



# SIX-STEP VSI – 120° CONDUCTION MODE

WAVEFORM & HARMONIC ANALYSIS

## Team 4:

Kiosi Anastasia  
Papazioga Dimitra  
Tichalas Andreas

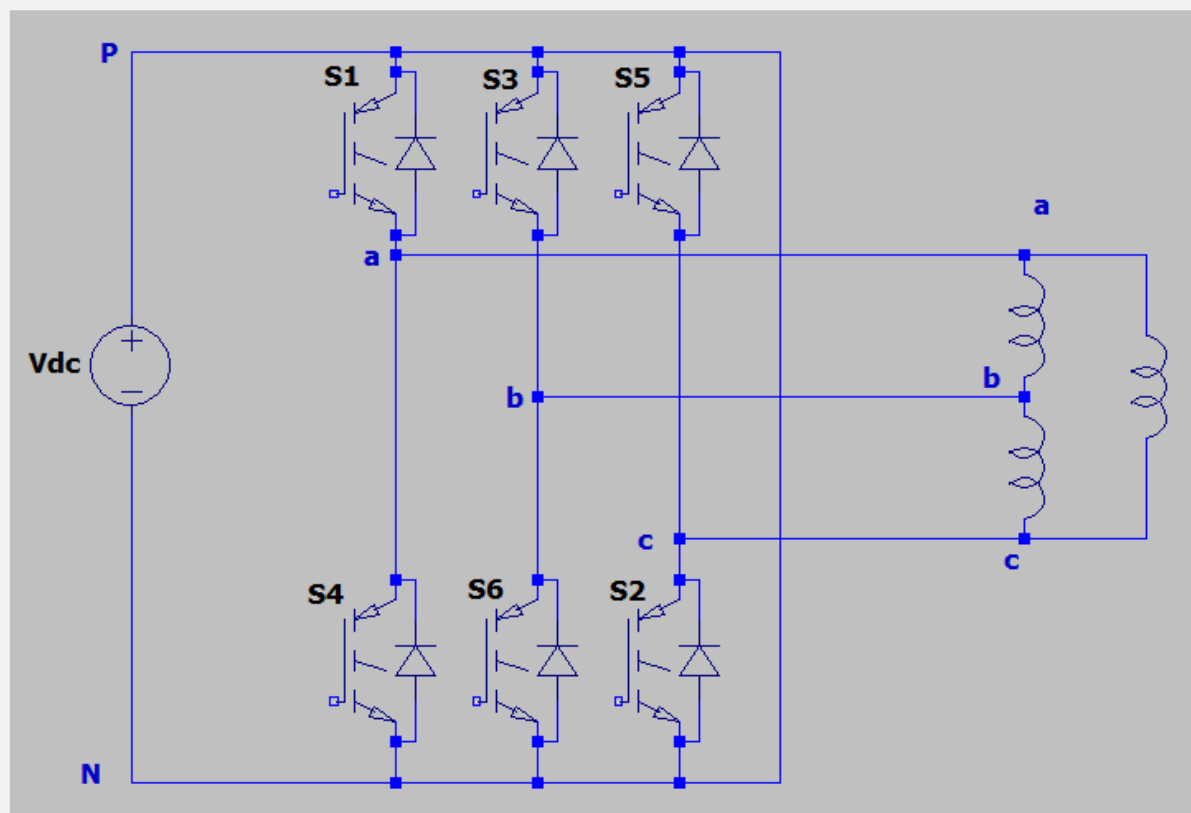
## Team 5:

