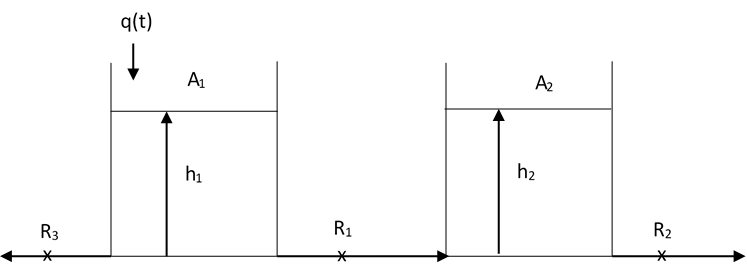
1η Πρόοδος Δυναμική και Έλεγχος Διεργασιών 14 Νοεμβρίου 2017

ΘΕΜΑ 1 (10 μονάδες)



Σε χρόνο 0, προστίθενται στιγμιαία 2 m3. Να υπολογιστεί και να παρασταθεί γραφικά (σε χρόνο 0, 0,5, 1, 2 και 5 ώρες) η απόκριση της στάθμης στην 1η και τη 2η δεξαμενή. Οι αντιστάσεις είναι γραμμικές: R1 = 1 και R2 = R3 = 0,5. Οι επιφάνειες των δεξαμενών είναι: Α1 = 1 m2 και A2=2m2. Σε μόνιμη κατάσταση η παροχή qs = 1 m3/h.

@SS 1 – (h1s – h2s)/1 – h1s/0,5 = 0 ⬄ 0,5 – 0,5h1s + 0,5h2s – h1s = 0 ⬄ 0,5 = 1,5h1s – 0,5h2s (1)

(h1s – h2s)/1 – h2s/0,5 = 0 ⬄ 0,5h1s – 0,5h2s – h2s = 0 ⬄ 0,5h1s – 1,5h2s = 0 ⬄ h1s = 3h2s (2)

1. ⬄ 4,5h2s – 0,5h2s = 0,5 ⬄ 4h2s = 0,5 ⬄ h2s = 0,125 m
2. ⬄ h1s = 0,375

Ισοζύγιο μάζας στην 1η δεξαμενή

qo(t) – (h1(t) – h2(t))/R1 – h1(t)/R3 = A1\*dh1(t)/dt ⬄ Qo(t) – H1(t) + H2(t) – H1(t)/0,5 = 1\*dH1(t)/dt ⬄

⬄ 0,5\*Qo(t) – 1,5\*H1(t) + 0,5\*H2(t) = 0,5\*dH1(t)/dt ⬄ Qo(s) – 3\*H1(s) + H2(s) = sH1(s) (3)

Ισοζύγιο μάζας στην 2η δεξαμενή

(h1(t) – h2(t))/R1 – h2(t)/R2 = A2\*dh2(t)/dt ⬄ h1(t) – h2(t) – h2(t)/0,5 = 2\*dh2(t)/dt ⬄ 0,5\*H1(t) – 1,5\*H2(t) = dH2(t)/dt ⬄

⬄ 0,5H1(s) – 1,5H2(s) = sH2(s) ⬄ 0,5H1(s) = (s + 1,5)H2(s) ⬄ H1(s) = (2s + 3)H2(s) ⬄ H2(s) = 0,5H1(s)/(s + 1,5) (4)

(3) Qo(s) – 3H1(s) + 0,5H1(s)/(s + 1,5) = sH1(s) ⬄ Qo/H1 – 3 + 0,5/(s + 1,5) = s ⬄ Qo/H1 = s – 0,5/(s + 1,5) + 3 ⬄

Qo/H1 = (s2 + 1,5s – 0,5 + 3s + 4,5)/(s + 1,5) ⬄ H1/Qo = (s + 1,5)/(s2 + 4,5s + 4) ⬄

**H1/Qo = s/(s2 + 4,5s + 4) + 1,5/( s2 + 4,5s + 4)**

(3) Qo(s) – (6s + 9)H2(s) + H2(s) = s(2s + 3)H2(s) ⬄ Qo/H2 – (6s +9) +1 = s(2s + 3) ⬄ Qo/H2 = 6s + 9 – 1 + 2s2 + 3s ⬄

