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Drought Severity Assessment Based on Bivariate Probability Analysis

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Abstract Conventionally drought severity is assessed based on drought indices. Recently the Reconnaissance Drought Index (RDI) was proposed to assess drought severity based on the precipitation to potential evapotranspiration ratio (P/PET). In this paper RDI is studied as a bivariate index under a set of assumptions and simplifications. The paper presents a simple computational procedure for estimating the P/PET ratio for selected reference periods varying from 3 to 12 months, for any return period of drought. Alternatively, based on this procedure, the severity of any drought episode is rationally assessed. A bivariate probability analysis is employed based on the assumption that P and PET values are normally distributed and often negatively correlated. Examples for the application of the proposed procedure are presented using data from several meteorological stations in Greece. It is shown that the assumption of normality of both P and PET holds for long periods at all examined stations.

Keywords Drought indices · Drought severity assessment · Reconnaissance drought index · Preparedness planning · Bivariate analysis · Drought return period

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1 Introduction

Droughts have been dramatically increased in number and intensity in many parts of the world. It is estimated that in EU countries the number of affected people was increased by 20% over the last three decades. Recent droughts in Europe include the persistent drought of 1989–1993 and the drought of 2003. This last drought only, affected about 100 million people creating a cost of damage about 8.7 billion Euros. The total cost of droughts in Europe for the last three decades amounts to 100 billion Euros (Commission of the European Communities 2007).

As known, drought is a natural hazard but with temporarily imbalanced water availability caused mainly by low precipitation and high evapotranspiration, thus resulting in reduced water availability.

To face drought, a preparedness planning process is required to reduce the vulnerability of the affected system so as to become capable to withstand drought (Tsakiris and Spiliotis 2010). It is easily understood that a totally unprotected system is more vulnerable to drought than a well protected system. Modern preparedness plans involve all stakeholders from decision makers up to the interested NGOs and the public. Therefore it is essential for the preparedness plans to be practical, simple and transparent.

In this context modelling of drought should be as simple as possible to assist all different groups to jointly assess the severity level of each drought episode as well as to prioritise actions for reducing the vulnerability of the affected systems and mitigate the drought impacts.

This paper presents a simple computational procedure for characterising the severity of drought conditions based on a bivariate drought index with the assistance of bivariate probability analysis.

2 Drought Severity Modelling

Drought is conceived as a multidimensional phenomenon which is difficult to model. Drought events are characterised by their severity, duration and areal extent. Several propositions have been made recently for limiting the dimensions of drought from the above three dimensions to one. The simplifications proposed are to replace duration and areal extent variables by two predetermined constant quantities so that the uni-dimensional frequency analysis of severity could lead to meaningful and practical results (Tsakiris 2008a).

The above simplifications were adopted also in this study. The river basin or sub-basin is used as the territorial unit for the severity analysis, replacing the variable "areal extent". This territorial unit is also used in the water resources management plans according to the Water Framework Directive 2000/60/EC. Further, the predetermined "reference period" is introduced for replacing the duration of drought on the temporal scale. For standardisation purposes reference periods of 3, 6, 9 and 12 months have been proposed. Considering that the hydrological year for the Mediterranean region starts on the 1st of October, the above reference periods have the same onset which is the beginning of the hydrological year.

Regarding the severity of drought, this is represented by drought indices. Drought indices have been developed during the last century as tools to detect, monitor and evaluate drought events. Drought indices are usually categorised according to the domain that they are referring to. Several reviews and classifications of indices have been proposed by various researchers, providing a good overview mainly in the domains of meteorology, hydrology and agriculture. Furthermore, a number of indices have been also developed recently in the domains of remote sensing and water resources management.

The development of a drought index is always a challenging task from a scientific point of view. The exploitation of new technologies as well as the application of advanced methodologies are important drivers for developing drought indices. Emphasis is given on the reliability and robustness of an index, but also on the aspect of data availability. The availability of suitable data to construct a drought index was the main reason that the first generation of indices relied on meteorological data and are known as meteorological drought indices. Examples of indices in this category are the Rainfall Anomaly Index (RAI; Van Rooy 1965), the Bhalme and Mooley Drought Index (BMDI; Bhalme and Mooley 1980), the Standardised Anomaly Index (SAI; Katz and Glantz 1986), the Drought Severity Index (DSI; Bryant et al. 1992) and the most popular drought index, the Standardised Precipitation Index (SPI; McKee et al. 1993; Tsakiris and Vangelis 2004; Edossa et al. 2010). Additionally several indices have been proposed in this category such as the Effective Drought Index (EDI; Byun and Wilhite 1999) and the Reconnaissance Drought Index (RDI; Tsakiris and Vangelis 2005 and Tsakiris et al. 2007b).

