

CHAPTER 1: INTRODUCTION TO INSTANTANEOUS POWER THEORY

ΔΗΜΗΤΡΙΑΔΟΥ ΚΩΝΣΤΑΝΤΙΝΑ-ΚΑΛΛΙΟΠΗ
ΡΟΪΔΟΣ ΙΩΑΝΝΗΣ-ΡΑΦΑΗΛ
ΒΑΓΙΑΝΝΗΣ ΝΙΚΟΛΑΟΣ
ΠΑΤΣΙΟΥΡΑΣ ΓΕΩΡΓΙΟΣ
ΧΑΤΖΗΑΓΓΕΛΟΥ ΓΕΩΡΓΙΟΣ-ΘΕΟΔΩΡΟΣ

CHAPTER 1

- 1.1. PROBLEMS RELATED TO HARMONIC POLLUTION IN POWER SYSTEMS
- 1.2. HARMONIC VOLTAGES IN POWER SYSTEMS
- 1.3. THREE-PHASE DIODE RECTIFIER WITH AN INDUCTIVE LOAD AND WITH A CAPACITIVE LOAD
- 1.4. STAND-ALONE SHUNT ACTIVE FILTER AND SERIES ACTIVE FILTER
- 1.5. SIMPLIFIED TRANSMISSION SYSTEM

Problems Related to Harmonic Pollution in Power Systems

- ❖ Overheating of transformers and electrical motors
- ❖ Overheating of capacitors for power-factor correction
- ❖ Voltage waveform distortion
- ❖ Voltage flicker
- ❖ Interference with communication systems

The pq theory that defines the instantaneous real and imaginary powers is a flexible tool, not only for harmonic compensation, but also for reactive-power compensation.

Harmonic Voltages in Power Systems (Japan)

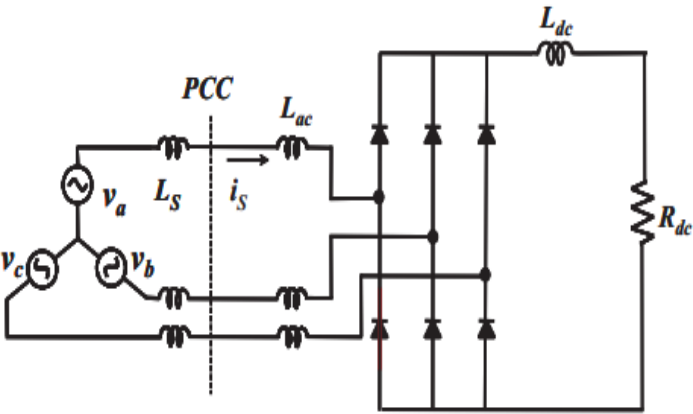
THD in voltage and 5th harmonic voltage
in a high-voltage power transmission
system

THD in voltage and 5th harmonic voltage
in a medium-voltage power distribution
system (6 kV)

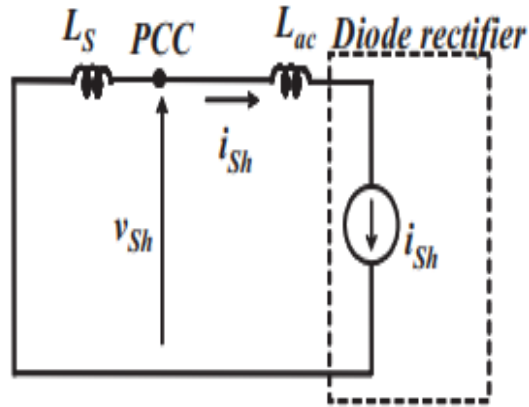
	Over 187 kV		22-154 kV			Residential Area		Commercial Area	
	THD	5 th Harmonic	THD	5 th Harmonic		THD	5 th Harmonic	THD	5 th Harmonic
Maximum	2,8 %	2,8 %	3,3 %	3,2 %	Maximum	3,5 %	3,4 %	4,6 %	4,3 %
Minimum	1,1 %	1 %	1,4 %	1,3 %	Minimum	3 %	2,9 %	2,1 %	1,2 %