Ioakeimidou Panagiota  
Papadopoulos Anastasios  
Touna Panagiota

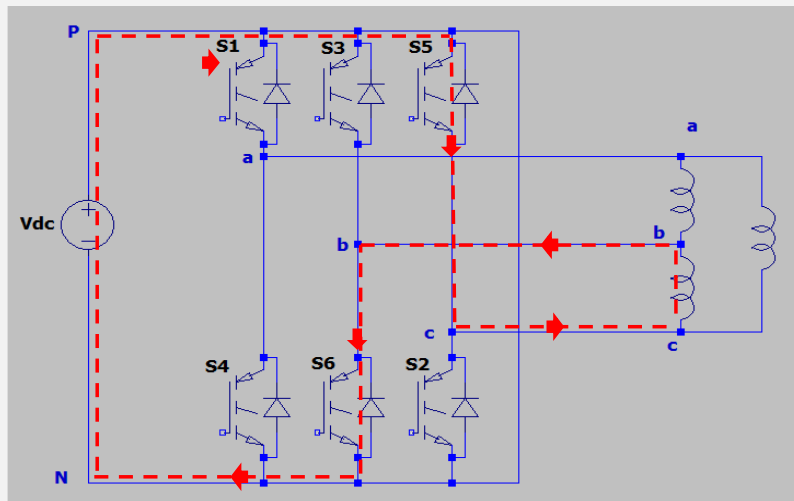
## Team 6:

Fragkiadakis Michail  
Kontidou Athanasia Niovi

# CIRCUIT



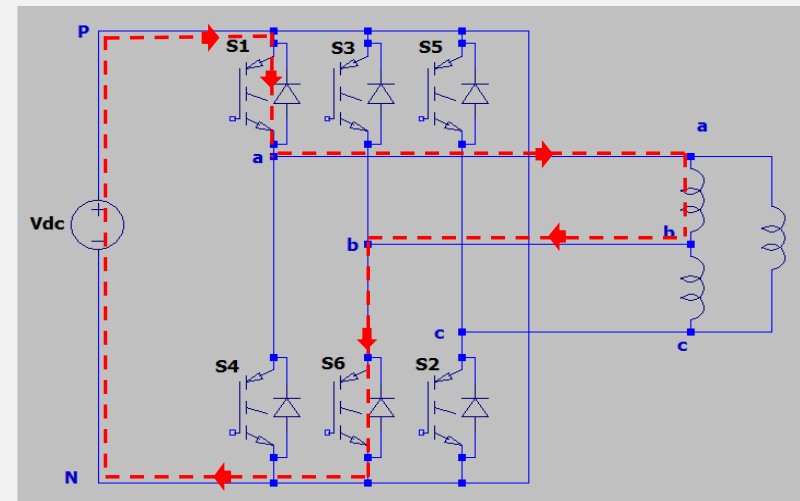
# STATES



**56 (S5 and S6 conduct)**

$$V_{bc} = -V_{dc}$$

$$V_{ab} = V_{ca} = V_{dc}/2$$

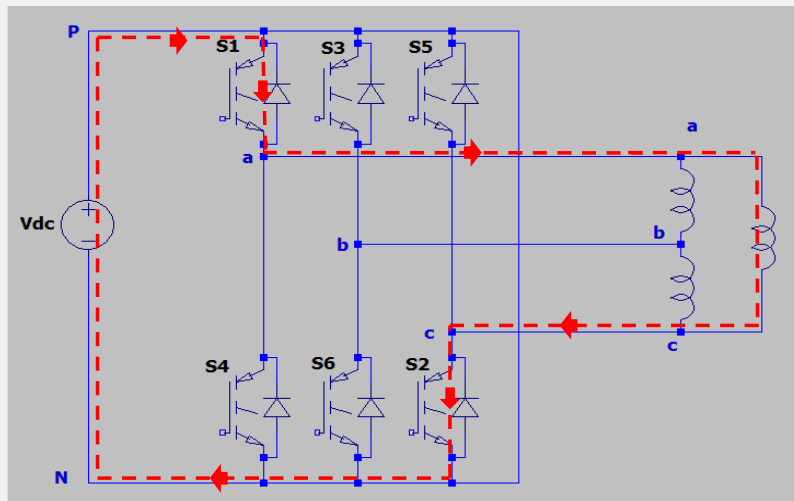


**61 (S6 and S1 conduct)**

$$V_{ab} = V_{dc}$$