Hydrological drought indices belong to another category of indices. In order to present a comprehensive picture of the water balance in a catchment area using the entire water cycle perspective, hydrological indices based on discharge were developed. Besides the popular Palmer Hydrological Drought Index (PHDI), another well known hydrological index is the Surface Water Supply Index (SWSI; Shafer and Dezman 1982). For the detection of regional drought events, the Regional Streamflow Deficiency Index (RSDI) was developed by Stahl (2001).

One of the most popular drought indices in the category of Agricultural Drought Indices is the Palmer Drought Severity Index (PDSI; Palmer 1965). The index provides a comprehensive picture of the water cycle and its elements, based on soil moisture and actual evapotranspiration. Typical indices of this category are also the Crop Moisture Index (CMI; Palmer 1968), the Soil Moisture Drought Index (SMDI; Hollinger et al. 1993) and the Crop Specific Drought Index (CSDI; Meyer et al. 1993). Two recently developed indices in this category are the Soil Moisture Deficit Index (SMDI) and the Evaporation Deficit Index (ETDI), both proposed by Narasimhan and Srinivasan (2005).

The new technologies and mainly the development of Earth observation satellites opened new roads for drought detection. The remote sensing based drought indices category was born, with the development of numerous indices describing the state of the land surface, mainly of vegetation, with the potential to detect anomalies such as droughts. The Normalised Difference Vegetation Index (NVDI; Tucker 1979) is probably the most prominent vegetation index and the most famous index of this category. What may be considered the latest trend in the development of drought indices is the incorporation of the maximum of the available information. This is achieved through the combination of meteorological data with remote sensing derived information. In any case, an index should retain a comprehensive character and a straightforward mathematical formulation that allows operational applications.

3 The Reconnaissance Drought Index

The Reconnaissance Drought Index (RDI) which is the basis for the analysis of this paper, can be characterised as a general meteorological index for drought assessment (Tsakiris et al. 2007a; Alexakis and Tsakiris 2010; Nalbantis and Tsakiris 2009; Kanellou et al. 2008). It was initially proposed in the framework of MEDROPLAN research project and was improved during the implementation of PRODIM research project (Iglesias et al. 2009; Tsakiris 2008b). As a new index, it has been tested in a large number of watersheds in the Mediterranean region, North Africa, Middle East and elsewhere. Although in some cases RDI behaves in a similar manner as SPI (in case the same period is examined), in other cases it deviates substantially from SPI, providing a more sound representation of drought conditions. The differences between SPI and RDI may be clearly illustrated when drought severity maps are produced. RDI may also be considered as a more suitable index than SPI for studying drought severity under climate change, since it incorporates both precipitation and potential evapotranspiration, which are directly affected by climate change. Also a strong advantage of RDI is that it offers a rational comparison of drought conditions between areas with different climatic characteristics. This enables the universal applicability of RDI in contrast with indices such as SPI, for which the same deviation from the normal precipitation does not necessarily mean the same severity in different climatic regions.

Two expressions of RDI are in use; the initial, α_k and the standardised form, $RDI_{st(k)}$. The initial expression (α_k) is presented in an aggregated form using a monthly time step and may be calculated on a monthly, seasonal or annual basis. The α_k for the year i and a reference period of k (months) is calculated as:

$$a_k^{(i)} = \frac{\sum_{j=1}^{k} P_{ij}}{\sum_{j=1}^{k} PET_{ij}}, \ i = 1\ (1)\ \mathrm{N} \qquad (1)$$

where P_{ij} and PET_{ij} are the precipitation and potential evapotranspiration of month j of year i, starting from October, which is the onset of the hydrological year for most Mediterranean countries and N is the total number of years of the available data. The mean annual $\bar{a}_{12}$ is equal to the well-known Aridity Index of the area (UNEP 1992), which is used for climate classification.

The calculation of the standardised form of RDI_{st} can be performed by fitting a gamma or lognormal probability density function (pdf) to the given frequency distribution of α_k (Tsakiris 2008b; Tigkas 2008). Positive values of RDI_{st} indicate wet periods, while negative values indicate dry periods compared with the normal conditions of the area. The severity of drought events increases when RDI_{st} values decrease. Drought severity can be categorised as mild, moderate, severe and extreme, with corresponding thresholds of RDI_{st} (−0.5), (−1.0), (−1.5) and (−2.0), respectively. It should be noticed that these thresholds coincide with the thresholds of SPI.

This paper is focusing on the initial expression of RDI for 4 different reference periods of 3, 6, 9 and 12 months. Therefore the P/PET ratio, for the proposed time scales, is proposed to be estimated based on the assumption that both quantities P and PET follow separately and jointly a normal distribution. This assumption was tested at several meteorological stations in Greece and proved valid as presented in the "Section 5" of this paper.

It is obvious that these assumptions do not hold always in the general applications of RDI. However, in case they are valid, the proposed procedure in this paper is a rational approach towards the use of a bivariate index such as RDI.

