ΚΕΦΑΛΑΙΟ 9: ΑΡΜΟΝΙΚΗ ΑΠΟΚΡΙΣΗ

$$H(s) = \frac{K(s+z_1)(s+z_2)\ldots(s+z_m)}{(s+p_1)(s+p_2)\ldots(s+p_n)}, \qquad m < n$$

ΕΙΣΟΔΟΣ: $r(t) = R\eta\mu\omega t$

$$R(s) = \frac{R\omega}{s^2+\omega^2}$$

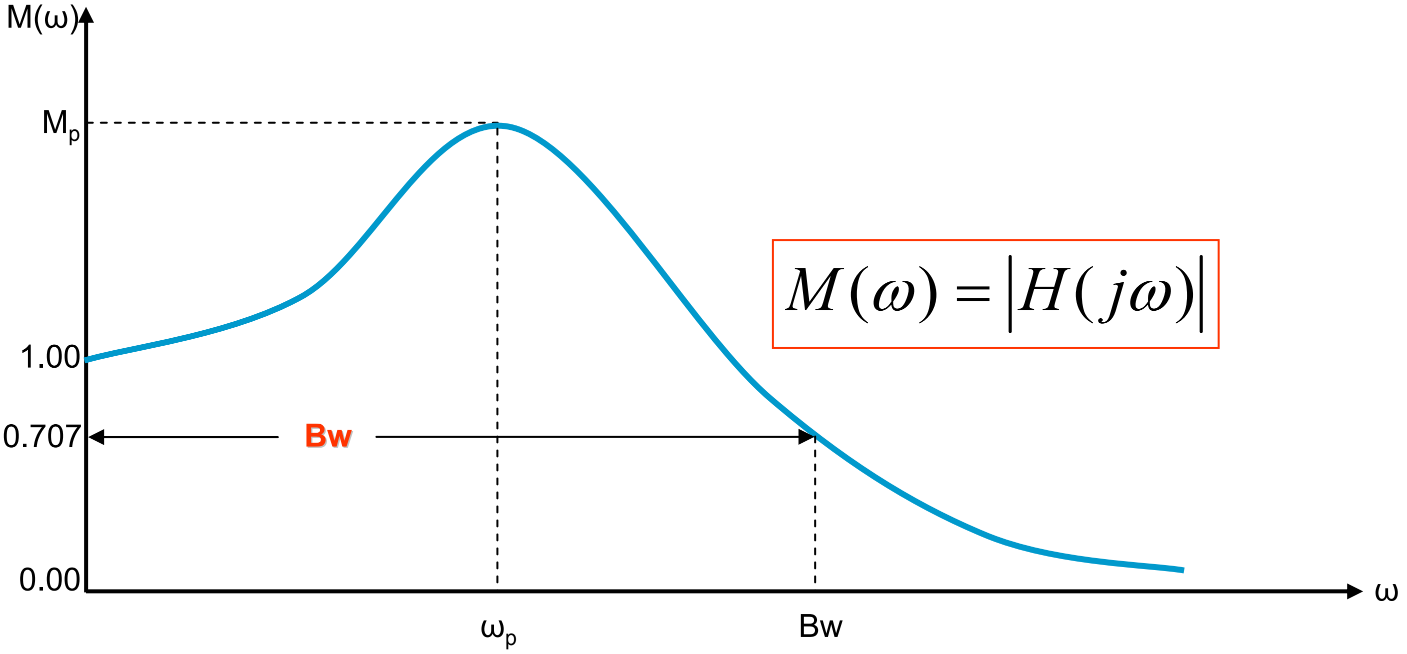
$$Y(s) = H(s)R(s) = \left[\frac{K(s+z_1)(s+z_2)\ldots(s+z_m)}{(s+p_1)(s+p_2)\ldots(s+p_n)}\right]\left[\frac{R\omega}{s^2+\omega^2}\right]$$

$$y(t) = L^{-1}\left[Y(s)\right] = k_1 e^{-p_1 t} + k_2 e^{-p_2 t} + \ldots + k_n e^{-p_n t} + \\ + k_{n+1} e^{-j\omega t} + k_{n+2} e^{j\omega t}$$

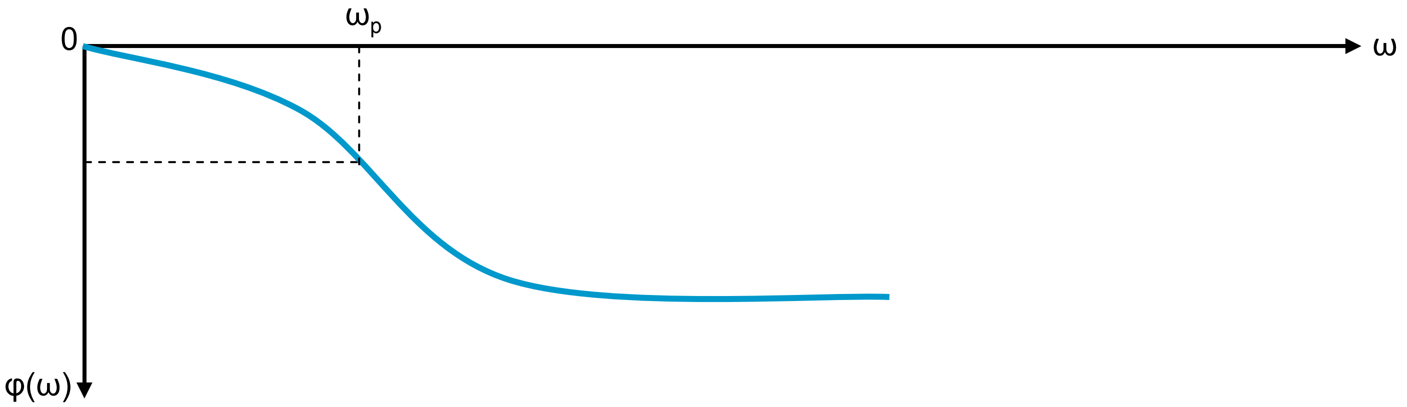
$$y_{\mu o \nu}(t) = R\left|H(j\omega)\right|\eta\mu(\omega t + \phi(\omega))$$