$$V_{ca} = V_{bc} = -V_{dc}/2$$

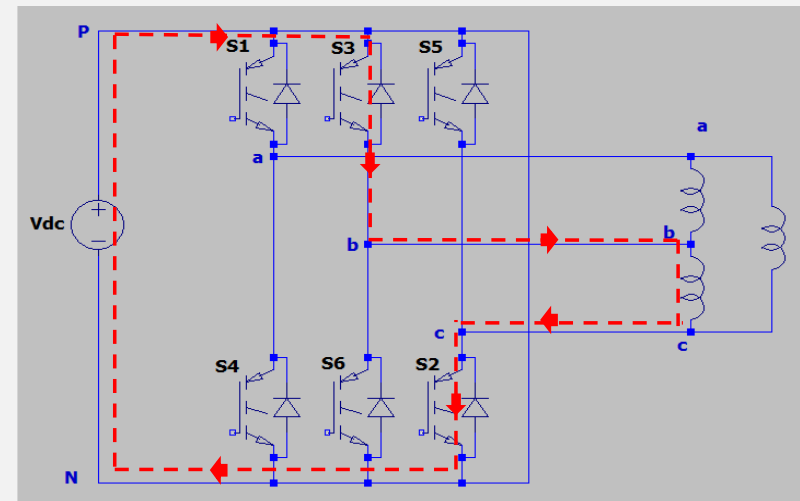
# STATES



**12(S1 and S2 conduct)**

$$V_{ca} = -V_{dc}$$

$$V_{bc} = V_{ab} = V_{dc}/2$$

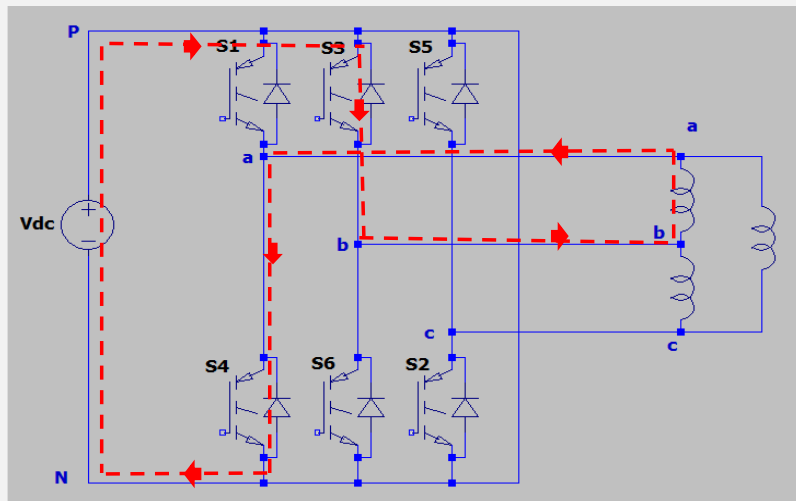


**23(S2 and S3 conduct)**

$$V_{bc} = V_{dc}$$