⬄ Qo/H2 = 2s2 + 9s +8 ⬄ **H2/Qo = 1/(s2 + 4,5s + 4)**

Κρουστική διαταραχή της παροχής: qo(t) = qos + 2δ(t) ⬄ qo(t) – qos = 2δ(t) ⬄ Qo(t) = 2δ(t) ⬄ Qo(s) = 2

Λύση τριωνύμου: Δ = 20,25-4\*1\*4 = 4,25 x1 = (-4,5+4,25^0,5)/2 = -1,219 x2 = (-4,5-4,25^0,5)/2 = -3,281

HEAVYSIDE

2s/(s + 1,219)(s + 3,281) = A/(s + 1,219) + B/(s + 3,281)

A = (-2\*1,219)/(-1,219+3,281) = -1,182

B = (-2\*3,281)/(-3,281+1,219) = 3,182

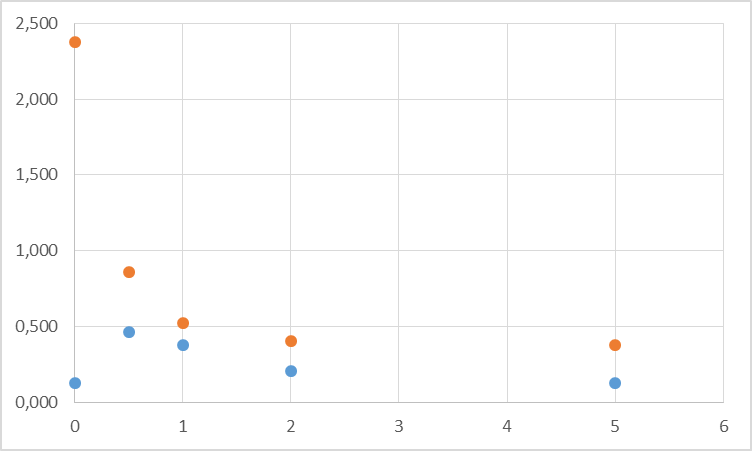
3/(s + 1,219)(s + 3,281) = Γ/(s + 1,219) + Ε/(s + 3,281) 2/(s + 1,219)(s + 3,281) = Z/(s + 1,219) + H/(s + 3,281)

Γ = 3/(-1,219+3,281) = 1,455 Z = 1/(-1,219+3,281) = 0,970

E = 3/(-3,281+1,219) = -1,455 H = 1/(-3,281+1,219) = -0,970

H1(s) = -1,182/(s + 1,219) + 3,182/(s + 3,281) + 1,455/(s + 1,219) – 1,455/(s + 3,281) = 0,273/(s + 1,219) + 1,727/(s + 3,281)

H1(t) = 0,273\*exp(-1,219\*t)+1,727\*exp(-3,281\*t)