$$\left.\begin{array}{lll} \left|H(j\omega)\right| & \text{: πλάτος} & H(s) \\ \phi(\omega) & \text{: φάση} & H(s) \end{array}\right\} \text{για} \qquad s = j\omega$$

ΧΑΡΑΚΤΗΡΙΣΤΙΚΑ ΑΡΜΟΝΙΚΗΣ ΑΠΟΚΡΙΣΗΣ



$$M(\omega) = \left| H(j\omega) \right|$$

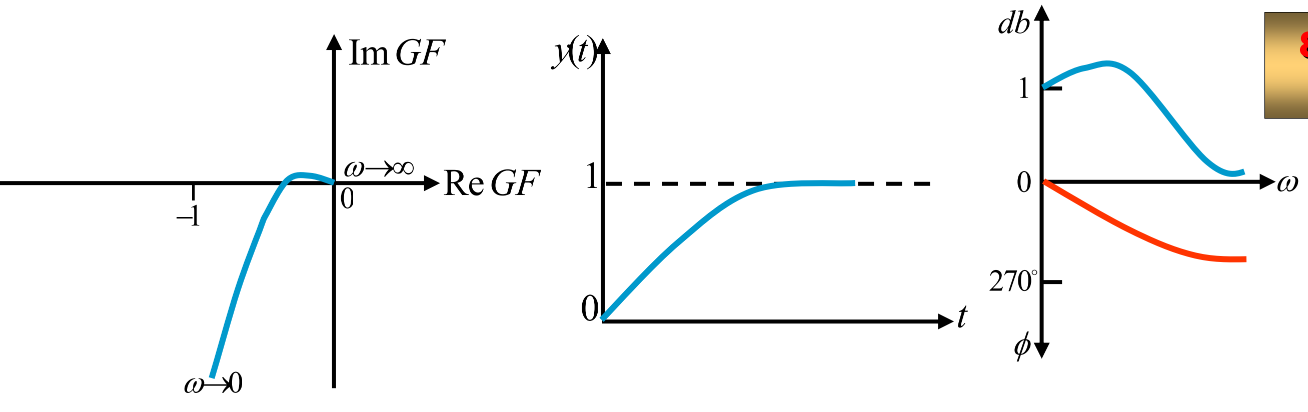


- **Μέγιστη τιμή M_p** (συνήθως $1.1<M_p<1.5$)
- **Συχνότητα συντονισμού ω_p**
- **Εύρος ζώνης ω_p=Bw** $\rightarrow M(\omega_b)=0.707$

