1η Πρόοδος Δυναμική και Έλεγχος Διεργασιών 11 Σεπτεμβρίου 2018

ΘΕΜΑ1 (10μονάδες)



Σε χρόνο 0 προστίθεται στιγμιαία 1 m3. Να υπολογιστεί και να παρασταθεί γραφικά (σε χρόνο 0, 1, 2, 3, 4, 5 min) η απόκριση της στάθμης στην 1η και τη 2η δεξαμενή.

@SS 1 – (h1s – h2s)/1 = 0 ⬄ (h1s – h2s) = 1 ⬄ h1s = 3

 (h1s – h2s)/1 – h2s/2 = 0 ⬄ h2s = 2

Ισοζύγιο μάζας στην 1η δεξαμενή

qo(t) – (h1(t) – h2(t))/R1 = A1\*dh1(t)/dt ⬄ Qo(t) – H1(t)/1 + H2(t)/1 = 1\*dH1(t)/dt

⬄ Qo(t) – H1(t) + H2(t) = dH1(t)/dt ⬄ Qo(s) – H1(s) + H2(s) = sH1(s) (1)

Ισοζύγιο μάζας στην 2η δεξαμενή

(h1(t) – h2(t))/R1 – h2(t)/R2 = A2\*dh2(t)/dt ⬄ h1(t)/1 – h2(t)/1 – h2(t)/2 = 1\*dh2(t)/dt ⬄

⬄ h1(t) – 1,5\*h2(t) = dh2(t)/dt ⬄ H1(t) – 1,5\*H2(t) = dH2(t)/dt ⬄ H1(s) – 1,5\*H2(s) = sH2(s) ⬄

⬄ H1(s) = (s + 1,5)\*H2(s) ⬄ H2(s) = H1(s)/(s + 1,5)

(1) Qo(s) – H1(s) + H1(s)/(s + 1,5) = sH1(s) ⬄ Qo/H1 – 1 + 1/(s + 1,5) = s ⬄ Qo/H1 = s – 1/(s + 1,5) + 1 ⬄

 Qo/H1 = (s2 + 2,5s + 0,5)/(s + 1,5) ⬄ H1/Qo = (s + 1,5)/(s2 + 2,5s + 0,5) ⬄

 **H1/Qo = s/(s2 + 2,5s + 0,5) + 1,5/(s2 + 2,5s + 0,5)**

(1) Qo(s) – (s + 1,5)H2(s) + H2(s) = s(s + 1,5)H2(s) ⬄ Qo/H2 = (s + 1,5) – 1 + s(s + 1,5) ⬄

 Qo/H2 = s2 + 2,5s +0,5 = ⬄ **H2/Qo = 1/(s2 + 2,5s + 0,5)**

Λύση τριωνύμου: **Δ = 2,5^2-4\*1\*0,5 = 4,25** x1 = -0,22 x2 = -2,28

Κρουστική διαταραχή της παροχής: δ(t) = 1 ⬄ Qo(s) = 1

HEAVYSIDE

s/(s + 0,22)(s + 2,28) = A/(s + 0,22) + B/(s + 2,28)

A = -0,22/(-0,22+2,28) = -0,106

B = -2,28/(0,22-2,28) = 1,106

1,5/(s + 0,22)(s + 2,28) = C/(s + 0,22) + D/(s + 2,28)

C = 1,5/(-0,22+2,28) = 0,728

D = 1,5/(0,22-2,28) = -0,728

1/(s + 0,22)(s + 2,28) = E/(s + 0,22) + F/(s + 2,28)

E = 1/(-0,22+2,28) = 0,485

F = 1/(0,22-2,28) = -0,485

H1 = -0,106/(s+0,22) + 1,106/(s+2,28) + 0,728/(s + 0,22) – 0,728/(s + 2,28) = 0,621/(s+0,22) + 0,379/(s+2,28)

⬄ H1 =0,621\*exp(-0,22\*t)+0,379\*exp(-2,28\*t)

H2 = 0,485/(s+0,22) – 0,485/(s+2,28) ⬄ H2 = 0,485\*exp(-0,22\*t)-0,485\*exp(-2,28\*t)