Three-Phase Diode Rectifier with an Inductive Load

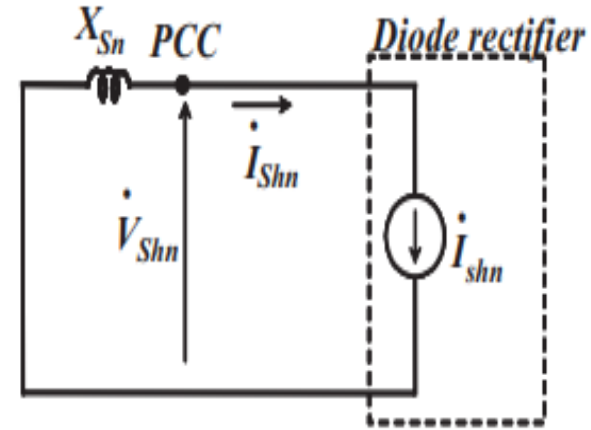
POWER CIRCUIT



EQUIVALENT CIRCUIT

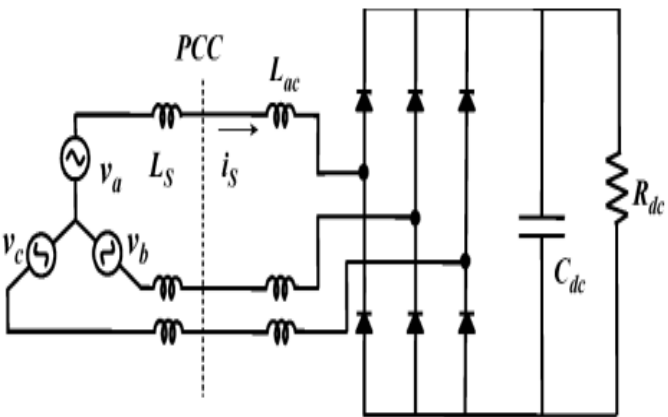


SIMPLIFIED CIRCUIT

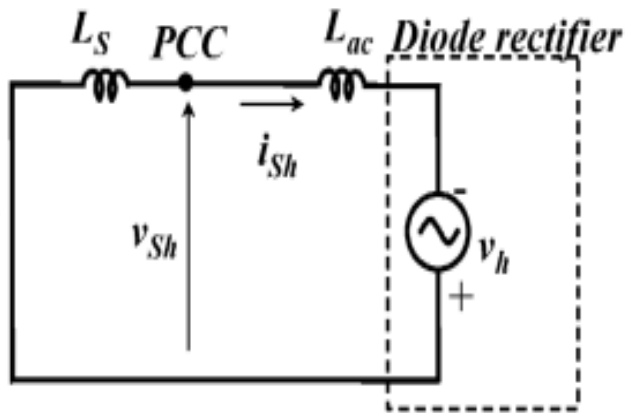


Three-Phase Diode Rectifier with a Capacitive Load

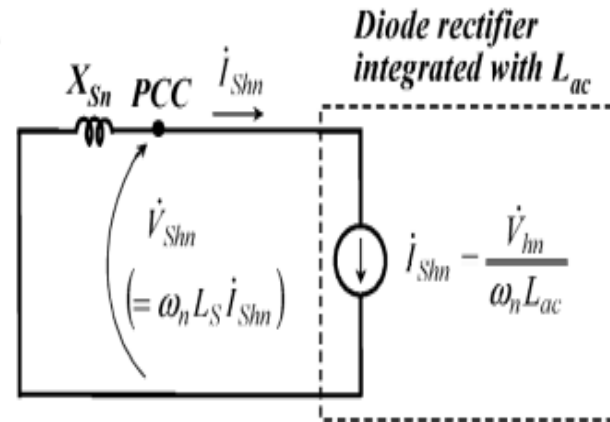
POWER CIRCUIT



EQUIVALENT CIRCUIT

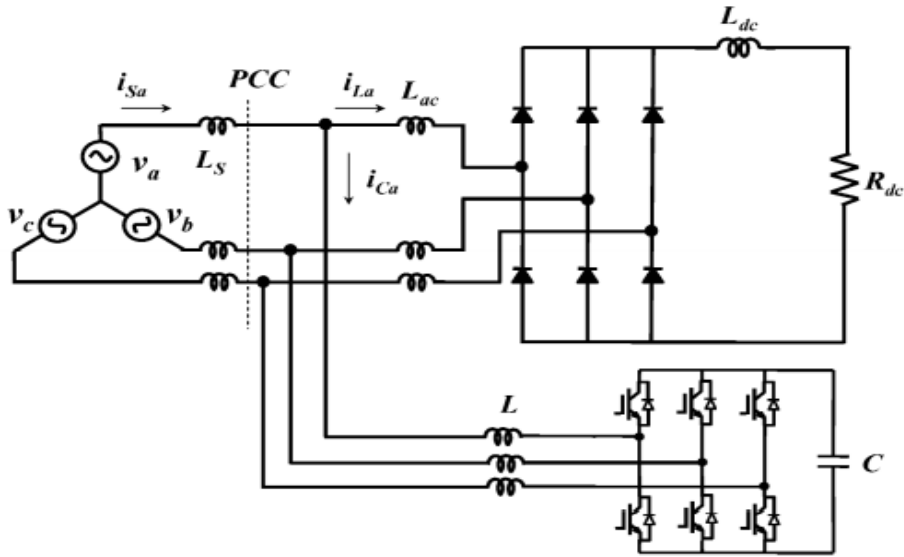


SIMPLIFIED CIRCUIT

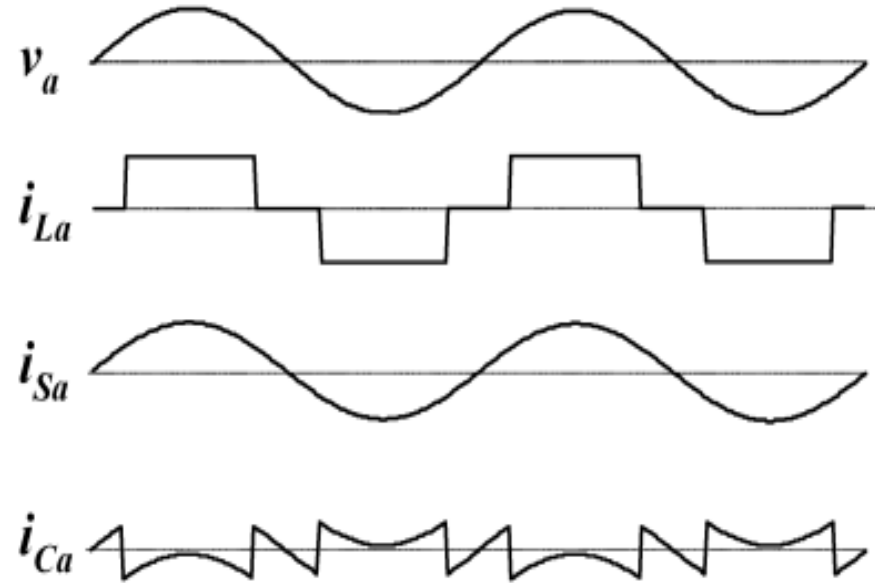


Stand-Alone Shunt Active Filter

CIRCUIT

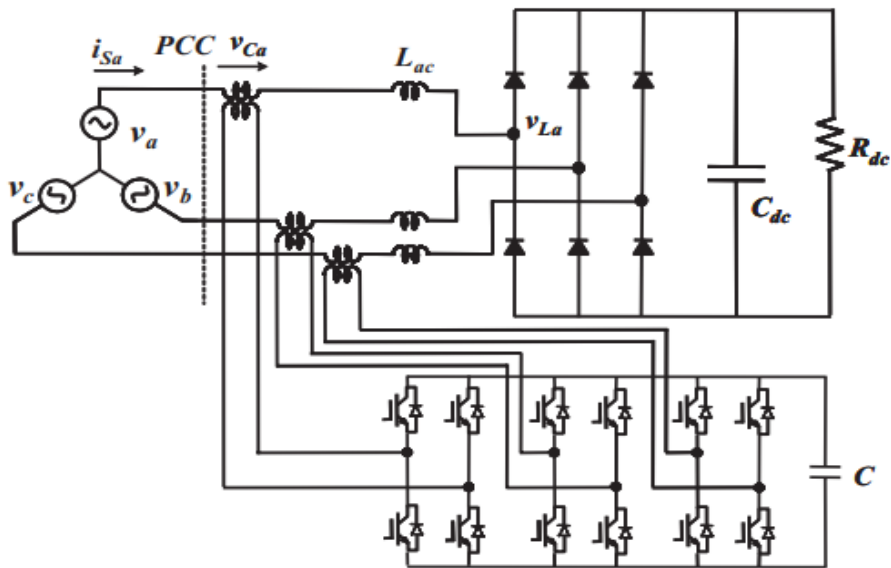


WAVEFORMS OF VOLTAGE AND CURRENTS

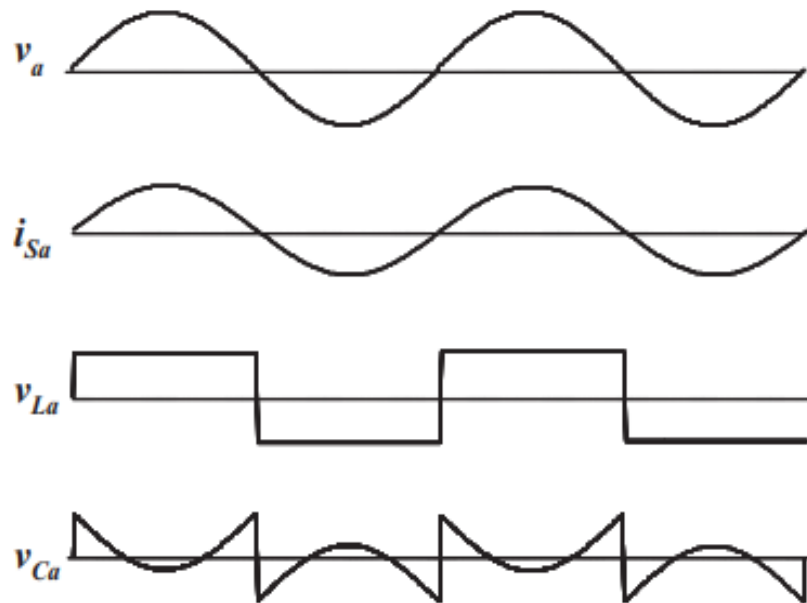