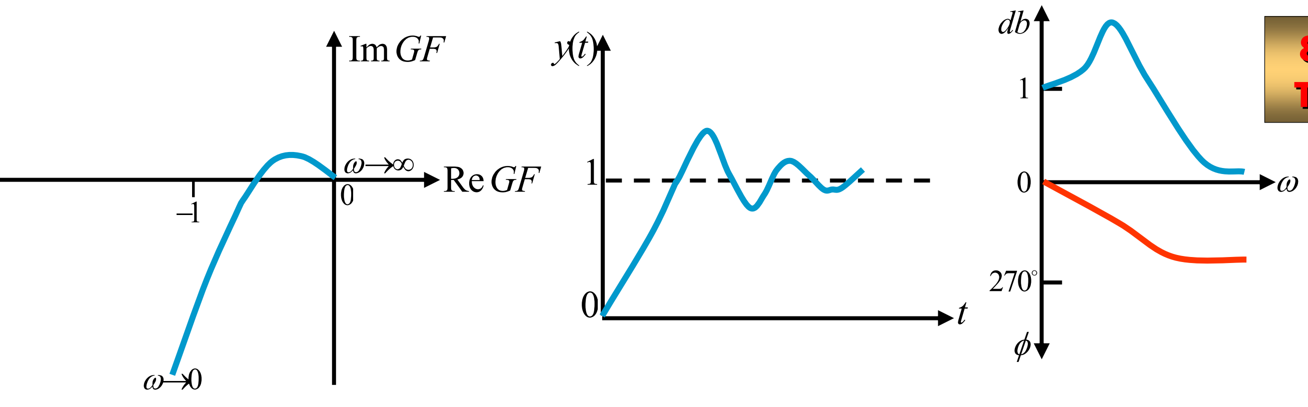
Αντιστρόφως ανάλογο του χρόνου ανύψωσης

ΘΕΣΕΙΣ ΠΟΛΩΝ

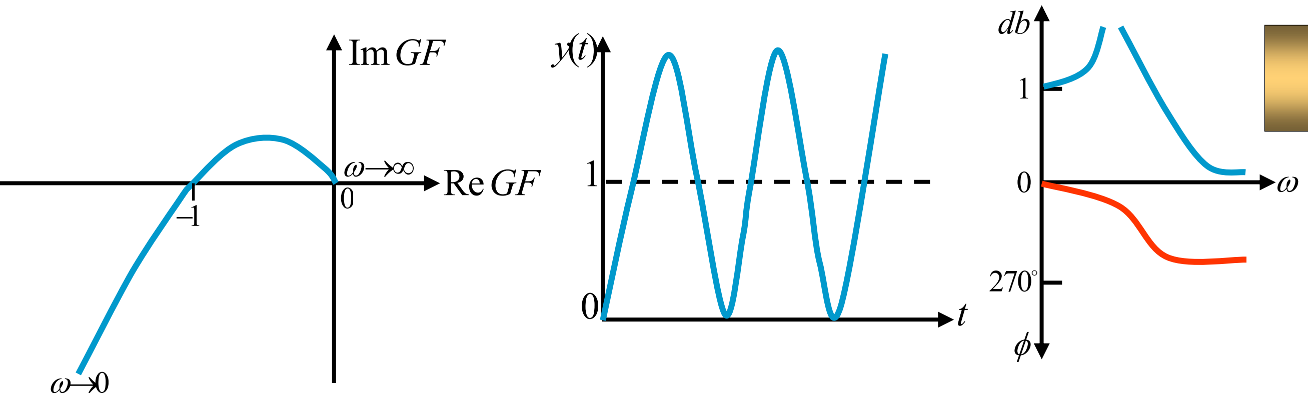
ΧΡΟΝΙΚΗ ΑΠΟΚΡΙΣΗ



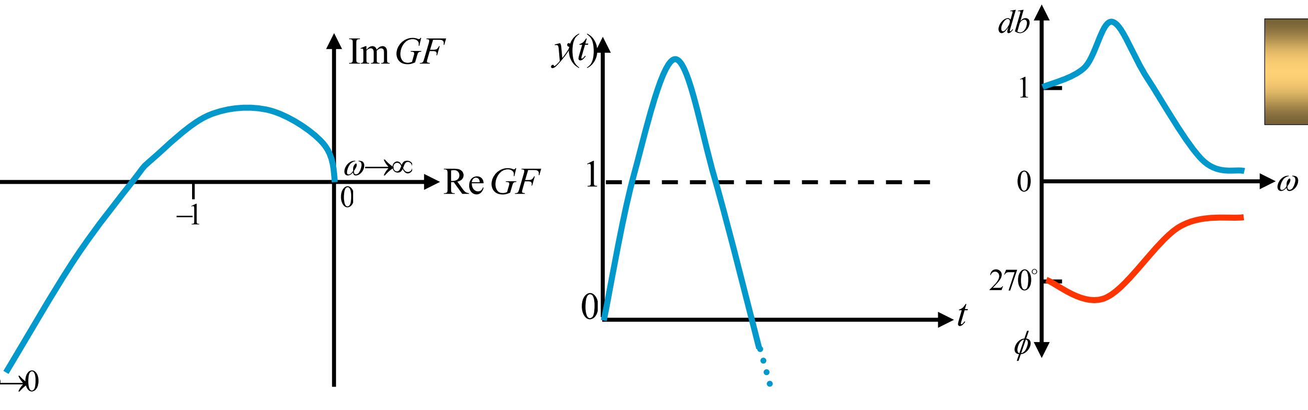
ευστάθεια – απόσβεση



ευστάθεια – ταλαντώσεις



οριακή ευστάθεια



αστάθεια

ΔΙΑΓΡΑΜΜΑΤΑ BODE

- $A = 20\log M = 20\log\left|H(j\omega)\right| = 20\log\left|G(j\omega)F(j\omega)\right|$
- $\phi(\omega)$

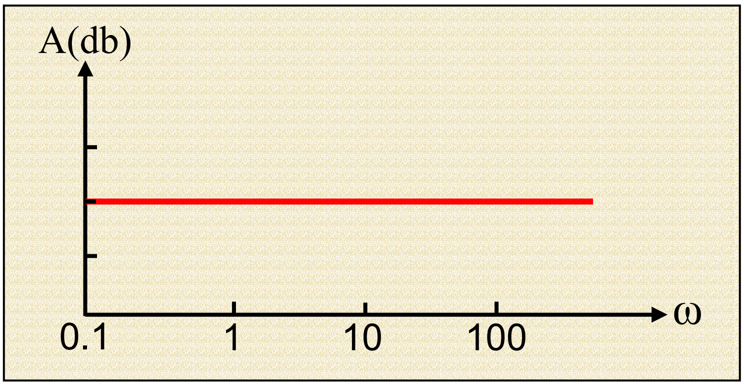
$$G(j\omega)F(j\omega) = \frac{K(j\omega T_1^{'}+1)(j\omega T_2^{'}+1)}{(j\omega)^2(j\omega T_1 \ +1)\left[(j\omega)^2+2\zeta\omega_0(j\omega)+\omega_0^2\right]}$$