4 Bivariate Probability Analysis

Let two random variables X and Y, which are normally distributed with mean values μ_X and μ_Y, respectively. Let also the variances of X and Y be σ_X^2 and σ_Y^2 and that X and Y are correlated with correlation coefficient ρ. The bivariate normal distribution has the following probability density function:

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} \exp\left[\frac{-1}{2(1-\rho^2)}\left\{\left(\frac{x-\mu_X}{\sigma_X}\right)^2 - 2\rho\left(\frac{x-\mu_X}{\sigma_X}\right)\left(\frac{y-\mu_Y}{\sigma_Y}\right) + \left(\frac{y-\mu_Y}{\sigma_Y}\right)^2\right\}\right]$$
$$\text{where: } -\infty < x < \infty \text{ and } -\infty < y < \infty, \tag{2}$$

For reference periods such as 3, 6, 9 and 12 months it is assumed that both precipitation and potential evapotranspiration can be adequately described by normal distributions.

Considering that $a_k = \frac{\sum_{j=1}^{k} P_j}{\sum_{j=1}^{k} PET_j}$, then the joint probability density function of the random variable $A_k = \frac{X}{Y}$ is obtained by the following integration:

$$f_{A_k}(a_k) = \int_{-\infty}^{+\infty} f_{X,Y}(a_k y, y)\,|y|dy \tag{3}$$

After integration the joint probability becomes (Fieller 1932):

$$f_{A_k}(a_k) = \frac{b(a_k)\,d(a_k)}{\sqrt{2\pi}\sigma_X\sigma_Y\omega^3(a_k)}\left[\Phi\left\{\frac{b(a_k)}{\sqrt{(1-\rho^2)}\omega(a_k)}\right\} - \Phi\left\{-\frac{b(a_k)}{\sqrt{(1-\rho^2)}\omega(a_k)}\right\}\right] + \\ + \frac{\sqrt{(1-\rho^2)}}{\pi\sigma_X\sigma_Y\omega^2(a_k)}\exp\left\{-\frac{c}{2(1-\rho^2)}\right\} \tag{4}$$

where

$$\left.\begin{array}{l} \omega\left(a_k\right)=\left(\dfrac{a_k^2}{\sigma_X^2}-\dfrac{2\rho a_k}{\sigma_X\sigma_Y}+\dfrac{1}{\sigma_Y^2}\right)^{1/2} \\ b\left(a_k\right)=\dfrac{\mu_X a_k}{\sigma_X^2}-\dfrac{\rho\left(\mu_X+\mu_Y a_k\right)}{\sigma_X\sigma_Y}+\dfrac{\mu_Y}{\sigma_Y^2} \\ c=\dfrac{\mu_X^2}{\sigma_X^2}-\dfrac{2\rho\mu_X\mu_Y}{\sigma_X\sigma_Y}+\dfrac{\mu_Y^2}{\sigma_Y^2} \\ d\left(a_k\right)=\exp\left\{\dfrac{b^2\left(a_k\right)-c\omega^2\left(a_k\right)}{2\left(1-\rho^2\right)\omega^2\left(a_k\right)}\right\} \end{array}\right\} \tag{4.b}$$

and Φ is the typical standard normal cumulative probability function.

Then, the cumulative probability $F(\alpha_k)$ can be determined as follows:

$$F\left(a_k\right)=\int_{-\infty}^{a_k} f_{A_k}\left(a_k\right)da_k \tag{5}$$

The $F(\alpha_k)$ can be calculated by using a numerical approach such as the trapezoid rule or any other rule with a very small integration step (e.g. Burden and Faires 1997).

In case $0\leq\sigma_Y\ll\mu_Y$ Hinkley (1969) proposed the following approximate cumulative probability function ($F^*\left(a_k\right)$):

$$F^*(a_k)=\Phi\left(\frac{\mu_Y a_k-\mu_X}{\sigma_X\sigma_Y\omega\left(a_k\right)}\right) \tag{6}$$

or by substituting $\omega(\alpha_k)$ from Eq. 4.b:

$$F^*(a_k)=\Phi\left(\frac{\mu_Y a_k-\mu_X}{\sqrt{a_k^2\sigma_Y^2-2\rho a_k\sigma_X\sigma_Y+\sigma_X^2}}\right) \tag{7}$$

For improving the above approximation, the following correction was proposed in case that $F^*(a_k)>F\left(a_k\right)$ (Hinkley 1969):

$$F^{**}(a_k)=\Phi\left(\frac{\mu_Y a_k-\mu_X}{\sigma_X\sigma_Y\omega\left(a_k\right)}\right)-\Phi\left(-\frac{\mu_Y}{\sigma_Y}\right) \tag{8}$$

On the opposite, the correction term in Eq. 8 is added if $F^*(a_k)>F\left(a_k\right)$.