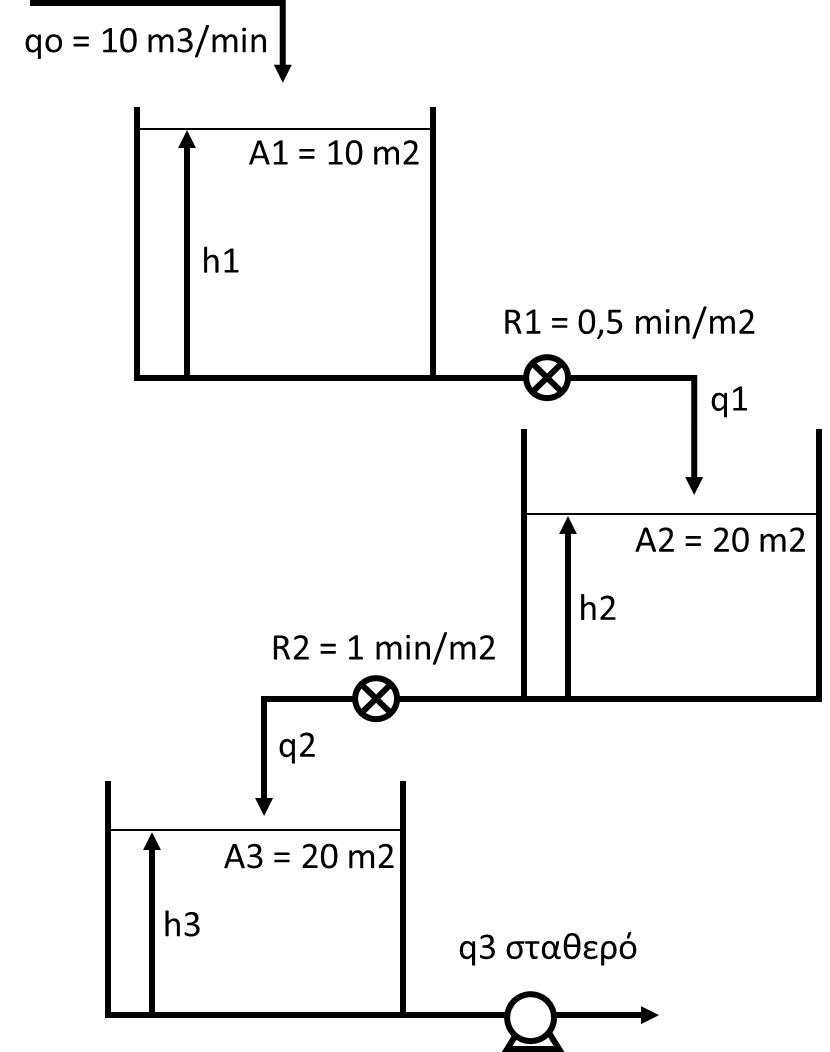


H2(s) = 0,970/(s + 1,219) - 0,970/(s + 3,281)

H2(t) = 0,970\*exp(-1,219\*t)-0,970\*exp(-3,281\*t)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **t** | **H1** | **h1** | **H2** | **h2** |
| 0 | 2,000 | 2,375 | 0,000 | 0,125 |
| 0,5 | 0,483 | 0,858 | 0,339 | 0,464 |
| 1 | 0,146 | 0,521 | 0,250 | 0,375 |
| 2 | 0,026 | 0,401 | 0,083 | 0,208 |
| 5 | 0,001 | 0,376 | 0,002 | 0,127 |

ΘΕΜΑ2 (10μονάδες)



Στο σύστημα δεξαμενών η παροχή αυξάνεται με ρυθμό 1 m3/min. Να υπολογιστεί η στάθμη των τριών δεξαμενών σε χρόνο 1 και 5 λεπτά.

ΧΩΡΙΣ ΤΡΙΤΗ ΔΕΞΑΜΕΝΗ

Γραμμική μεταβολή της qo: qo(t) = qos + t\*u(t) ⬄ qo(t) – qos = t\*u(t) ⬄

⬄ Qo(t) = t\*u(t) ⬄ **Qo(s) = 1/s2**

**Ισοζύγιο μάζας στην 1η δεξαμενή**

@SS 10 – h1s/0,5 = 0 ⬄ h1s = 5

qo(t) – h1(t)/R1 = A1\*dh1(t)/dt ⬄ Qo(t) – H1(t)/0,5 = 10\*dH1(t)/dt

⬄ 0,5\*Qo(t) – H1(t) = 5\*dH1(t)/dt ⬄ 0,5Qo(s) – H1(s) = 5sH1(s) ⬄ 0,5 – Η1/Qo = 5sH1/Qo ⬄ H1/Qo\*(5s + 1) = 0,5 ⬄

⬄ H1/Qo = 0,5/(5s + 1) ⬄ H1/Qo = 1/(10s + 2) (1)

H1 = 1/s2(10s + 2) (2)

HEAVYSIDE 1/s2(10s + 2) = A/s2 + B/s + Γ/(10s + 2) Γ = 25 A = 0,5

1/s2(10s + 2) = 0,5/s2 + B/s + 25/(10s + 2) = (0,5(10s + 2) + Bs(10s + 2) + 25s2)/s2(10s + 2) ⬄

5s + 1 + 10Bs2 + 2Bs + 25s2 = 1 ⬄ 25 + 10B = 0 ⬄ B = -2,5

5 + 2B = ⬄ B = -2,5

(2) H1 = 0,5/s2 – 2,5s + 25/(10s + 2) = 0,5/s2 – 2,5s + 2,5/(s + 0,2) ⬄ H1(t) = 0,5t-2,5+2,5exp(-0,2t)