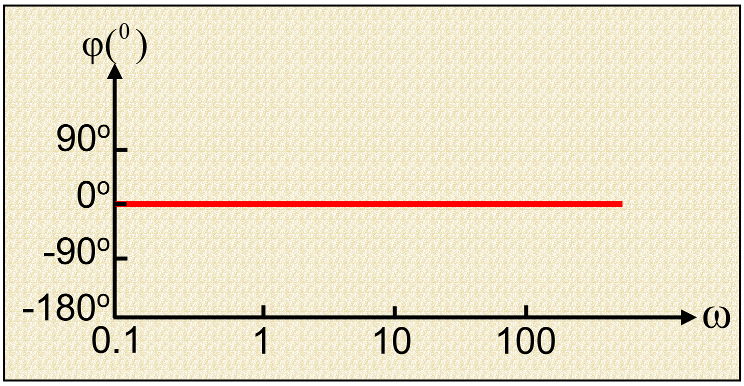
$$A = 20\log\left|K\right| + 20\log\left|j\omega T_1^{'}+1\right| + 20\log\left|j\omega T_2^{'}+1\right| -$$
$$-20\log\left|(j\omega)^2\right| - 20\log\left|j\omega T_1 \ +1\right| - 20\log\left|(j\omega)^2+2\zeta\omega_0(j\omega)+\omega_0^2\right|$$

$$\phi(\omega) = \underline{\left|K\right.} + \underline{\left|j\omega T_1^{'}+1\right.} + \underline{\left|j\omega T_2^{'}+1\right.} -$$
$$-\underline{\left|(j\omega)^2\right.} - \underline{\left|j\omega T_1 \ +1\right.} - \underline{\left|(j\omega)^2+2\zeta\omega_0(j\omega)+\omega_0^2\right.}$$

ΣΤΑΘΕΡΟΣ ΟΡΟΣ K

- $A = 20\log|K|$
- $\phi = \begin{cases} 0^\circ, & k > 0 \\ 180^\circ, & k < 0 \end{cases}$

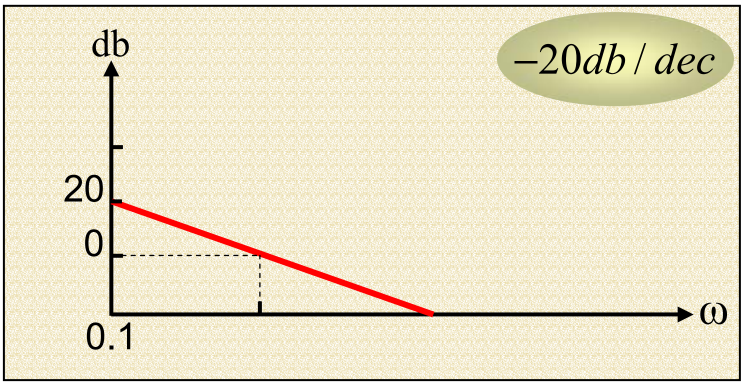


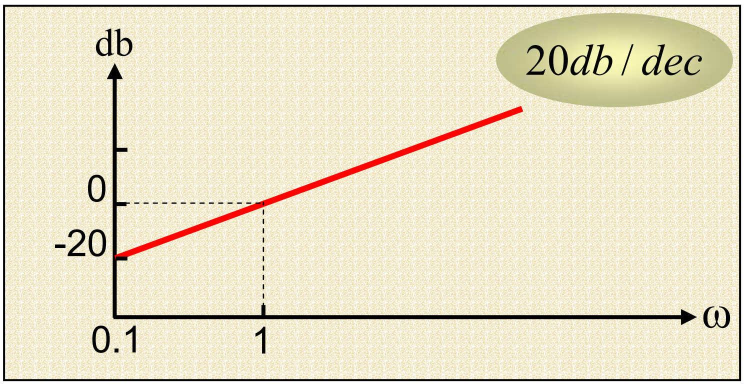


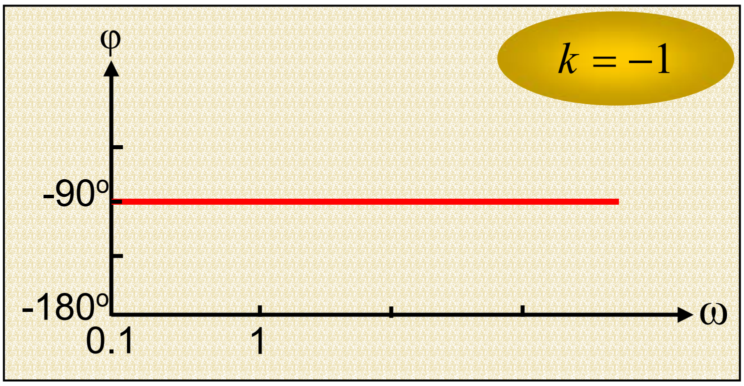
$$(j\omega)^{\pm k}$$

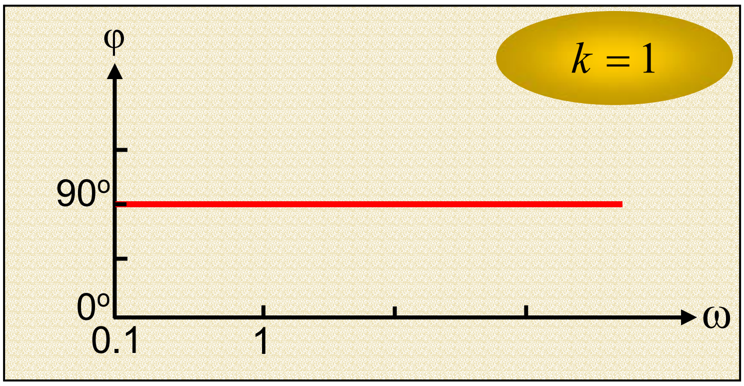
$$A = 20\log\left|(j\omega)^{\pm k}\right| = \pm 20k\log\omega$$