However, since $0\leq\sigma_Y\ll\mu_Y$ the correction term $\Phi(-\frac{\mu_Y}{\sigma_Y})$ is negligible. Therefore, the finally proposed cumulative distribution function concerning the ratio P/PET, provided that both P and PET (corresponding to X and Y in the previous analysis) are random variables following normal distributions, is Eq. 6.

From the above analysis it becomes evident that if the time series for a long historical record of P and PET are known, the cumulative probability and therefore the return period of drought can be estimated by:

$$T=\frac{1}{P\left(\mathrm{A}_k\leq a_k\right)}=\frac{1}{F\left(a_k\right)} \tag{9}$$

The solution of the inverse problem is also useful for the formulation of the preparedness plan to face droughts in a region. That is for each preselected return period reflecting the society's perception for the acceptable protection level against drought, the ratio P/PET level may be determined.

Following the above approximate approach and replacing X for P and Y for PET, Eq. 9 is solved:

$$\left(\sqrt{a_k^2\sigma_{PET}^2 - 2\rho a_k\sigma_P\sigma_{PET} + \sigma_P^2}\right)\Phi^{-1} = \mu_{PET}a_k - \mu_P \Rightarrow a_k^2\left(\sigma_{PET}^2\left(\Phi^{-1}\right)^2 - \mu_{PET}^2\right) + 2a_k\left(-\rho\sigma_P\sigma_{PET}\left(\Phi^{-1}\right)^2 + \mu_{PET}\mu_P\right) + \left(\sigma_P^2\left(\Phi^{-1}\right)^2 - \mu_P^2\right) = 0 \tag{10}$$

From the two solutions the lowest value is selected since the other solution has no practical meaning.

$$a_k = \frac{\left(+\rho\sigma_P\sigma_{PET}\left(\Phi^{-1}\right)^2 - \mu_{PET}\mu_P\right) \pm \sqrt{\begin{array}{l}\rho^2\sigma_P^2\sigma_{PET}^2\left(\Phi^{-1}\right)^4 - 2\rho\sigma_P\sigma_{PET}\left(\Phi^{-1}\right)^2\mu_{PET}\mu_P - \\ \sigma_{PET}^2\sigma_P^2\left(\Phi^{-1}\right)^4 + \sigma_{PET}^2\left(\Phi^{-1}\right)^2\mu_P^2 + \mu_{PET}^2\sigma_P^2\left(\Phi^{-1}\right)^2\end{array}}}{\left(\sigma_{PET}^2\left(\Phi^{-1}\right)^2 - \mu_{PET}^2\right)} \tag{11}$$

Finally the correlation hypothesis between P and PET is tested following the conventional procedure by using the t-distribution test with n-2 degrees of freedom (e.g. Haan 1977; Graybill 1961). In case $\rho = 0$, Eq. 11 is simplified accordingly.

5 Application and Discussion

For the application of the proposed methodology the data of monthly precipitation and average monthly temperature from four meteorological stations in Greece were used. Namely, data from the stations Helliniko (Athens), Larissa, Heraklion and Naxos were used. Monthly values of PET were then calculated using the Hargreaves method, a method based on average monthly temperatures (Hargreaves and Samani 1982).

Further the monthly P and PET values were aggregated into the reference periods of 3, 6, 9 and 12 months. The main statistical characteristics of time series of the above quantities of 47 years are presented in Table 1. The mean annual P/PET corresponds to the Aridity Index of the area. From Table 1 the Aridity Index is calculated 0.34 for Helliniko (Athens), 0.33 for Larissa, 0.47 for Heraklion and 0.42 for Naxos. Further both P and PET time series are successfully tested against the assumption that they are normally distributed (for various confidence levels selected), as presented in Table 2.

The test on the correlation between P and PET was performed for all the above reference periods and stations. The results presented in Table 3 show that the two variables are correlated apart from some cases in which correlation was not

Table 1 The main statistics of P and PET values (mm) for various reference periods for Helliniko, Larissa, Heraklion and Naxos