Stand-Alone Series Active Filter

CIRCUIT



WAVEFORMS OF VOLTAGES AND CURRENTS

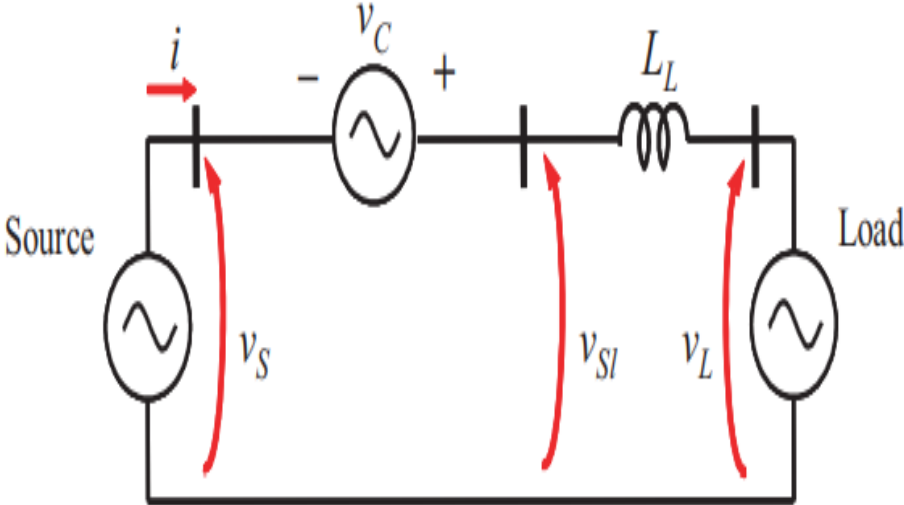


Comparisons Between Shunt and Series Active Filters

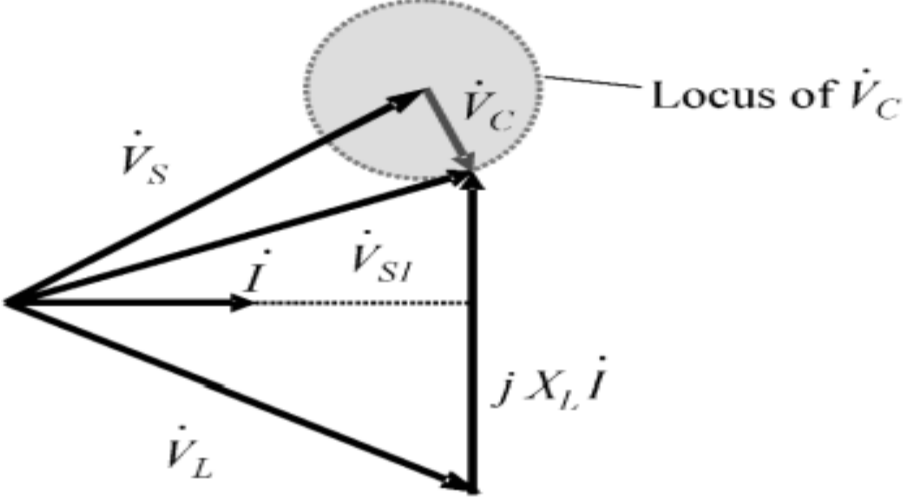
	POWER CIRCUIT	Operation	Suitable nonlinear load	Additional function
SHUNT ACTIVE FILTER	Voltage-source PWM converter with current minor loop	Current source: i_c	Diode / thyristor rectifier with inductive load	Reactive-power compensation
SERIES ACTIVE FILTER	Voltage-source PWM converter without current minor loop	Voltage source: v_c	Diode rectifier with capacitive load	AC voltage regulation