$$\phi = \pm k 90^\circ$$









$$(j\omega T+1)^{\pm k}$$

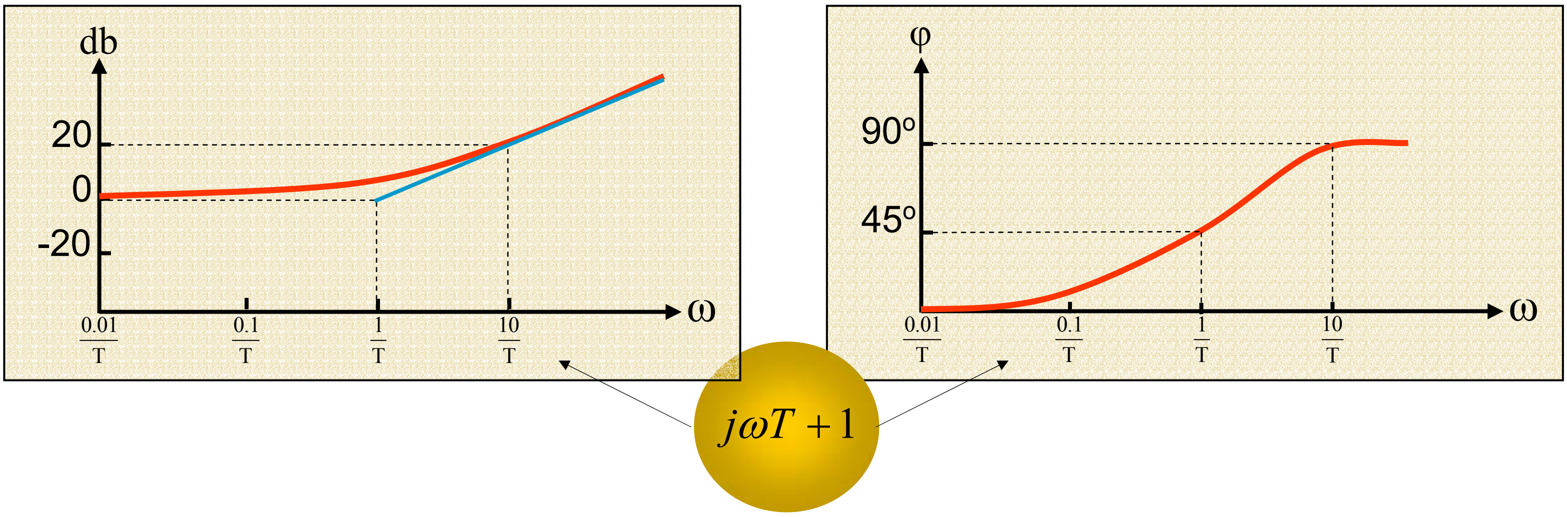
$$A=\pm 20k\log\left|j\omega T+1\right|=\pm 20k\log(\omega^2T^2+1)^{1/2}$$