Station		3-month period		6-month period		9-month period		12-month period	
		P	PET	P	PET	P	PET	P	PET
Helliniko	Average	167.2	146.1	296.0	290.1	341.2	657.1	363.6	1,069.1
	Median	164.9	146.0	299.4	291.5	357.2	655.1	371.3	1,066.6
	St. dev.	64.1	8.2	91.1	14.2	90.6	22.4	95.1	34.9
	Max	321.6	168.0	472.2	322.1	496.7	700.7	528.7	1,150.4
	Min	49.2	132.6	120.3	252.0	146.4	614.5	155.1	998.7
Larissa	Average	158.2	149.2	258.7	297.4	354.0	760.3	417.7	1,262.6
	Median	149.6	148.0	243.1	298.7	346.1	768.6	420.2	1,259.4
	St. dev.	65.9	9.8	91.0	20.6	105.4	32.6	110.0	40.1
	Max	380.7	172.1	614.7	337.7	751.4	828.7	763.5	1,350.4
	Min	40.3	128.5	100.6	249.8	185.4	691.8	216.2	1,190.4
Heraklion	Average	203.7	157.2	419.4	313.6	464.2	672.7	483.7	1,024.3
	Median	193.8	154.5	416.2	311.3	466.7	663.7	490.2	1,013.7
	St. dev.	83.4	12.9	105.6	19.1	122.6	33.8	123.2	53.0
	Max	402.9	195.9	739.3	357.3	890.2	744.5	890.2	1,149.0
	Min	78.1	136.0	242.0	278.6	271.0	601.0	294.4	925.5
Naxos	Average	155.5	127.3	322.1	258.6	350.7	567.7	360.1	854.8
	Median	146.1	125.8	300.3	255.3	351.3	569.9	360.4	849.4
	St. dev.	60.1	11.8	102.5	15.8	110.0	22.7	111.4	29.8
	Max	327.7	167.4	614.5	305.1	683.1	627.4	683.1	950.3
	Min	53.7	108.5	123.8	225.2	131.8	503.0	140.0	786.9

significantly different than zero at significance level $\alpha = 0.10$. From the stations examined the one of Larissa exhibits a strong negative correlation.

Yue (2000) solving a similar hydrological problem, proposed the test of the linear relation and the correlation between the observed and theoretical joint probabilities. The relationship between observed and theoretical joint probabilities of the annual precipitation and potential evapotranspiration (12 month reference period) for all the stations is presented in Fig. 1. The results show satisfactory agreement between observed and theoretical joint probabilities.

For illustration purposes the cumulative probability of annual P/PET in the Larissa station was calculated from the bivariate normal distribution according to Eq. 7 (Fig. 2).

Further, following the proposed procedure, based on the 3, 6, 9 and 12 month reference periods the a_k values were linked with the return periods of drought, for return periods of 5, 10, 20 and 50 years for all the selected stations. The results appear in Fig. 3. The aridity index for each station appears in the column of the 12 month period.

An important application of the proposed methodology lies in the fact that the calculation of the probability for each annual ratio a^i_{12} may enable us to assess the severity of drought of that particular year. For example, as it is shown in Fig. 4, the most significant drought episodes of the recent years occurred at the Helliniko station in 1999–2000 (T = 88 years), 1989–1990 (T = 49 years), 1956–1957 (T = 25 years), 1992–1993 (T = 17 years) and 2000–2001 (T = 11 years). Based on Fig. 4 it can be

Table 2 Test of normality of P and PET for various reference periods

Station	Goodness-of-fit test	3-month period	6-month period	9-month period	12-month period
Helliniko		*Precipitation* (P)			
	Chi-square value	21.659	5.574	4.979	1.106
	D_{max}	0.145	0.073	0.086	0.077
	Selected distribution	Normal	Normal	Normal	Normal
	Confidence level	$\alpha = 0.10$	$\alpha = 0.10$	$\alpha = 0.10$	$\alpha = 0.10$
		Potential evapotranspiration (PET)			
	Chi-square value	2.894	1.702	2.298	6.468
	D_{max}	0.099	0.072	0.080	0.122
	Selected distribution	Normal	Normal	Normal	Normal
	Confidence level	$\alpha = 0.10$	$\alpha = 0.10$	$\alpha = 0.10$	$\alpha = 0.10$
Larissa		*Precipitation* (P)			
	Chi-square value	5.872	2.595	5.277	4.978
	D_{max}	0.092	0.099	0.138	0.075
	Selected distribution	Normal	Normal	Normal	Normal
	Confidence level	$\alpha = 0.10$	$\alpha = 0.10$	$\alpha = 0.10$	$\alpha = 0.10$
		Potential evapotranspiration (PET)			
	Chi-square value	7.064	4.383	6.171	2.894
	D_{max}	0.109	0.069	0.129	0.089
	Selected distribution	Normal	Normal	Normal	Normal
	Confidence level	$\alpha = 0.10$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.10$
Heraklion		*Precipitation* (P)			
	Chi-square value	5.872	7.063	4.383	8.553
	D_{max}	0.078	0.114	0.096	0.106
	Selected distribution	Normal	Normal	Normal	Normal
	Confidence level	$\alpha = 0.10$	$\alpha = 0.10$	$\alpha = 0.10$	$\alpha = 0.05$
		Potential evapotranspiration (PET)			
	Chi-square value	9.149	8.851	11.234	4.085
	D_{max}	0.109	0.128	0.132	0.127
	Selected distribution	Normal	Normal	Normal	Normal
	Confidence level	$\alpha = 0.05$	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.10$
Naxos		*Precipitation* (P)			
	Chi-square value	5.872	4.978	7.361	4.084
	D_{max}	0.092	0.099	0.106	0.155
	Selected distribution	Normal	Normal	Normal	Normal
	Confidence level	$\alpha = 0.10$	$\alpha = 0.10$	$\alpha = 0.10$	$\alpha = 0.10$
		Potential evapotranspiration (PET)			
	Chi-square value	4.085	7.362	8.851	7.064
	D_{max}	0.131	0.095	0.086	0.124
	Selected distribution	Normal	Normal	Normal	Normal
	Confidence level	$\alpha = 0.10$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.10$

argued that the most extreme drought episodes during the period examined occurred at the Helliniko (Athens) rather that the other 3 stations.