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **t** | **H1** | **h1** | **H2** | **h2** | **t** | **H1** | **h1** | **H2** | **h2** |
| **0** | 1 | 4 | 0 | 2 | **3** | 0,321 | 3,321 | 0,250 | 2,250 |
| **1** | 0,537 | 3,537 | 0,340 | 2,340 | **4** | 0,258 | 3,258 | 0,201 | 2,201 |
| **2** | 0,404 | 3,404 | 0,307 | 2,307 | **5** | 0,207 | 3,207 | 0,161 | 2,161 |

ΘΕΜΑ 2 (10 μονάδες)



Στο σύστημα δεξαμενών η παροχή αυξάνεται με ρυθμό 1 m3/min. Να υπολογιστεί η στάθμη των δύο δεξαμενών σε χρόνο 5 και 10 λεπτά.

Γραμμική μεταβολή της qo: qo(t) = 10\*u(t) + t\*u(t) ⬄ qo(t) – qos = t\*u(t) ⬄

⬄ Qo(t) = t\*u(t) ⬄ **Qo(s) = 1/s2**

**Ισοζύγιο μάζας στην 1η δεξαμενή**

@SS 10 – h1s/2 = 0 ⬄ h1s = 20 m

qo(t) – h1(t)/R1 = A1\*dh1(t)/dt ⬄ Qo(t) – H1(t)/2 = 5\*dH1(t)/dt

⬄ 2\*Qo(t) – H1(t) = 10\*dH1(t)/dt ⬄ 2Qo(s) – H1(s) = 10sH1(s) ⬄

⬄ 2 – Η1/Qo = 10sH1/Qo ⬄ H1/Qo\*(10s + 1) = 2 ⬄ H1/Qo = 2/(10s + 1) ⬄ H1/Qo = 1/(5s + 0,5) (1) H1 = 1/s2(5s + 0,5) (2)

HEAVYSIDE 1/s2(5s + 0,5) = A/s2 + B/s + Γ/(5s + 0,5) Γ = 100 A = 2

 1/s2(5s + 0,5) = 2/s2 + B/s + 100/(5s + 0,5) = (2(5s + 0,5) + Bs(5s + 0,5) + 100s2)/s2(5s + 0,5) ⬄

 10s + 1 + 5Bs2 + 0,5Bs + 100s2 = 1 ⬄ 100 + 5B = 0 ⬄ B = -20

 10 + 0,5B = 0 ⬄ B = -20

(2) H1 = 2/s2 – 20/s + 100/(5s + 0,5) = 2/s2 – 20/s + 20/(s + 0,1) ⬄ H1(t) = 2t-20+20exp(-0,1t)

H1(5) = 2\*5-20+20\*exp(-0,1\*5) = 2,13 m h1(5) = 20+2,13 = 22,13 m

H1(10) = 2\*10-20+20\*exp(-0,1\*10) = 7,36 m h1(10) = 20+7,36 = 27,36 m

q1(t) = h1(t)/R1 ⬄ Q1(t) = H1(t)/2 ⬄ Q1(s) = H1(s)/2 ⬄ H1 = 2Q1

(1) 2Q1/Qo = 1/(5s + 0,5) ⬄ Q1/Qo = 0,5/(5s + 0,5) ⬄ Q1/Qo = 1/(10s + 1)

**Ισοζύγιο μάζας στην 2η δεξαμενή** q1(t) – h2(t)/R2 = A2\*dh2(t)/dt ⬄ Q1(t) – H2(t)/1 = 20\*dH2(t)/dt ⬄

@SS 10 – h2s/1 = 0 ⬄ h2s = 10 ⬄ Q1(s) – H2(s) = 20sH2(s) ⬄ 1 – Η2/Q1 = 20sH2/Q1 ⬄

 ⬄ H2/Q1\*(20s + 1) = 1 ⬄ H2/Q1 = 1/(20s + 1) (3)

H2/Qo = (H2/Q1)\*(Q1/Qo) = (1/(20s + 1))\*(1/(10s + 1)) = 1/(20s + 1)(10s + 1)

H2 = 1/s2(20s + 1)(10s + 1) (4)

HEAVYSIDE 2/s2(20s + 1)(5s + 0,5) = A/s2 + B/s + C/(20s + 1) + D/(5s + 0,5) A = 4 C = 3200 D = -200