Simplified Transmission System

CIRCUIT

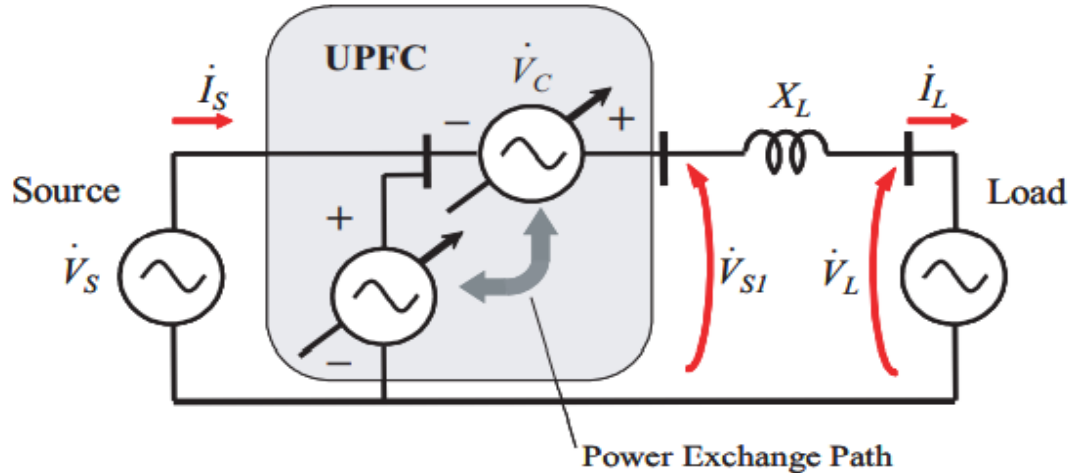


PHASOR DIAGRAM



Unified Power Flow Controller (UPFC)

The UPFC is a device which consists series and shunt compensators based on voltage-fed converters. This device can eliminate voltage and current harmonics and control active and reactive power in a transmission line.



**CHAPTER 2:
ELECTRIC POWER
DEFINITIONS
BACKGROUND**

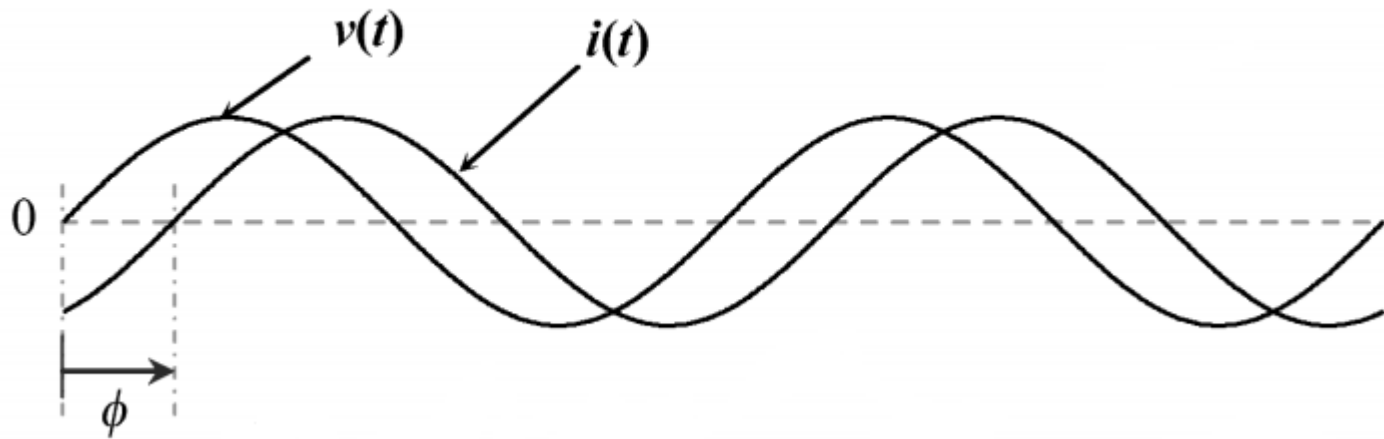
CHAPTER 2

- 2.1. POWER DEFINITIONS UNDER SINUSOIDAL CONDITIONS
- 2.2. POWER DEFINITIONS BY BUDEANU
- 2.3. POWER DEFINITIONS BY FRYZE
- 2.4. CLASSIFICATIONS OF THREE-PHASE SYSTEMS
- 2.5. SYMMETRICAL COMPONENTS THEORY
- 2.6. POWER IN BALANCED THREE-PHASE SYSTEMS
- 2.7. POWER IN UNBALANCED THREE-PHASE SYSTEMS

Power Definitions under Sinusoidal Conditions

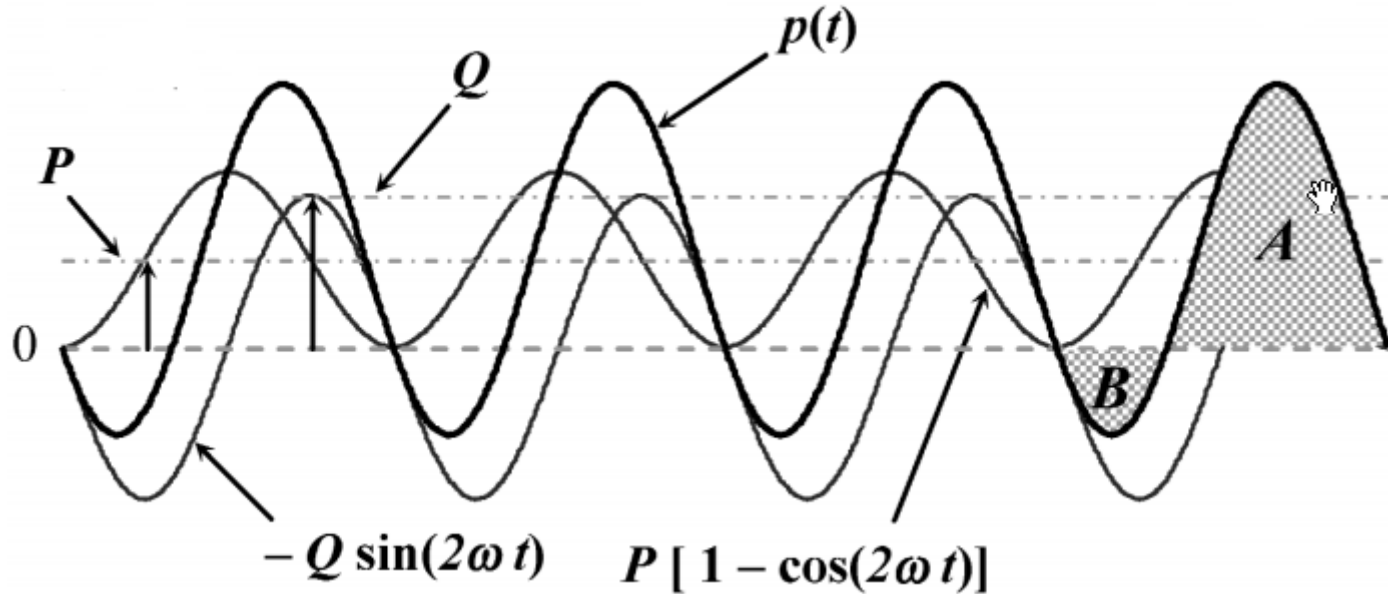
An ideal single-phase system with sinusoidal voltage source and a linear resistive-inductive load has voltage and current that are represented by:

$$v(t) = \sqrt{2}V \sin(\omega t) \quad i(t) = \sqrt{2}I \sin(\omega t - \phi)$$



The product of these two is the instantaneous power:

$$p(t) = \underbrace{VI \cos \phi [1 - \cos(2\omega t)]}_{(I)} - \underbrace{VI \sin \phi \sin(2\omega t)}_{(II)} = \underbrace{P [1 - \cos(2\omega t)]}_{(I)} - \underbrace{Q \sin(2\omega t)}_{(II)}$$



Conventionally, the instantaneous power is represented by three “constant” powers:

- ❖ Active power: $P = VI \cos(\phi)$ [W], the portion of power that is actually consumed by the load in ac circuits
- ❖ Reactive power: $Q = VI \sin(\phi)$ [VAr], the portion of power that does not realize work or oscillating power
- ❖ Apparent power: $S = VI$ [VA], maximum reachable active power at unity power factor

Phasors and Complex Impedance(Frequency Domain)

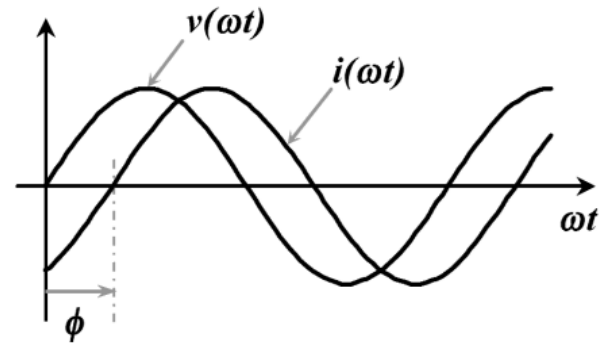
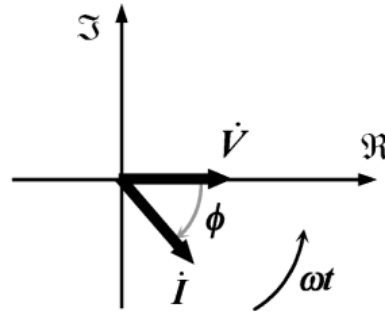
$$f(t) = \sqrt{2}A \sin(\omega t + \phi) = \text{Im}\{\dot{F} \cdot e^{j\omega t}\}$$

Thus, $\dot{F} = \sqrt{2}A \angle \phi$

$$\dot{V} = V \angle \theta_V$$

$$\dot{I} = I \angle \theta_I$$

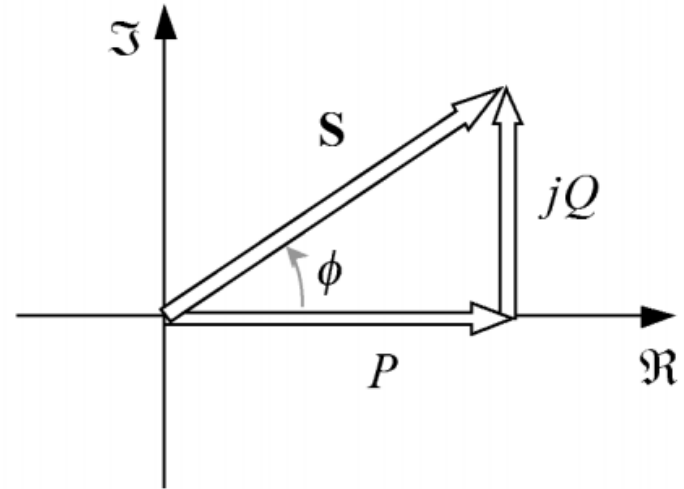
$$\mathbf{Z} = \frac{\dot{V}}{\dot{I}} = \frac{V \angle \theta_V}{I \angle \theta_I} = \frac{V}{I} \angle (\theta_V - \theta_I)$$



Complex Power and Power Factor

❖ Complex power S :
$$\mathbf{S} = \dot{V}I^* = (V \angle \theta_V)(I \angle -\theta_I) = \underbrace{VI \cos(\theta_V - \theta_I)}_P + j \underbrace{VI \sin(\theta_V - \theta_I)}_Q$$

❖ Power factor λ (PF):
$$\lambda = \text{PF} = \cos \phi = \frac{P}{S}$$



Power Definitions under Nonsinusoidal Conditions

- ❖ The conventional concepts of reactive and apparent power lose their usefulness in nonsinusoidal cases. This problem has existed for many years and is still with us.
- ❖ The problems related to nonlinear loads became increasingly significant at the beginning of advances in power electronics devices, which draw a significant amount of harmonic current from the power system.
- ❖ Two important approaches to power definitions under nonsinusoidal conditions were introduced by Budeanu in 1927 and by Fryze in 1932. Budeanu worked in the frequency domain, whereas Fryze defined the power in the time domain.

Power Definitions by Budeanu

- ❖ In frequency domain (in steady-state analysis).
- ❖ He used Fourier series to decompose the voltage and current waveforms of a generic load and to derive the phasor for each harmonic component.
- ❖ This theory treats electric circuits under nonsinusoidal conditions as a sum of several independent circuits excited at different frequencies.