It is worth noticing that by using a numerical approach of integration (e.g. Simpson integration; Gerald and Wheatley 1994) or using the "complete" equation of the probability density function (Eq. 4), the results are practically the same.

Table 3 Testing the hypothesis of zero correlation between P and PET

Station	3-month period	6-month period	9-month period	12-month period
	t			
Helliniko	−1.056	−1.506	−1.356	−1.212
Larissa	−3.473	−5.588	−4.822	−3.639
Heraklion	−0.859	1.780	1.001	2.085
Naxos	0.809	0.658	1.003	0.640
α	0.10	0.10	0.10	0.10
$t_{1-\alpha/2,N-2}$	1.68	1.68	1.68	1.68
Helleniko	Not significant $\rho \neq 0$	Not significant $\rho \neq 0$	Not significant $\rho \neq 0$	Not significant $\rho \neq 0$
Larissa	Dependent	Dependent	Dependent	Dependent
Heraklion	Not significant $\rho \neq 0$	Dependent	Not significant $\rho \neq 0$	Dependent
Naxos	Not significant $\rho \neq 0$	Not significant $\rho \neq 0$	Not significant $\rho \neq 0$	Not significant $\rho \neq 0$

Another interesting point is the fact that following Hinkley's approach, the mean value a_k according to Eq. 11 is equal to the ratio of the mean values of P and PET because $\sigma_{PET} \ll \mu_{PET}$. It is known that this does not always hold (e.g. Ang and Tung 1975).

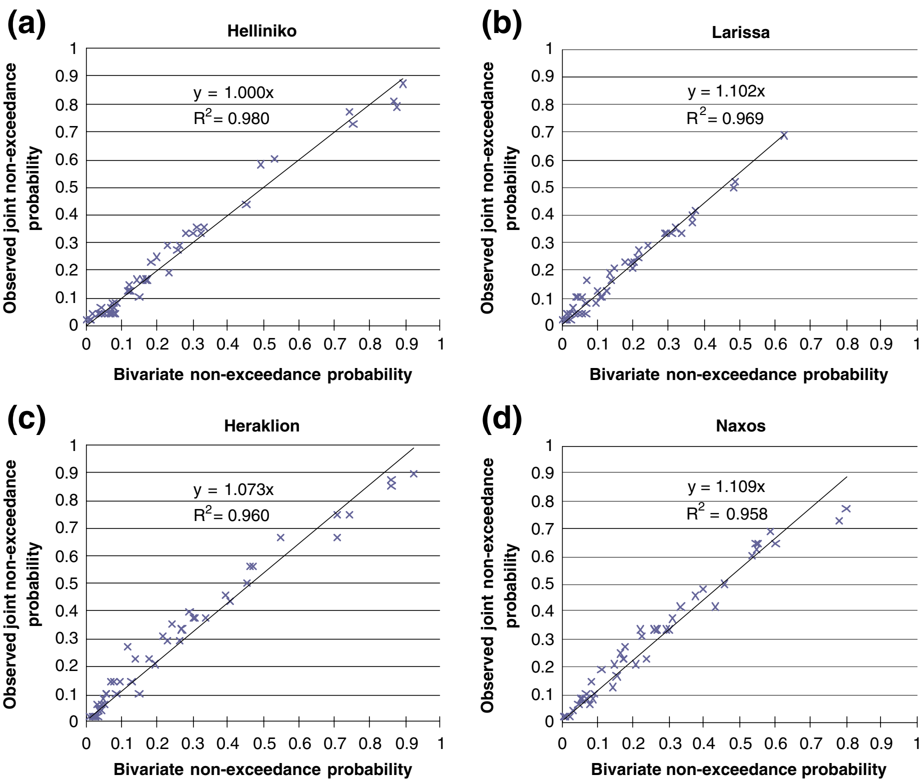

Fig. 1 Relation between joint observed and theoretical non-exceedence probability of the annual precipitation and annual potential evapotranspiration at **a** Helliniko, **b** Larissa, **c** Heraklion and **d** Naxos