$$V_{ab} = V_{ca} = -V_{dc}/2$$

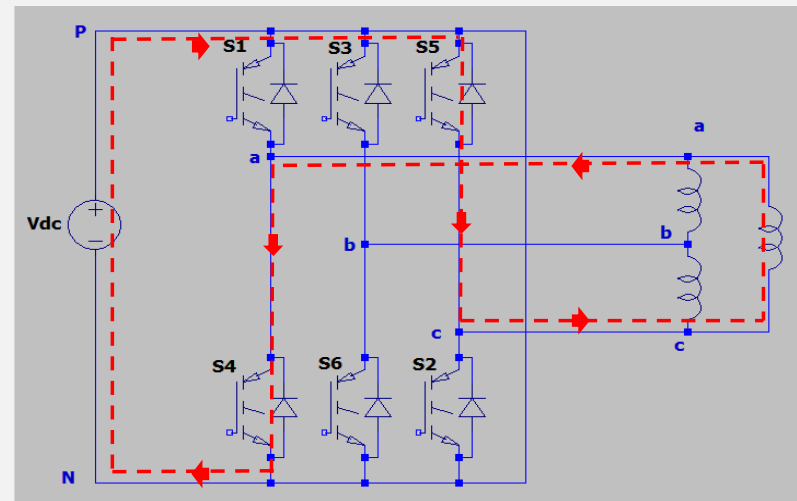
# STATES



**34**(S3 and S4 conduct)

$$V_{ab} = -V_{dc}$$

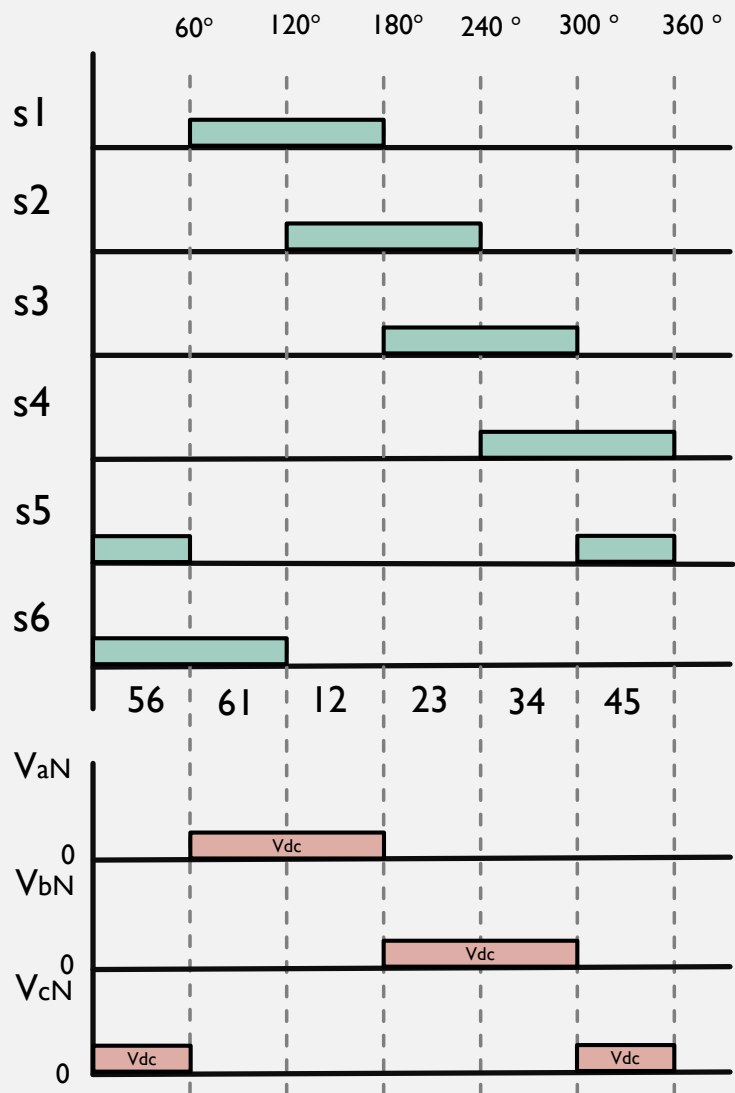
$$V_{ca} = V_{bc} = V_{dc}/2$$



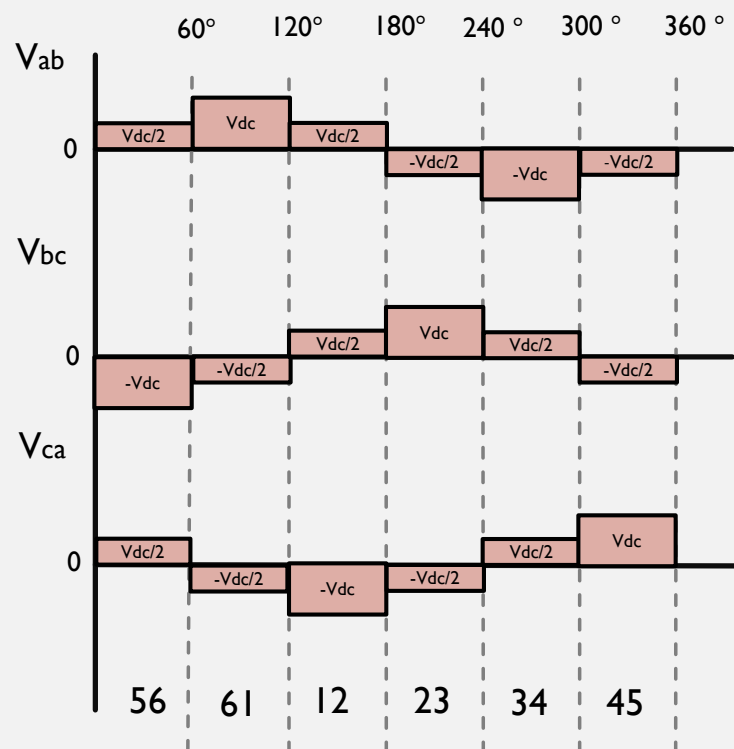
**45**(S4 and S5 conduct)

