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Technical Discussion …

SIMPLIFIED METHOD FOR ESTIMATING HEAT SINK THERMAL RESISTANCE - Θ_{SA} FORCED CONVECTION APPLICATIONS

Heat Sink Thermal Performance in Forced Convection Applications is heavily dependant on Heat Sink Surface Area. In fact the main purpose of a Heat Sink is to increase the surface area "exposed" to the air flowing in the area of the device being cooled – effectively removing the heat by convection heat transfer.

In an effort to simply the complex calculations typically required when estimating the expected thermal performance of a heat sink profile for a specific application – thermal engineers have developed a very convenient equation to determine the Heat Sink Thermal Resistance – from Heat Sink to Ambient Air – Theta "Sink-to-Air" - Θ_{SA} as a function of a "derived" Performance Factor and the Profile Perimeter.

Simplified Performance Factor Equation:

$$\Theta_{SA} = \frac{\text{Performance Factor}}{\text{Profile Perimeter}}$$

$$\text{Performance Factor} = \frac{916}{(V \times L)^{.5}}$$

- **L = Heat Sink Length in inches**
- **V = Air Velocity in Linear Feet per Minute – LFM**

MATHEMATICAL FORMULA ANALYSES VALIDATION OF SIMPLIFIED METHOD

To validate this simplified Performance Factor equation and justify it's use – we will now provide the accepted thermal equations governing the heat transfer of forced convection cooling – as applied to heat sinks.

The primary formula used in the determination of boundary air layer conditions surrounding a heat sink is Equation #10B from Laminar Boundary Layer on a Flat Plate from Transmission of Heat by Conduction and Convection by McAdams, Williams and Smith published in The Standard Handbook for Mechanical Engineers by Baumeister and Marks. This formula provides method of evaluating the heat transferred through the boundary layer to the cooling air passing the heat sink. The resultant boundary layer transfer coefficient is

an average for the length of heat transfer surface. Convective heat transfer removes essentially all of the energy from a heat sink under forced air cooling.

Conduction and radiation provide little additional cooling.

This equation is confirmed from many other references including Equation #6.33 from Thermal Analysis and Control of Electronic Equipment by Kraus and Bar-Cohen.

$$\frac{Hc}{D \times Cp \times V} \times (Cp \times Dv / K)^{.66} = \frac{.664}{(D \times V \times L / Dv)^{.5}}$$

Details of the input parameters are:

- **Hc** = heat transfer coefficient (BTU / hr sq. ft deg F)
- **K** = thermal conductivity of air at 77 deg F; .014 BTU / hr deg F sq ft / ft
- **Cp** = specific heat at 77 deg F; .241 BTU / lb deg F
- **Dv** = dynamic viscosity of air at 77 deg F; .044 lb / hr ft
- **D** = Density of air at 77 deg F; .075 lbs. Per cubic foot
- **V** = velocity of air in feet per hour
- **L** = extrusion length of the heat sing in feet

Components of this formula can be expressed as three generally accepted dimensionless numbers:

Stanton Number = (Hc / D x Cp x V)

This number is the heat absorbed by the cooling media as compared to the heat available for absorption:

Prandtl Number = (Cp x Dv / K)

The Prandtl number measures the amount of temperature gradient in a fluid at a given temperature.

The Reynolds number represents a ratio of internal forces to viscous forces due to the velocity of the fluid flow. This number is used to indicate fluid flow types (laminar, transition or turbulent).

Reynolds Number = (D x V x L / Dv) - This formula is valid within the following conditions:

1. Standard atmospheric pressure of 30.0 mmHg

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2. **The heat sink is assumed to be isothermal and isoflux having a constant temperature and constant heat input along it's length and width – no spreading resistance is being accounted for.**
3. **There is assumed to be no static pressure drop, hence no change in velocity, of the air moving along the length of the plate. This may not be a realistic condition, however, in general the impact on the thermal results is minimal.**
4. **The airflow at the leading edge if the heat sink must be in a condition having a Reynolds number of less than 4×10^5.**
5. **The airflow along the heat sink surface must be parallel with the length of the extrusion.**

THERMAL RESISTANCE FORMULA

The thermal resistance formula uses the result of the first equation to arrive at a thermal resistance. The film coefficient (Hc) and the total surface are of the heat sink are used to derive a thermal resistance for the entire heat sink. Conversion constants are added to correct the input values to usable units.

$$\Theta_{SA} = \frac{60}{Hc \times A \times 17.57 \times 1.8}$$

Details of these input parameters are:

- Θ_{SA} **– thermal transfer coefficient; Deg C / Watt**
- **Hc – film coefficient from Equation # 1**
- **A = total surface area of the heat sink in square feet**
    - **17.57 converts BTU / minute to watts**
    - **1.8 converts Deg. F to Deg. C**
    - **60 converts hours to minutes**

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COMBINATION OF FORMULAS

In order to solve for total thermal resistance a composite formula was derived from the previous two equations. Rearranging these equations, the two formulas will result in a simplified when combined with specific length and velocity variables.

The combination of the basic formulas appears here:

$$\frac{60}{17.57 \times 1.8 \times A \times Ø} = \frac{.664 \times (Cp \times Dv / K)^{.66}}{(D \times V \times L / Dv)^{.5} \times D \times Cp \times V}$$

This formula is cumbersome – we will use values of air at standard temperature and pressure along with the previously listed constants to reduce the basic formula to include just two input variables.