H1(1) = 0,5\*1-2,5+2,5exp(-0,2\*1)= 0,046 m h1(1) = 0,50+0,046 = 5,046 m

H1(5) = 0,5\*5-2,5+2,5exp(-0,2/85)= 0,920 m h1(5) = 0,50+0,920 = 5,920 m

q1(t) = h1(t)/R1 ⬄ Q1(t) = H1(t)/0,5 ⬄ Q1(s) = H1(s)/0,5 ⬄ H1 = 0,5Q1

(1) 0,5Q1/Qo = 1/(10s + 2) ⬄ Q1/Qo = 2/(10s + 2)

**Ισοζύγιο μάζας στην 2η δεξαμενή** q1(t) – h2(t)/R2 = A2\*dh2(t)/dt ⬄ Q1(t) – H2(t)/1 = 20\*dH2(t)/dt ⬄

@SS 10 – h2s/1 = 0 ⬄ h2s = 10 ⬄ Q1(s) – H2(s) = 20sH2(s) ⬄ 1 – Η2/Q1 = 20sH2/Q1 ⬄

⬄ H2/Q1\*(20s + 1) = 1 ⬄ H2/Q1 = 1/(20s + 1) (3)

H2/Qo = (H2/Q1)\*(Q1/Qo) = (1/(20s + 1))\*(2/(10s + 2)) = 2/(20s + 1)(10s + 2)

H2 = 2/s2(20s + 1)(10s + 2) (4)

HEAVYSIDE 2/s2(20s + 1)(10s + 2) = A/s2 + B/s + C/(20s + 1) + D/(10s + 2) A = 1 C = 533,33 D = -16,67

2/s2(20s + 1)(10s + 2) = 1/s2 + B/s + 533,33/(20s+1) – 16,67/(10s+2) =

= (20s+1)(10s+2) + Bs(20s+1)(10s+2) + 533,33s2(10s+2) – 16,67s2(20s+1) ⬄

⬄ 200s2 + 50s + 2 + 200Bs3 + 50Bs2 + 5333,3s3 + 1066,66s2 – 333,33s3 – 16,67s2 = 2 ⬄

⬄ 5333,3 -333,33 + 200B = 0 ⬄ B = -25

⬄ 200 + 50B + 1066,66 – 16,66 = 0 ⬄ B = -25

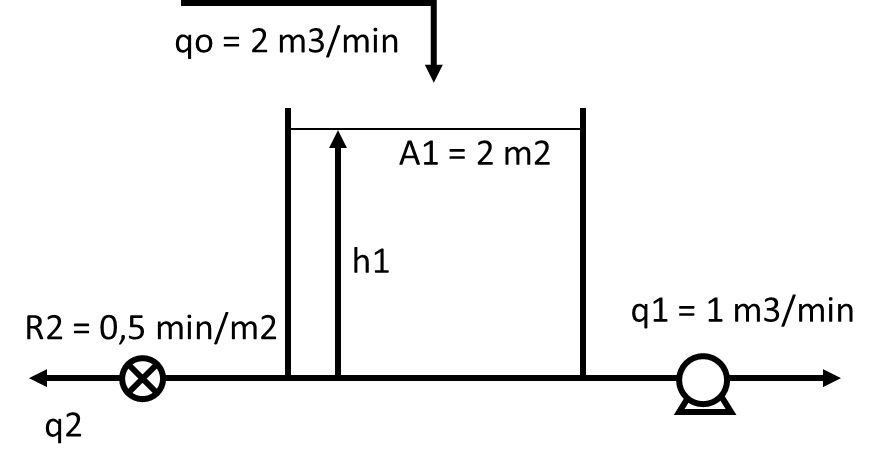
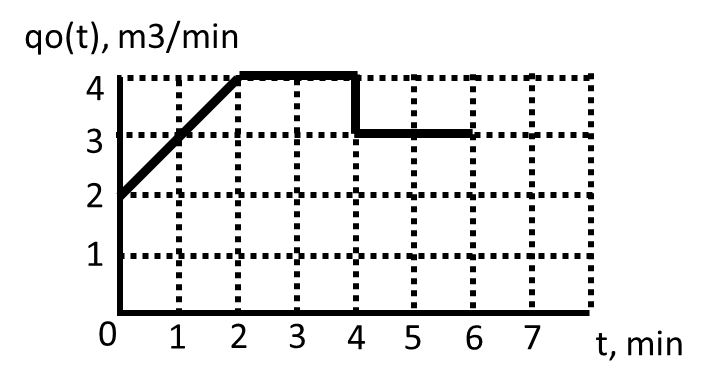
(4) H2 = 1/s2 – 25/s + 533,33/(20s+1) – 16,67/(10s+2) = 1/s2 – 25/s + 26,67/(s+0,05) – 1,667/(s+0,2) ⬄