$$V_{ca} = V_{dc}$$

$$V_{bc} = V_{ab} = -V_{dc}/2$$



### Delta 120° from the start



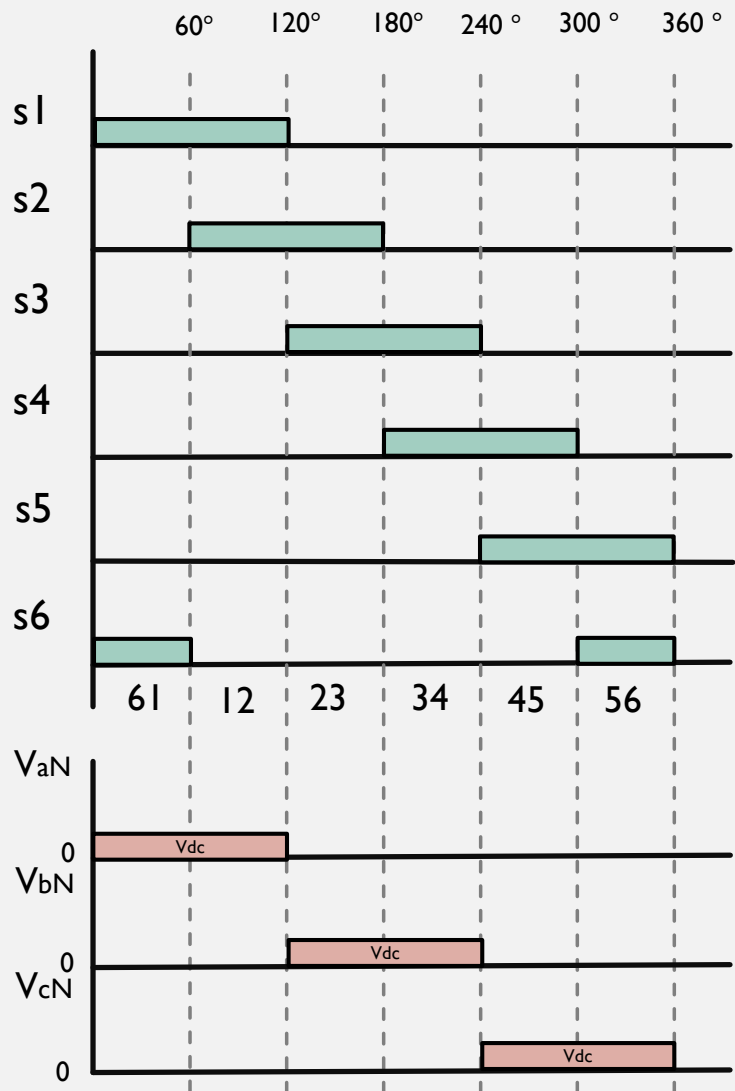
### **Delta-connection:**

$$V_{\text{phase}} = V_{\text{line}}$$

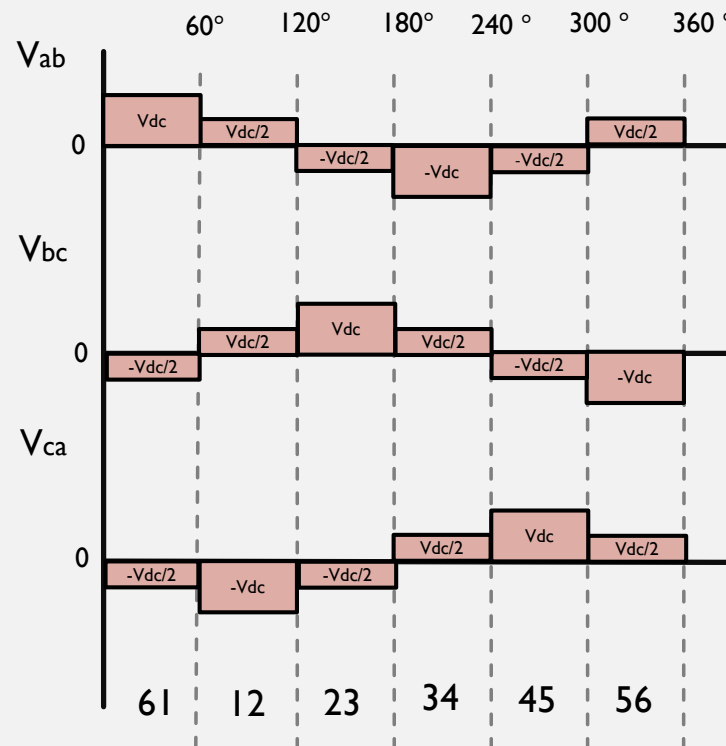
$$V_{\text{an}} = V_{\text{ab}}$$

$$V_{\text{bn}} = V_{\text{bc}}$$

$$V_{\text{cn}} = V_{\text{ca}}$$



### Delta 120° from the end



### **Delta-connection:**

$$V_{\text{phase}} = V_{\text{line}}$$

$$V_{an} = V_{ab}$$

$$V_{bn} = V_{bc}$$

$$V_{cn} = V_{ca}$$

# HARMONIC ANALYSIS

➤ Delta 120° from the start

$T_o = T$ ,  $\omega_o = 2\pi/T$ ,  $\omega_n = n\omega_o$  with  $V_{ab}$  odd function

$$b_n = \frac{2}{T_o} \int_0^{T_o} V_{ab(t)} \sin(\omega_n t) dt = \frac{4}{T} \int_0^{T/2} V_{ab(t)} \sin(\omega_n t) dt$$

$$= \frac{4}{T} \left[ \int_0^{T/6} \frac{V_d}{2} \sin(\omega_n t) dt + \int_{T/6}^{T/3} V_d \sin(\omega_n t) dt + \int_{T/3}^{T/2} \frac{V_d}{2} \sin(\omega_n t) dt \right]$$

$$= \frac{4}{T} \left\{ \frac{V_d}{2} \left[ \frac{-\cos(\omega_n t)}{\omega_n} \right]_0^{T/6} + V_d \left[ \frac{-\cos(\omega_n t)}{\omega_n} \right]_{T/6}^{T/3} + \frac{V_d}{2} \left[ \frac{-\cos(\omega_n t)}{\omega_n} \right]_{T/3}^{T/2} \right\}$$

$$= \frac{4V_d}{T\omega_n} \left\{ \frac{1}{2} [-\cos(\omega_n t)]_0^{T/6} + [-\cos(\omega_n t)]_{T/6}^{T/3} + \frac{1}{2} [-\cos(\omega_n t)]_{T/3}^{T/2} \right\}$$