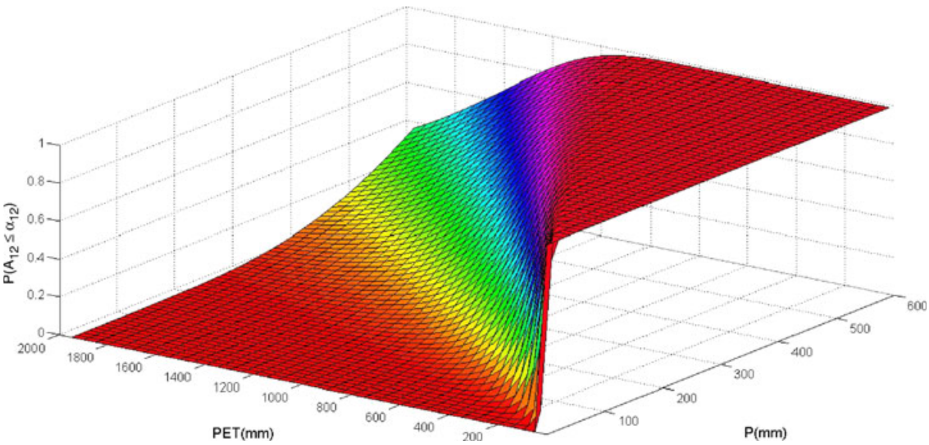

Fig. 2 The two dimensional graph of cumulative probability of the ratio a_{12} projected against the variables P and PET for Larissa

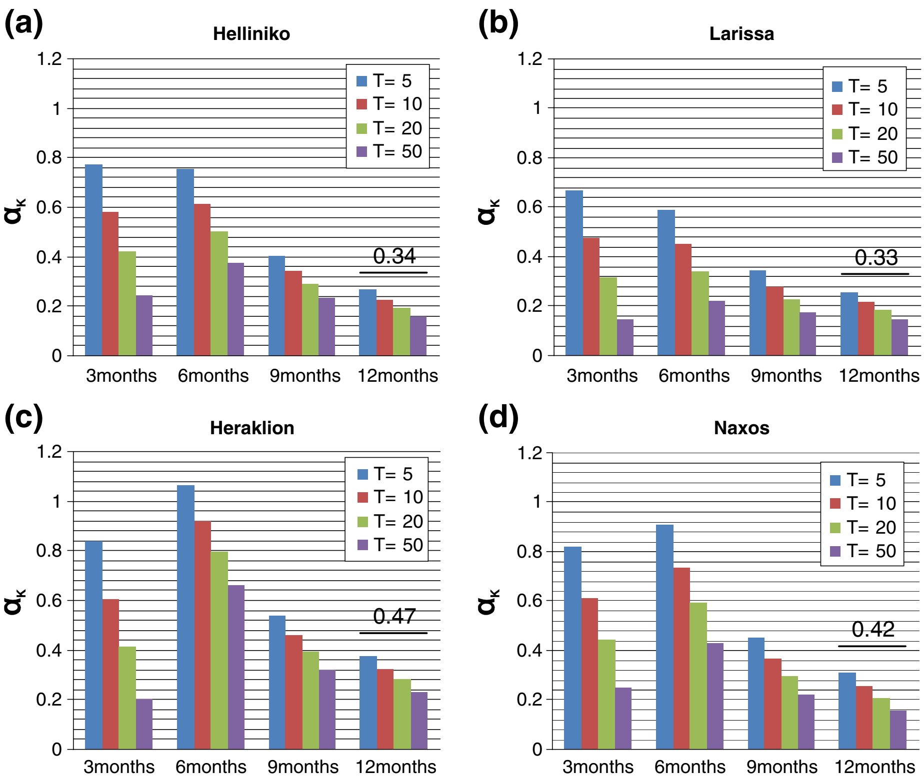

Fig. 3 The α_k ratio for return periods of 5, 10, 20 and 50 years at **a** Helliniko, **b** Larissa, **c** Heraklion and **d** Naxos