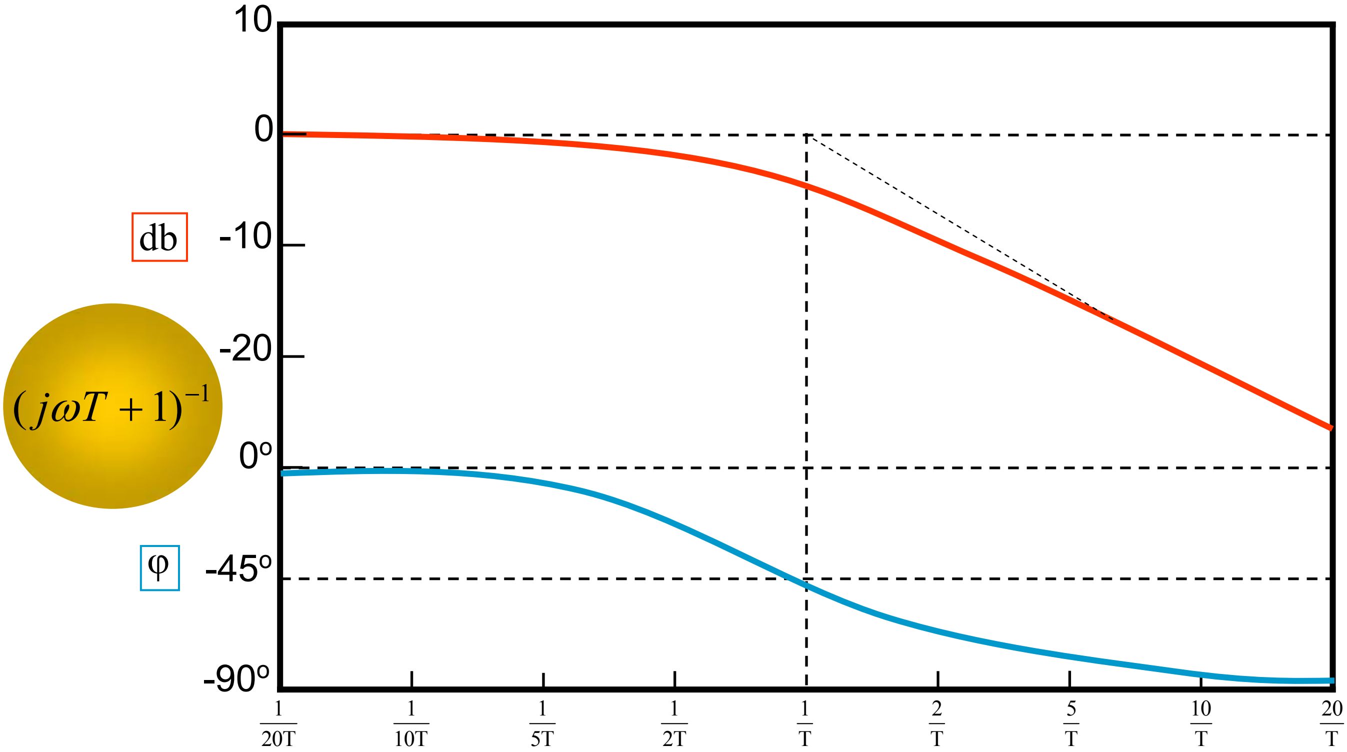
$$\phi=\pm k\varepsilon\phi^{-1}\omega T$$

$$\omega \ll 1/T, \quad A=0 \qquad\qquad \omega=0, \quad \phi=0^\circ$$

$$\omega = 1/T, \quad A\simeq \pm 3k \qquad\qquad \omega=1/T, \quad \phi=\pm k45^\circ$$

$$\omega \gg 1/T, \quad A=\pm 20k\log\omega T \qquad\qquad \omega\to\infty, \quad \phi=\pm k90^\circ$$





$$\left[(j\omega)^2 + 2\zeta\omega_0(j\omega) + \omega_0^2\right]^{\pm k}$$

$$A = \pm 20k \log((1-u^2)^2 + 4\zeta^2u^2)^{1/2}$$

$$\phi = \pm k\varepsilon\phi^{-1}\frac{2\zeta u}{1-u^2}$$

$$u = {}^{\omega}\!/_{\omega_0}$$

$$u \ll 1, \quad A \simeq \pm 20k \log 1 = 0$$

$$u \gg 1, \quad A = 40k \log u$$

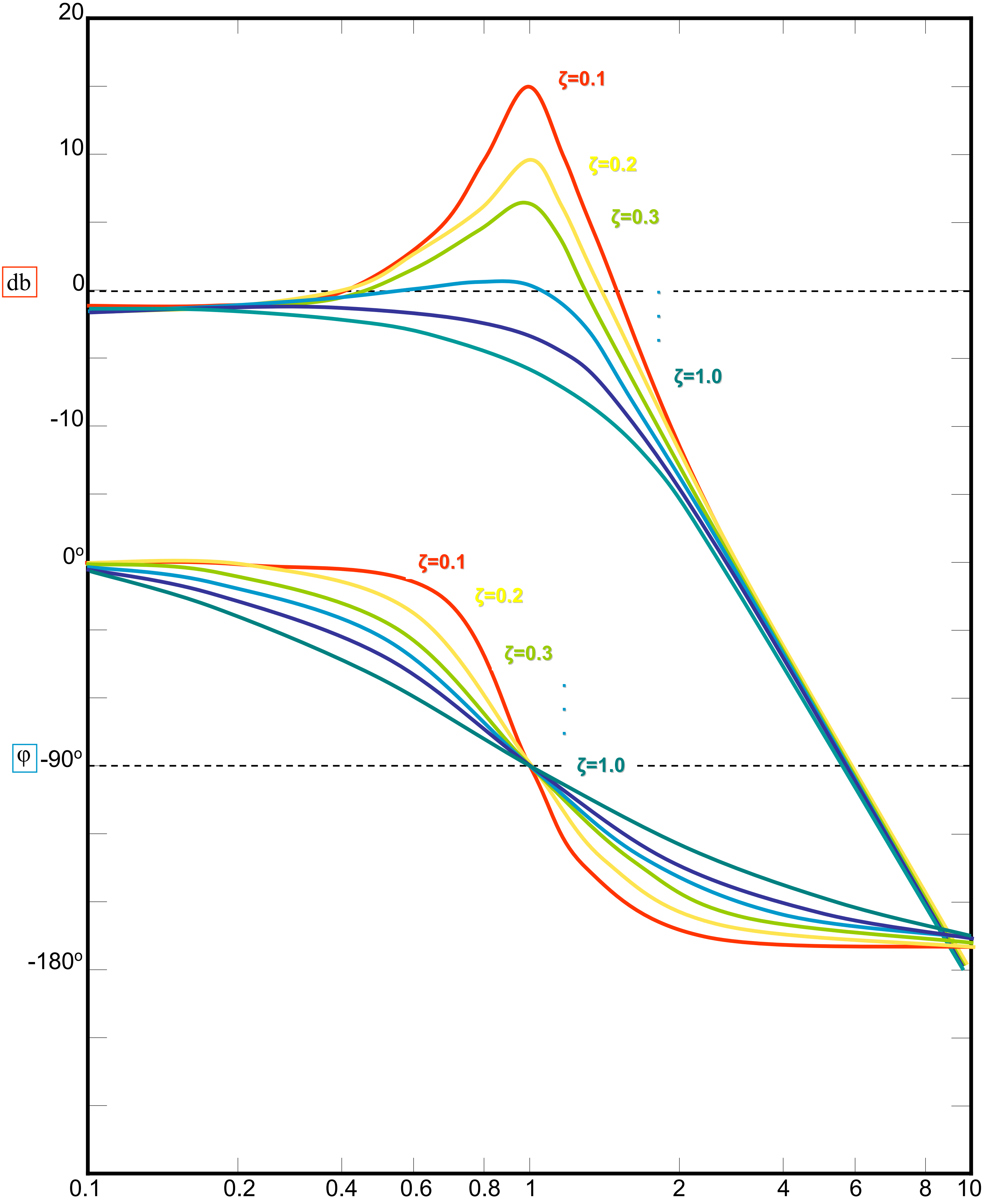
$$\zeta = 1, \quad A = \pm 20k \log\left|1+u^2\right|$$