❖ Apparent Power S : $S = VI$

V and I represent the rms values of the voltage and current waveforms, which are calculated by the following formula:

$$V = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt} = \sqrt{\sum_{n=1}^{\infty} V_n^2} \quad I = \sqrt{\frac{1}{T} \int_0^T i^2(t) dt} = \sqrt{\sum_{n=1}^{\infty} I_n^2}$$

Here, V_n and I_n correspond to the rms value of the n th order harmonic components of the Fourier series (no dc component is being considered in this analysis).

❖ Active Power P:
$$P = \sum_{n=1}^{\infty} P_n = \sum_{n=1}^{\infty} V_n I_n \cos \varphi_n$$

❖ Reactive Power Q:
$$Q = \sum_{n=1}^{\infty} Q_n = \sum_{n=1}^{\infty} V_n I_n \sin \varphi_n$$

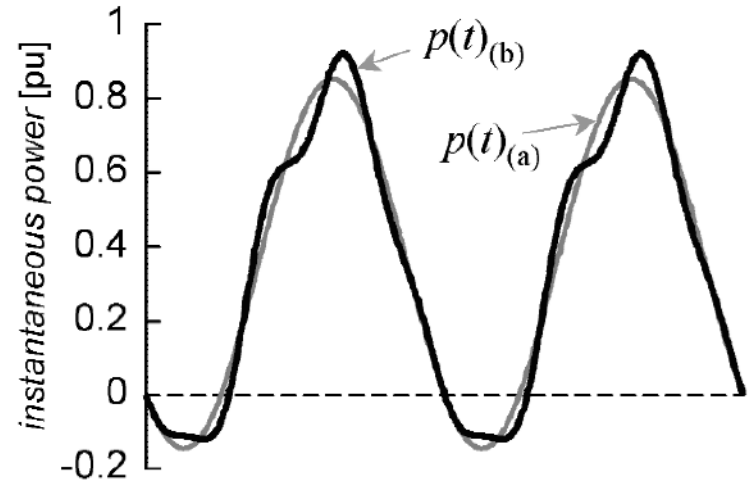
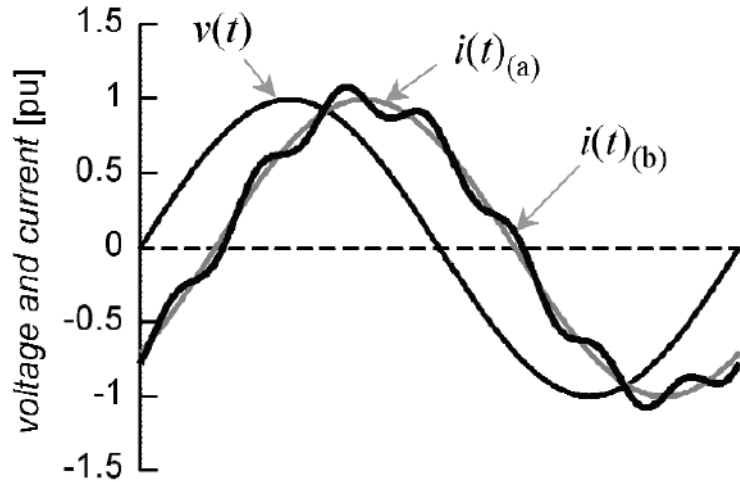
The reactive and apparent power are means to quantify the portion of power that does not realize in sinusoidal systems, but are proved inadequate to characterize the loss of power quality under nonsinusoidal conditions.

Another power definition was introduced for that purpose:

❖ Distortion Power D:

$$D^2 = S^2 - P^2 - Q^2$$

- ❖ A drawback of the theory is that a common instrument for power measurement based on the power definitions in the frequency domain cannot indicate easily a loss of power quality in practical cases.



Power Tetrahedron and Distortion Factor

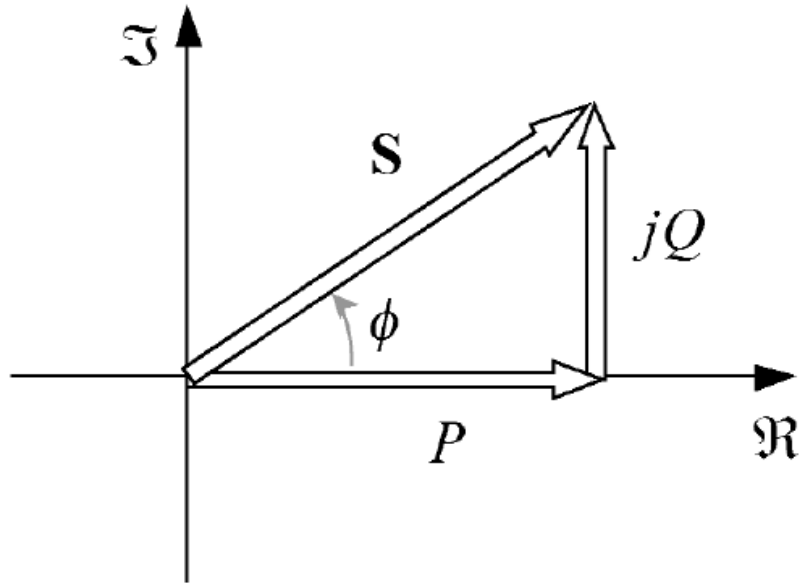
The presence of the distortion power “D” under nonsinusoidal conditions changes the way we realize the power flow into the grid.