Therefore, the final form used to generate the Forced Air Resistance Performance Factors can be found:

$$\text{Performance Factor} = \frac{916}{(V \times L)^{.5}}$$

Details of the remaining variable parameters are:

- **L = Heat Sink Length in inches**
- **V = Air Velocity in Linear Feet per Minute - LFM**

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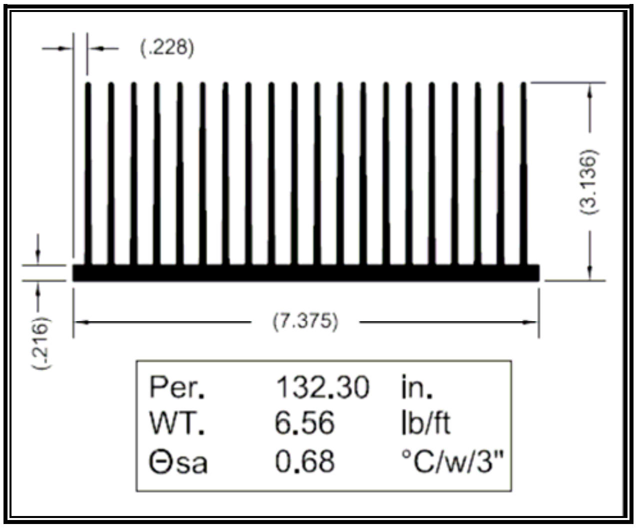
COMPARING THE RESULTING CONDENSED FORMULA WITH THE ORIGINAL EQUATION

An example will serve to demonstrate the accuracy of the Performance Factors.

This example will use the following specifics:



Wakefield Extrusion Profile – XX5113

A 10” long heat sink to be used with an air velocity of 400 fpm and having an extrusion surface area of 132.3 square inches per inch, total surface area of 1323 square inches, (9.19 sq ft).

From the Original formula:

$$\frac{Hc}{D \times Cp \times V} \times (Cp \times Dv / K)^{.66} = \frac{.664}{(D \times V \times L / Dv)^{.5}}$$

$$\frac{Hc}{.075 \times .241 \times 24000} \times (.241 \times .044 / .014)^{.66} = \frac{.664}{(.075 \times 24000 \times .833 / .044)^{.5}}$$

Hc = 1.88 BTU per hr sq ft deg F

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$$\Theta_{SA} = \frac{60}{Hc \times A \times 17.57 \times 1.8}$$

$$\Theta_{SA} = \frac{60}{1.88 \times 9.19 \times 17.57 \times 1.8}$$

Θ_{SA} = 0.110 deg C per Watt

Forced Air Performance Factors Table:

	VELOCITY IN LINEAR FEET PER MINUTE									
HEAT SINK LENGTH IN INCHES	100	200	300	400	500	600	700	800	900	1000
1.00	91.60	64.77	52.89	45.80	40.96	37.40	34.62	32.39	30.53	28.97
2.00	64.77	45.80	37.40	32.39	28.97	26.44	24.48	22.90	21.59	20.48
3.00	52.89	37.40	30.53	26.44	23.65	21.59	19.99	18.70	17.63	16.72
4.00	45.80	32.39	26.44	22.90	20.48	18.70	17.31	16.19	15.27	14.48
5.00	40.96	28.97	23.65	20.48	18.32	16.72	15.48	14.48	13.65	12.95
6.00	37.40	26.44	21.59	18.70	16.72	15.27	14.13	13.22	12.47	11.83
7.00	34.62	24.48	19.99	17.31	15.48	14.13	13.09	12.24	11.54	10.95
8.00	32.39	22.90	18.70	16.19	14.48	13.22	12.24	11.45	10.80	10.24
9.00	30.53	21.59	17.63	15.27	13.65	12.47	11.54	10.80	10.18	9.66
10.00	28.97	20.48	16.72	14.48	12.95	11.83	10.95	10.24	9.66	9.16
11.00	27.62	19.53	15.95	13.81	12.35	11.28	10.44	9.76	9.21	8.73
12.00	26.44	18.70	15.27	13.22	11.83	10.80	9.99	9.35	8.81	8.36
15.00	23.65	16.72	13.65	11.83	10.58	9.66	8.94	8.36	7.88	7.48
16.00	22.90	16.19	13.22	11.45	10.24	9.35	8.66	8.10	7.63	7.24
17.00	22.22	15.71	12.83	11.11	9.94	9.07	8.40	7.85	7.41	7.03
18.00	21.59	15.27	12.47	10.80	9.66	8.81	8.16	7.63	7.20	6.83
10.00	28.97	20.48	16.72	14.48	12.95	11.83	10.95	10.24	9.66	9.16
20.00	20.48	14.48	11.83	10.24	9.16	8.36	7.74	7.24	6.83	6.48

From this Forced Air Performance Factors Table:

The Performance Factor found:

At the intersection of 10 inch length and 400 feet per minute is 14.48.

From the Wakefield Extrusion Profile Sketch of Wakefield Extrusion Profile XX5113:

Obtain the Profile Perimeter – Per. = 132.30 inches.

Estimated Heat Sink Thermal Resistance – Theta Heat Sink to Air – 10 inch HS Length @ 400 LFM:

$$\Theta_{SA} = \frac{\text{Performance Factor}}{\text{Profile Perimeter}}$$

$$\Theta_{SA} = \frac{14.48}{132.30} = 0.109 \text{ Deg C per W}$$

Conclusion:

Excellent correlation between the detailed calculation, (Θ_{SA} = 0.110 C/W) and the simplified Performance Factor Method, (Θ_{SA} = 0.109 C/W) – analysis technique has validated the method and established the value in using this "quick-and-easy" method.

WAKEFIELD THERMOVATIONS **can support customers during the any stage of new product development – using the combined knowledge of over 100 years of designing and manufacturing experience – our Thermal and Mechanical Engineers are most effective when they are engaged early in the development process.**

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