$$\zeta = 0, \quad A = \pm 20k \log\left|1-u^2\right|$$

$$u = 0, \quad \phi = 0^\circ$$

$$u = 1, \quad \phi = \pm k90^\circ$$

$$u \to \infty, \quad \phi = \pm k180^\circ$$


20
10
0
-10
db
0º
-90º
-180º
φ
ζ=0.1
ζ=0.2
ζ=0.3
ζ=1.0
ζ=0.1
ζ=0.2
ζ=0.3
ζ=1.0
0.1
0.2
0.4
0.6
0.8
1
2
4
6
8
10

Παράδειγμα

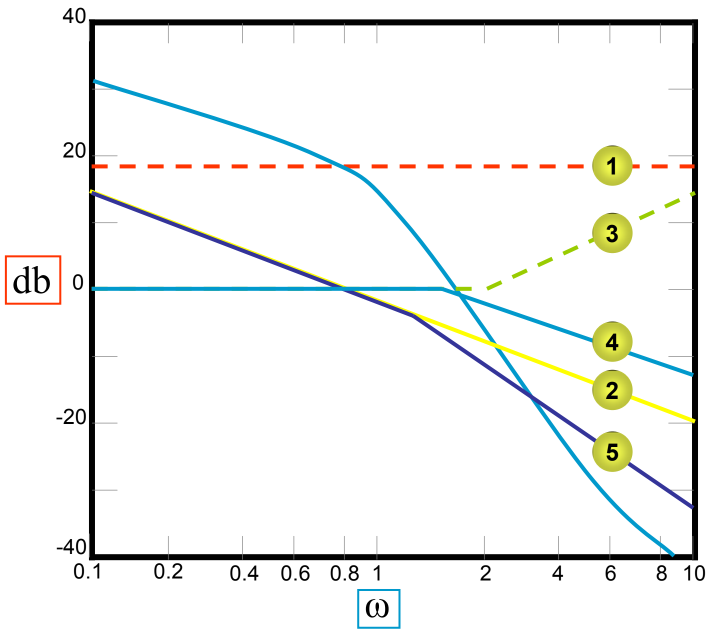
$$G(j\omega)F(j\omega)=\frac{10(j\omega+3)}{(j\omega)(j\omega+2)\left[(j\omega)^2+(j\omega)+2\right]}=$$

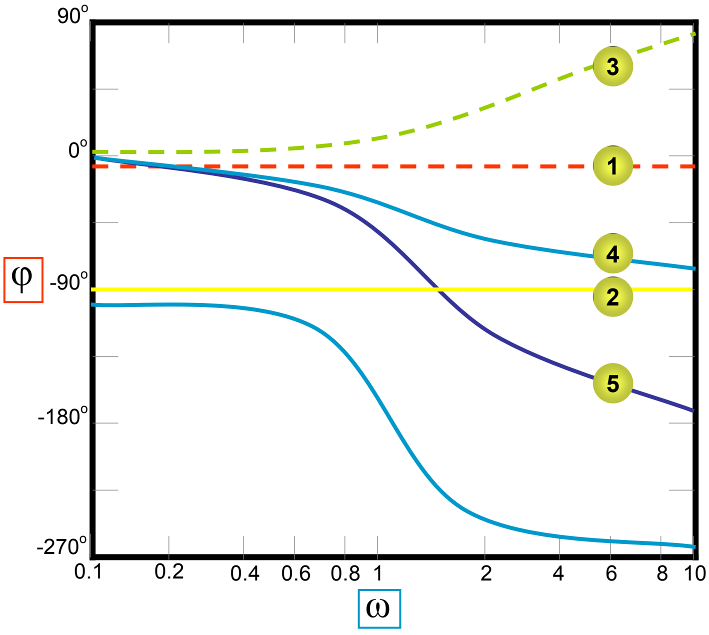
$$=\frac{\frac{3}{4}10(j\frac{\omega}{3}+1)}{(j\omega)(j\frac{\omega}{2}+1)\left[\frac{(j\omega)^2}{2}+\frac{(j\omega)}{2}+1\right]}$$