⬄ H2(t) = t-25+26,67exp(-0,05t)-1,667exp(-0,2t)

H2(1) = 1-25+26,67exp(-0,05\*1)-1,667exp(-0,2\*1) = 0,0045 m h2(1) = 10+0,0045 = 10 m

H2(5) = 5-25+26,67exp(-0,05\*5)-1,667exp(-0,2/\*5)= 0,1574 m h2(5) = 10+0,16 = 10,16 m

ΘΕΜΑ3 (10μονάδες)



Στη δεξαμενή επιβάλλεται η μεταβολή του σχήματος. Να υπολογιστεί η στάθμη στο 2ο, 3ο, 4ο και 6ο λεπτό.

Ισοζύγιο μάζας: @SS 2 – 1 – hs/0,5 = 0 ⬄ 1 = hs/0,5 ⬄ hs = 0,5

q(t) – q1 – h(t)/R2 = Adh(t)/dt ⬄ Q(t) – H(t)/0,5 = 2dH(t)/dt ⬄ Q(s) – 2H(s) = 2sH(s) ⬄

⬄ 1 –2H/Q = 2sH/Q ⬄ 1 = H/Q(2s + 2) ⬄ H/Q = 1/(2s + 2) ⬄ H/Q = 0,5/(s+1) (1)

q(t) = qs + t\*u(t) – (t-2)\*u(t-2) – u(t-4) ⬄ q(t) - qs = t\*u(t) – (t-2)\*u(t-2) – u(t-4) ⬄ Q(t) = t\*u(t) – (t-2)\*u(t-2) – 1u(t-4) ⬄ Q(s) = 1/s2 – exp(-2s)/s2 – exp(-4s)

(1) H = 0,5/s2(s+1) – 0,5exp(-2s)/s2(s+1) – 0,5exp(-4s)/(s+1) (2)

HEAVYSIDE: 0,5/s2(s + 1) = A/s2 + B/s + C/(s + 1) A = 0,5 C = 0,5

0,5/s2(s + 1) = 0,5/s2 + B/s + 0,5/(s + 1) ⬄ 0,5(s+1) + Bs(s+1) + 0,5s2 = 0,5 ⬄

⬄ 0,5s + 0,5 + Bs2 + Bs + 0,5s2 = 0,5 ⬄ B = -0,5

(2) H = 0,5/s2 – 0,5/s + 0,5/(s+1) – 0,5exp(-2s)/s2 + 0,5exp(-2s)/s – 0,5exp(-2s)/(s+1) – 0,5exp(-4s)/(s+1) ⬄

H(t) = 0,5t-0,5+0,5exp(-t)-0,5(t-2)u(t-2)+0,5u(t-2)-0,5exp(-t+2)u(t-2)-0,5exp(-t+4)u(t-4)

H(2) = 0,5\*2-0,5+0,5\*exp(-2)-0,5\*(2-2)+0,5-0,5\*exp(-2+2) = 0,568 h(2) = 0,5 + 0,57 =1,57 m

H(3) = 0,5\*3-0,5+0,5\*exp(-3)-0,5\*(3-2)+0,5-0,5\*exp(-3+2) = 0,841 h(3) = 0,5 + 0,84 =1,34 m

H(4) = 0,5\*4-0,5+0,5\*exp(-4)-0,5\*(4-2)+0,5-0,5\*exp(-4+2)-0,5\*exp(-4+4) = 0,441 m h(4) = 0,5 + 0,44 =0,94 m

H(6) = 0,5\*6-0,5+0,5\*exp(-6)-0,5\*(6-2)+0,5-0,5\*exp(-6+2)-0,5\*exp(-6+4) = 0,924 m h(6) = 0,5 + 0,92 =1,42 m