❖ Complex Power:
$$\mathbf{S}_{\mathbf{PQ}} = P + jQ = \sum_{n=1}^{\infty} P_n + j \sum_{n=1}^{\infty} Q_n = \sum_{n=1}^{\infty} V_n I_n \cos \varphi_n + j \sum_{n=1}^{\infty} V_n I_n \sin \varphi_n$$

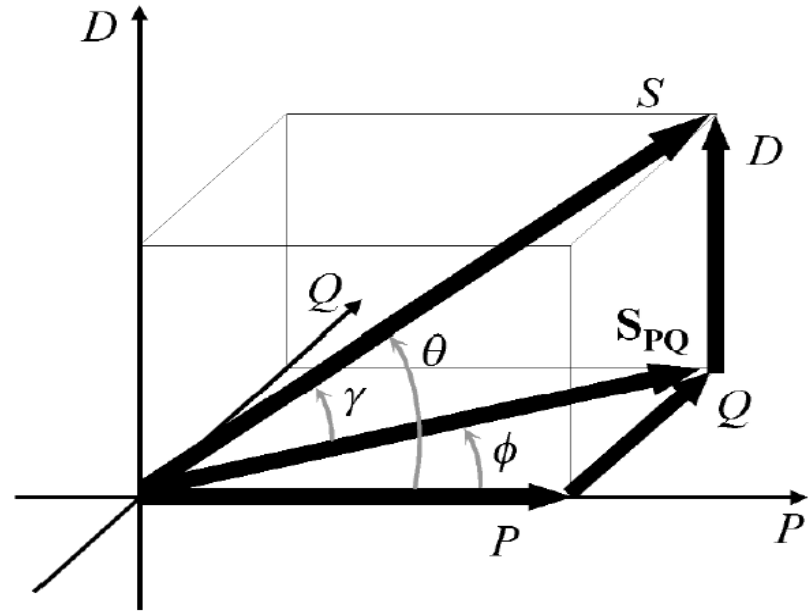
❖ Apparent Power:
$$S = VI = \sqrt{P^2 + Q^2 + D^2} = \sqrt{|\mathbf{S}_{\mathbf{PQ}}|^2 + D^2}$$

Graphic Difference Between Power Vector Diagrams

Sinusoidal



Nonsinusoidal



Characteristic Factors in Nonsinusoidal cases

- ❖ Power factor λ : $\lambda = \cos \theta = \frac{P}{S}$
- ❖ Displacement factor $\cos(\phi)$: $\cos \phi = \frac{P}{|\mathbf{S}_{PQ}|}$
- ❖ Distortion factor $\cos(\gamma)$: $\cos \gamma = \frac{|\mathbf{S}_{PQ}|}{S}$
- ❖ Valid Relation: $\lambda = \cos \theta = \frac{P}{S} = \cos \phi \cdot \cos \gamma$

Fryze's Power Definitions/Time Domain(1/2):

Equations for Fryze's approach:

❖ Active power P_w :
$$P_w = \frac{1}{T} \int_0^T p(t) dt = \frac{1}{T} \int_0^T v(t)i(t) dt = V_w I = VI_w$$

❖ Apparent power P_s :
$$P_s = VI$$

❖ Active power factor:
$$\lambda = \frac{P_w}{P_s} = \frac{P_w}{VI}$$

❖ Reactive power P_q :
$$P_q = \sqrt{P_s^2 - P_w^2} = V_q I = VI_q$$

Fryzes Power Definitions/Time Domain (2/2):

❖ Reactive power factor q : $\lambda_q = \sqrt{1 - \lambda^2}$

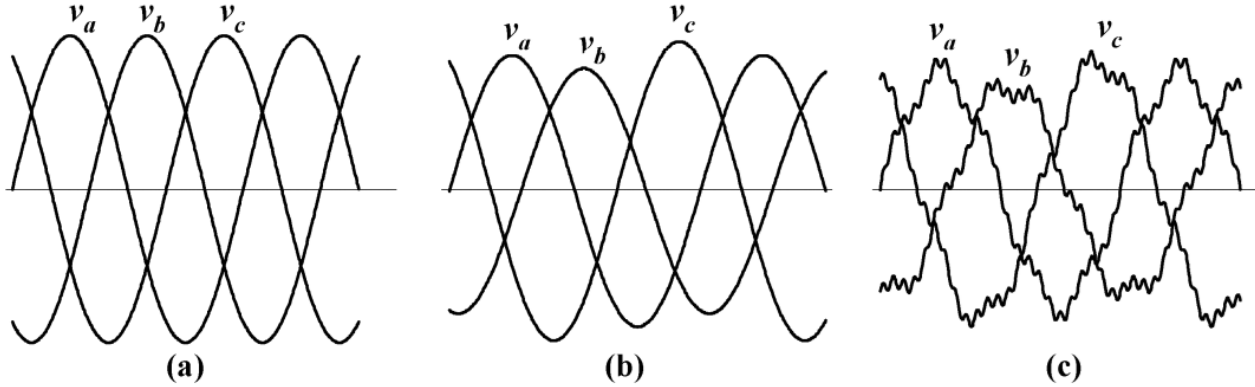
❖ Active Voltage (V_w) and Active Current (I_w): $V_w = \lambda \cdot V$ $I_w = \lambda \cdot I$

❖ Reactive Voltage (V_q) and Reactive Current (I_q): $V_q = \lambda_q \cdot V$ $I_q = \lambda_q \cdot I$

Electric Power in Three-Phase Systems

- ❖ Three-phase circuits are analyzed as a sum of three separate single-phase circuits. This is a simplification, especially in cases involving power electronic devices or nonlinear load.
- ❖ Only single-phase systems have been considered and classified as systems “under sinusoidal conditions” or “under nonsinusoidal conditions”
- ❖ Distorted system used to refer to a system under nonsinusoidal conditions
- ❖ Three-phase systems under sinusoidal conditions have a particular characteristic regarding the amplitude and phase angle of each phase voltage or line current. If the amplitudes are equal and the displacement angles between the phases are equal to 120 degrees, the three-phase system is said to be “balanced” or “symmetrical.” Otherwise, the three-phase system is “unbalanced” or “unsymmetrical.”

Classifications of Three-Phase Systems



- ❖ The above figures show examples of three-phase balanced (a), unbalanced voltages (b) and three-phase distorted and unbalanced voltages (c) that are obtained by superposing harmonic components on the unbalanced voltages given in (b).

