$$y(x_1, x_2) \to \boxed{\max} \quad (1)$$

$$\text{s.t.} \quad f(x_1, x_2) = 0 \quad (2)$$

Λύση:

$$(2) \Leftrightarrow f = 0 \Leftrightarrow df = 0 \Leftrightarrow 0 = \frac{\partial f}{\partial x_1} dx_1 + \frac{\partial f}{\partial x_2} dx_2 \Rightarrow (\text{Taylor})$$

$$\Leftrightarrow \boxed{dx_2 = \frac{-\dfrac{\partial f}{\partial x_1}}{\dfrac{\partial f}{\partial x_2}} dx_1} \quad (3)$$

$$(1) \ \overset{\theta.}{\Leftrightarrow} \ dy = \frac{\partial y}{\partial x_1} dx_1 + \frac{\partial y}{\partial x_2} dx_2 \Leftrightarrow$$

Taylor

$$\Leftrightarrow dy = \frac{\partial y}{\partial x_1} dx_1 + \left(\frac{-\dfrac{\partial f}{\partial x_1}}{\dfrac{\partial f}{\partial x_2}} \right) dx_1 \frac{\partial y}{\partial x_2} +$$

$$\Leftrightarrow dy = \cancel{\frac{\partial y}{\partial x_1} dx_1} \left[\frac{\partial y}{\partial x_1} + \frac{-\dfrac{\partial y}{\partial x_2}}{\dfrac{\partial f}{\partial x_2}} \frac{\partial f}{\partial x_1} \right] dx_1$$

Όπου, $\lambda = \dfrac{-\frac{\partial y}{\partial x_2}}{\frac{\partial f}{\partial x_2}}$

$$\downarrow$$

$$dy = \left[\frac{\partial y}{\partial x_1} + \lambda \frac{\partial f}{\partial x_2}\right] dx_1$$

y^* ακρότατο $\Rightarrow$ ψάχνω λύση ώστε

$$dy = 0 \Leftrightarrow \frac{\partial y}{\partial x_1} + \lambda \frac{\partial f}{\partial x_1} = 0 \Rightarrow$$

$$\Rightarrow \boxed{\frac{\partial (y + \lambda f)}{\partial x_1} = 0}$$

Αν ορίσω $L = y + \lambda f$,

x ακρότατο L, (1) $\frac{\partial L}{\partial x_1} = 0$

(2) $\frac{\partial L}{\partial x_2} = 0$

και $\frac{\partial L}{\partial \lambda} = 0$ (3)

για να ισχύει η ισότητα

Ερμηνεία πολλ/στών

Lagrange!

$$\max \varphi(x_1, x_2)$$

$$\text{s.t.} \quad f(x_1, x_2) = k$$

f=0
f'=f-k=k
no problem

$$\boxed{\frac{\partial \varphi}{\partial k} = \frac{\partial \varphi}{\partial x_1}\frac{\partial x_1}{\partial k} + \frac{\partial \varphi}{\partial x_2}\frac{\partial x_2}{\partial k}}, \quad (1)$$

Δα Ισότητα ισχύει!

$$f = k(\cdot)$$

$$\boxed{\frac{\partial f}{\partial k} = 1 \Rightarrow \frac{\partial f}{\partial x_1}\frac{\partial x_1}{\partial k} + \frac{\partial f}{\partial x_2}\frac{\partial x_2}{\partial k} - 1 = 0}$$

② : λ

① +

$$\frac{\partial y}{\partial k} = -\lambda + \lambda\left(\frac{\partial f}{\partial x_1}\frac{\partial x_1}{\partial k} + \frac{\partial f}{\partial x_2}\frac{\partial x_2}{\partial k}\right) + \frac{\partial y}{\partial x_1}\frac{\partial x_1}{\partial k} + \frac{\partial y}{\partial x_2}\frac{\partial x_2}{\partial k}$$

$=0$ στ ακρο

$$\Rightarrow \frac{\partial y}{\partial k} = -\lambda + \left(\frac{\partial y}{\partial x_1} + \lambda\frac{\partial f}{\partial x_1}\right)\frac{\partial x_1}{\partial k} + \left(\frac{\partial y}{\partial x_2} + \lambda\frac{\partial f}{\partial x_2}\right)\frac{\partial x_2}{\partial k} \Rightarrow$$

$=0$ στ. ακρότατο

$\Rightarrow$ στ ακρότατο

$$\frac{\partial y}{\partial k} = -\lambda =$$

δηλ δείχνει

ε Ů ŰŰbɞǝæ Ű ɖɠ ů ɡɜ Ɏɉ Ű ɖůɖɠ ů Űɧɞɞɡ (Ŭ ɜɖůɖ y ε Ů Űɖɜ Ŭ ɜɖůɖ ǝ (ů Ű ŰɗŲ ɧɠ ɧɉ ɞɠ ů Űɞɜ ´ Ų ɞɉ ɞ ε ɧ ɞ ɧŰɖŰŰɠ))
= ε Ůɞɜ Űɞɜ ´ ɞǝŬ´ ǝŬůɞŬů Űɞ Lagrange