 2/s2(20s + 1)(5s + 0,5) = 4/s2 + B/s + 3200/(20s+1) – 200/(5s+0,5) =

 = 4(20s+1)(5s+0,5) + Bs(20s+1)(5s+0,5) + 3200s2(5s+0,5) – 200s2(20s+1) ⬄

 ⬄ 400s2 + 60s + 2 + 100Bs3 + 15Bs2 + 0,5Bs + 16000s3 + 1600s2 – 4000s3 – 200s2 = 2 ⬄

 ⬄ 16000 - 4000 + 100B = 0 ⬄ B = -120

 ⬄ 400 + 15B + 1600 – 200 = 0 ⬄ B = -120

 ⬄ 60+0,5B = 0 ⬄ B = -120

(4) H2 = 4/s2 – 120/s + 3200/(20s+1) – 200/(5s+0,5) = 4/s2 – 120/s + 160/(s+0,05) – 40/(s+0,1) ⬄

 ⬄ H2(t) = 4t-120+160exp(-0,05t)-40exp(-0,1t)

H2(5) = 4\*5-120+160\*exp(-0,05\*5)-40\*exp(-0,1\*5) = 0,35 m h2(5) = 10+0,35 = 10,35 m

H2(10) = 4\*10-120+160\*exp(-0,05\*10)-40\*exp(-0,1\*10) = 2,33 m h2(10) = 10+2,33 = 12,33 m

ΘΕΜΑ 3 (10 μονάδες)



Στη δεξαμενή επιβάλλεται η μεταβολή του σχήματος. Να υπολογιστεί η στάθμη στο 2ο, 3ο, 4ο και 5ο λεπτό.

Ισοζύγιο μάζας: @SS 1 – hs – hs/2 = 0 ⬄ 1 = 3hs/2 ⬄ hs = 2/3

q(t) – h(t)/R1 – h(t)/R2 = Adh(t)/dt ⬄ Q(t) – H(t)/2 – H(t) = dH(t)/dt ⬄ Q(s) – 3/2H(s) = sH(s) ⬄

⬄ 1 – 1,5H/Q = sH/Q ⬄ 1 = H/Q(s + 1,5) ⬄ H/Q = 1/(s + 1,5) (1)

Q(t) = 1\*u(t) + t\*u(t-2) ⬄ Q(s) = 1/s + exp(-2s)/s2

(1) H = 1/s(s + 1,5) + exp(-2s)/s2(s + 1,5) (2)

HEAVYSIDE: 1/s(s + 1,5) = A/s + B/(s + 1,5) = (2/3)/s – (2/3)/(s + 1,5)

1/s2(s + 1,5) = C/s2 + D/s + E/(s + 1,5) C = 2/3 E = 1/(-3/2)^2 = 1/(9/4) = 4/9

1 = (2/3)\*(s + 3/2) + D\*s\*(s + 3/2) + (4/9)\*s2 = (2/3)s + 1 + D\*s2 + (3D/2)s + (4/9)\*s2

D = -4/9 2/3 = - 3D/2 ⬄ D = -4/9

(2) H = (2/3)/s – (2/3)/(s + 1,5) + exp(-2s)[(2/3)/s2 – (4/9)/s + (4/9)/(s + 1,5)] ⬄

 H(t) = 2/3 – (2/3)exp(-1,5t) + [(2/3)(t – 2) – (4/9) + (4/9)exp(-1,5(t-2))]u(t – 2)

H(2) = 2/3-(2/3)\*exp(-1,5\*1) = 0,52 h(2) = 0,67 + 0,52 =1,19 m

H(3) = 2/3-(2/3)\*exp(-1,5\*2) = 0,63 h(3) = 0,67 + 0,63 =1,30 m

H(4) = 2/3-(2/3)\*exp(-1,5\*3) + (2/3)\*(3-2)-(4/9)+(4/9)\*exp(-1,5\*(3-2)) = 0,98 m h(4) = 0,67 + 0,98 =1,65 m

H(5) = 2/3-(2/3)\*exp(-1,5\*4) + (2/3)\*(4-2)-(4/9)+(4/9)\*exp(-1,5\*(4-2)) = 1,58 m h(5) = 0,67 + 1,58 =2,24 m