier to solve
  - Number of operations is a function of system size n.
- **Iterative solution methods**
  - Computes succesive approximations of the solution vector for a given $\boldsymbol{A}$ and $\boldsymbol{b}$, starting from an initial point $\boldsymbol{x_0}$.
  - Total number of operations is uncertain, may not converge.

- **Direct solution methods**
  - Gauss elimination
  - Gauss-Jordan Method (variation of Gauss elimination)
  - LU Decomposition

- **Iterative solution methods**
  - Jacobi Method
  - Gauss-Seidel Method
  - Successive Over Relaxation (SOR)