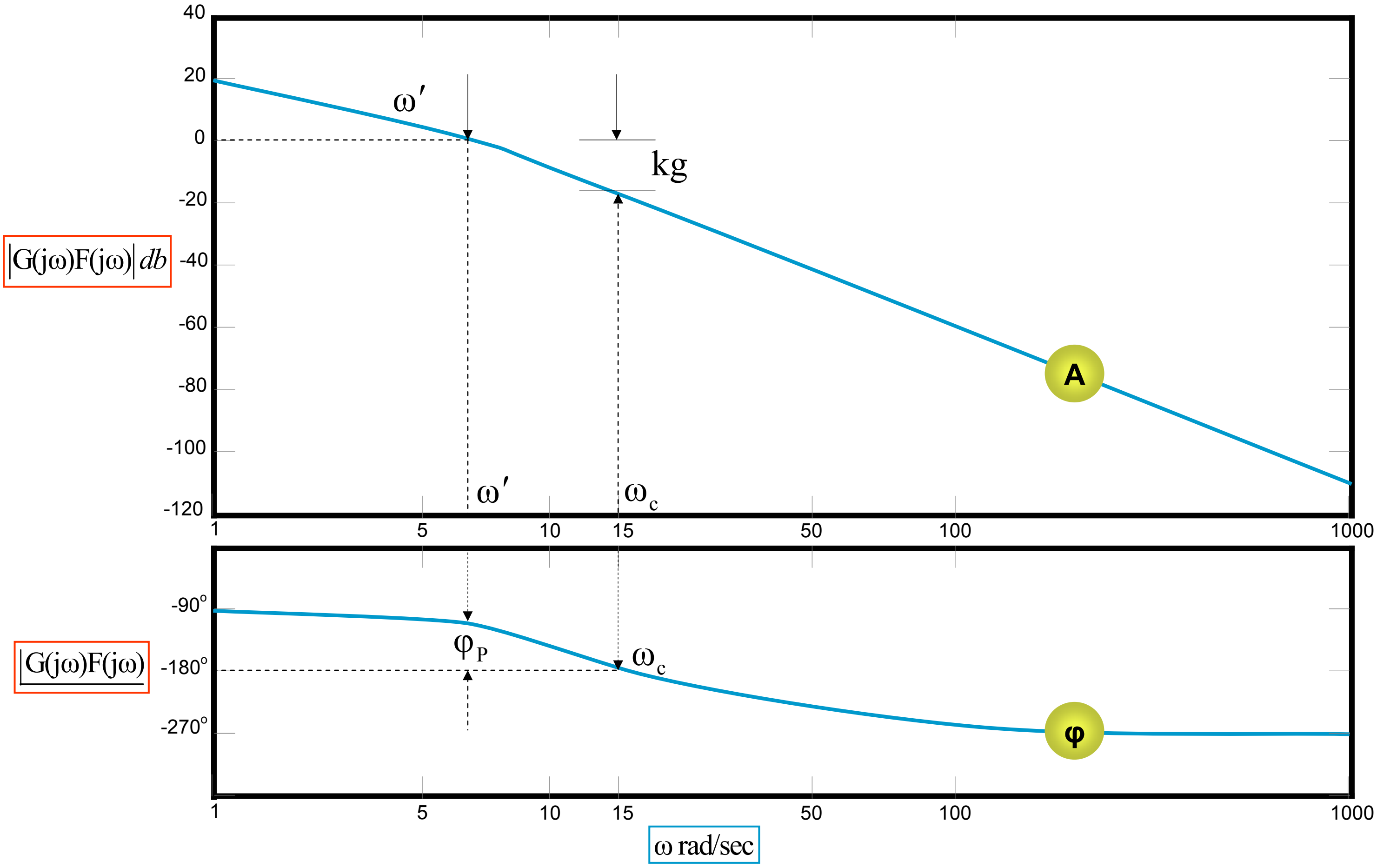
ΠΑΡΑΓΟΝΤΕΣ:

$$7.5,\quad (j\omega)^{-1},\quad (j\frac{\omega}{3}+1),\quad (j\frac{\omega}{2}+1)^{-1},\quad \left[\frac{(j\omega)^2}{2}+\frac{(j\omega)}{2}+1\right.$$

$$\omega=3 \qquad \omega=2 \qquad \omega=\sqrt{2}$$







για $\omega = \omega' \Rightarrow \left|G(j\omega)F(j\omega)\right| = 1 \quad \rightarrow 0db$

ϕ_P : από $(\omega', -180^\circ)$ μέχρι καμπύλη φάσης →

$$\phi_P = 180^\circ + \underline{\left|G(j\omega')F(j\omega')\right.}$$

για $\omega = \omega_c \Rightarrow \underline{\left|G(j\omega)F(j\omega)\right.} = -180^\circ$

Kg : από $(\omega_c, 0db)$ μέχρι καμπύλη A: →

$Kg = +\left|G(j\omega_c)F(j\omega_c)\right|$ ή $-20\log\left|G(j\omega_c)F(j\omega_c)\right|db$

Ευστάθεια: **Kg, φ_P** θετικά