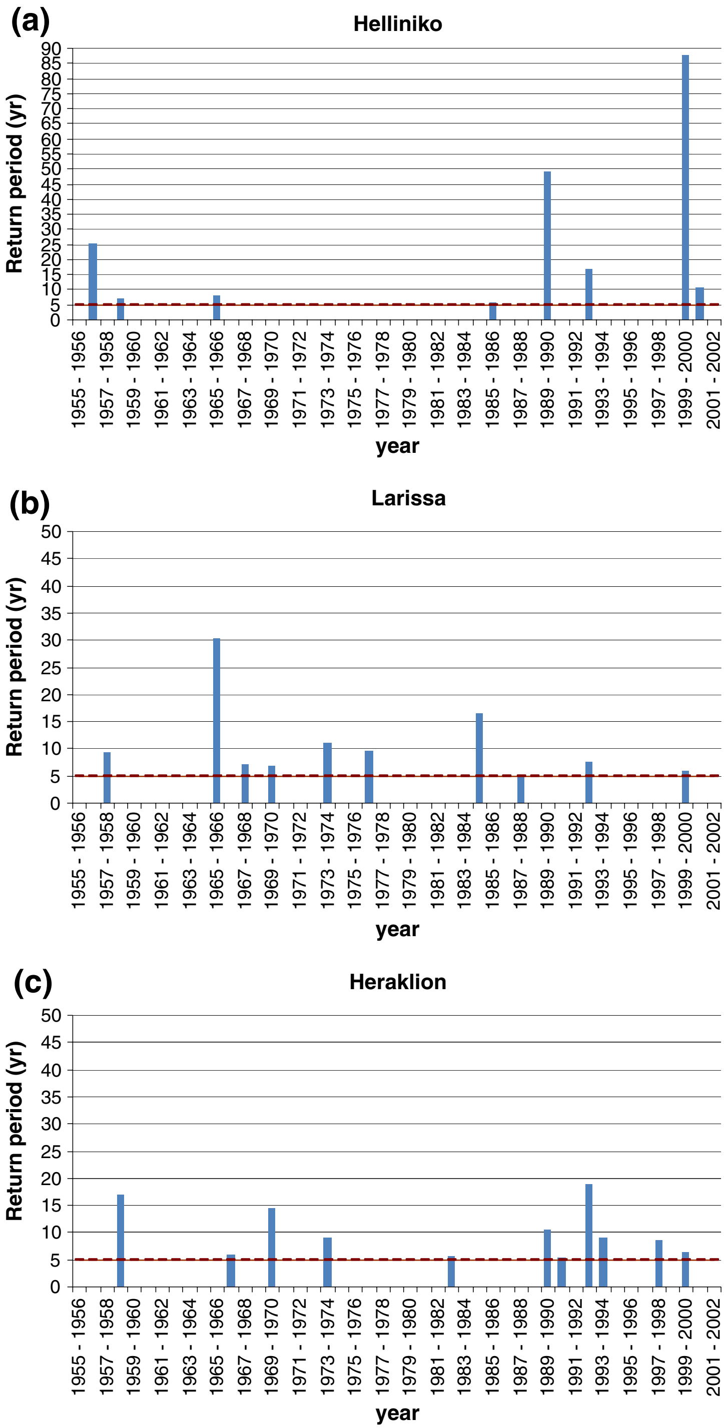

Fig. 4 The return periods of the major drought years (T > 5 years) at **a** Helliniko, **b** Larissa, **c** Heraklion and **d** Naxos

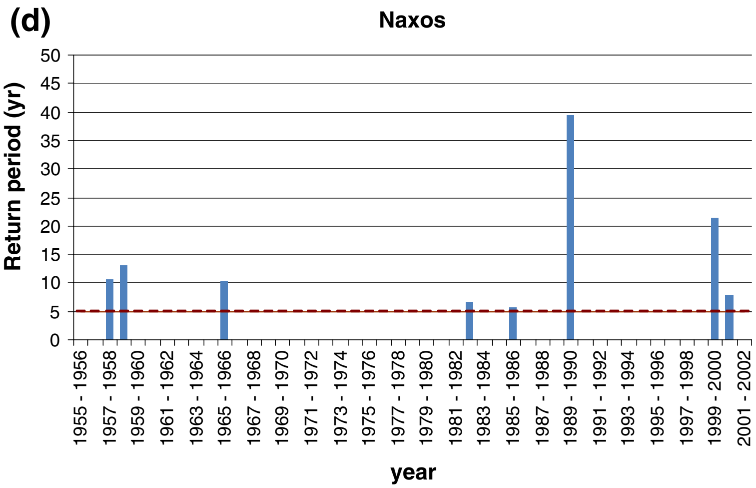

Fig. 4 (continued)

6 Conclusions

The Reconnaissance Drought Index (RDI) incorporating precipitation and potential evapotranspiration is one of the most recent developments for the assessment of drought severity through drought indices.

In this paper RDI which is based on the P/PET ratio is considered as a bivariate index which is studied through a strict probabilistic analysis in case both P and PET follow a normal distribution. As known in several meteorological stations in Greece this assumption holds for annual values.

The paper presents a simple computation procedure based on the bivariate probability analysis for the rational estimation of the annual P/PET ratio for any return period of drought in the regions under study. Alternatively, based on this procedure, the severity of any drought episode is rationally assessed.

The result is the cornerstone for determining the level of desire protection against drought, which the preparedness plan can adopt. It is also essential for the prioritisation of actions, measures and engineering works, which should be implemented in a proactive framework for facing droughts and water shortages.

The proposed procedure can be also applied for reference periods smaller than a year (e.g. 3, 6, 9 months), for which the assumptions of normality of the distribution of precipitation and potential evapotranspiration separately and jointly hold. In case these assumptions are not valid, the users should continue to use the standardised RDI in its initial proposal, which assumes that P/PET ratio generally follows a skewed distribution.

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