Symmetrical Components Theory

- ❖ The three-phase unbalanced phasors ($\mathbf{V}_a, \mathbf{V}_b, \mathbf{V}_c$) are transformed into three other phasors: the positive-sequence phasor \mathbf{V}_+ , the negative-sequence phasor \mathbf{V}_- , and the zero-sequence phasor \mathbf{V}_0

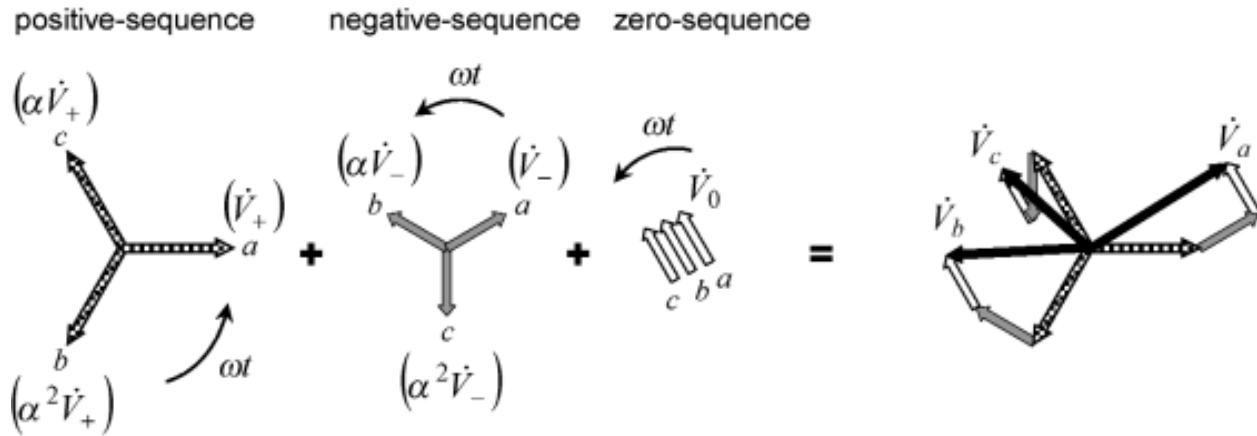
$$\begin{bmatrix} \dot{V}_0 \\ \dot{V}_+ \\ \dot{V}_- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} \dot{V}_a \\ \dot{V}_b \\ \dot{V}_c \end{bmatrix}$$

- ❖ The constant α is a complex number that acts as a 120° -phase shift operator

$$\alpha = 1 \angle 120^\circ = e^{j(2\pi/3)} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

- ❖ The zero-sequence phasor contributes to the phase voltages without 120° phase shifting
- ❖ The positive-sequence phasor contributes to the voltage in the form of an abc sequence
- ❖ The negative-sequence phasor in the form of an acb sequence
- ❖ A three-phase system consisting only of a positive-sequence component or only of a negative-sequence component is a balanced system

- ❖ For instance, three-phase, generic, periodic voltages and currents can be decomposed in Fourier series. From these series, the third (3ω), fifth (5ω), seventh (7ω), and subsequent harmonics in a three-phase generic voltage or current can be separated in groups of abc harmonics that are at a given frequency.



- ❖ Figure shows phasors in the same angular frequency, corresponding to the decomposition of an unbalanced three-phase system into symmetrical components.

Power in Balanced Three-Phase Systems

- ❖ Note that the reactive power Q does not describe the same phenomenon in three-phase and in single-phase circuits
- ❖ Three-phase voltage and line current that contain only the positive-sequence fundamental component (sinusoidal and balanced system) are given by these equations:

$$\begin{cases} v_a(t) = \sqrt{2}V_+ \sin(\omega t + \phi_{V_+}) \\ v_b(t) = \sqrt{2}V_+ \sin\left(\omega t + \phi_{V_+} - \frac{2\pi}{3}\right) \\ v_c(t) = \sqrt{2}V_+ \sin\left(\omega t + \phi_{V_+} + \frac{2\pi}{3}\right) \end{cases}$$

$$\begin{cases} i_a(t) = \sqrt{2}I_+ \sin(\omega t + \phi_{I_+}) \\ i_b(t) = \sqrt{2}I_+ \sin\left(\omega t + \phi_{I_+} - \frac{2\pi}{3}\right) \\ i_c(t) = \sqrt{2}I_+ \sin\left(\omega t + \phi_{I_+} + \frac{2\pi}{3}\right) \end{cases}$$

The three-phase instantaneous active power given by:

$$p_{3\phi}(t) = v_a(t)i_a(t) + v_b(t)i_b(t) + v_c(t)i_c(t) = p_a(t) + p_b(t) + p_c(t)$$



The three-phase active (average) power:

$$p_{3\phi}(t) = 3V_+I_+ \cos(\phi_{V_+} - \phi_{I_+}) = 3P$$

- ❖ The instantaneous active three-phase power is constant. It is time-independent
- ❖ In contrast, the single-phase active power contains a time dependent term
- ❖ The three-phase apparent power is defined as: $S_{3\phi} = 3S = 3V_+I_+$
- ❖ The three-phase complex power is defined as:

$$\begin{aligned}
 \mathbf{S}_{3\phi} &= 3\dot{V}_+ \dot{I}_+^* = 3V_+ \angle \phi_{V_+} I_+ \angle -\phi_{I_+} \\
 &= 3V_+I_+ \underbrace{\cos(\phi_{V_+} - \phi_{I_+})}_{P_{3\phi}} + j3V_+I_+ \underbrace{\sin(\phi_{V_+} - \phi_{I_+})}_{Q_{3\phi}}
 \end{aligned}$$

- ❖ The three-phase reactive power is defined as: $Q_{3\phi} = 3Q = 3V_+I_+ \sin(\phi_{V_+} - \phi_{I_+})$

Also can be derived from the imaginary part of the definition of this three-phase complex power

Power in Three-Phase Unbalanced Systems

- ❖ Based on rms values of voltage and current there are two definitions of three-phase apparent power:

i) “Per phase” calculation:

$$S_{3\phi} = \sum_k S_k = \sum_k V_k I_k, \quad k = (a, b, c)$$

ii) “aggregate rms value” calculation:

$$S_{\Sigma} = \sqrt{\sum_k V_k^2} \sqrt{\sum_k I_k^2}, \quad k = (a, b, c)$$

- ❖ Under nonsinusoidal or unbalanced conditions S_{Σ} less than or equal to $S_{3\phi}$, while for a balanced and sinusoidal case S_{Σ} equal to $S_{3\phi}$
- ❖ The (rms) aggregate voltage and current are:

$$V_{\Sigma} = \sqrt{V_a^2 + V_b^2 + V_c^2} \quad \text{and} \quad I_{\Sigma} = \sqrt{I_a^2 + I_b^2 + I_c^2}$$