Because of the odd symmetry

=>  $a_n=0$



1.	0	S1	S1	0	0	0
2.	0	0	S2	S2	0	0
3.	0	0	0	S3	S3	0
4.	0	0	0	0	S4	S4
5.	S5	0	0	0	0	S5
6.	S6	S6	0	0	0	0
	0-60°	60°-120°	120°-180°	180°-240°	240°-300°	300°-360°
$V_{ab}$	$V_d/2$	$V_d$	$V_d/2$	$-V_d/2$	$-V_d$	$-V_d/2$
$V_{bc}$	$-V_d$	$-V_d/2$	$V_d/2$	$V_d$	$V_d/2$	$-V_d/2$
$V_{ca}$	$V_d/2$	$-V_d/2$	$-V_d$	$-V_d/2$	$V_d/2$	$V_d$



## HARMONIC ANALYSIS

$$\begin{aligned} &= \frac{2V_d}{n\pi} \left\{ \frac{1}{2} \left[ -\cos\left(\frac{2n\pi T}{T} \frac{1}{6}\right) + 1 \right] + \left[ -\cos\left(\frac{2n\pi T}{T} \frac{1}{3}\right) + \cos\left(\frac{2n\pi T}{T} \frac{1}{6}\right) \right] + \frac{1}{2} \left[ -\cos\left(\frac{2n\pi T}{T} \frac{1}{2}\right) + \cos\left(\frac{2n\pi T}{T} \frac{1}{3}\right) \right] \right\} \\ &= \frac{2V_d}{n\pi} \left\{ \frac{1}{2} \left[ -\cos\left(\frac{n\pi}{3}\right) + 1 \right] + \left[ -\cos\left(\frac{2n\pi}{3}\right) + \cos\left(\frac{n\pi}{3}\right) \right] + \frac{1}{2} \left[ -\cos(n\pi) + \cos\left(\frac{2n\pi}{3}\right) \right] \right\} \\ &= \frac{2V_d}{n\pi} \left( -\frac{1}{2} \cos\left(\frac{n\pi}{3}\right) + \cos\left(\frac{n\pi}{3}\right) - \cos\left(\frac{2n\pi}{3}\right) + \frac{1}{2} \cos\left(\frac{2n\pi}{3}\right) + \frac{1}{2} - \frac{1}{2} \cos(n\pi) \right) \\ &= \frac{2V_d}{n\pi} \left( \frac{1}{2} \cos\left(\frac{n\pi}{3}\right) - \frac{1}{2} \cos\left(\frac{2n\pi}{3}\right) + \frac{1}{2} - \frac{1}{2} \cos(n\pi) \right) \\ &= \frac{V_d}{n\pi} \left( \cos\left(\frac{n\pi}{3}\right) - \cos\left(\frac{2n\pi}{3}\right) + 1 - \cos(n\pi) \right) \end{aligned}$$

# HARMONIC ANALYSIS

$$b_1 = \frac{3V_d}{\pi} = 0,9549V_d$$

$$b_2 = b_3 = b_4 = 0$$

$$b_5 = \frac{3V_d}{5\pi} = 0,19099V_d$$

$$b_6 = 0$$

$$b_7 = \frac{3V_d}{7\pi} = 0,13642V_d$$

$$\dots$$

$$b_{11} = \frac{3V_d}{11\pi} = 0,0868V_d$$


...

$$b_n = \sum_{n=6k\pm 1}^{\infty} \frac{3V_d}{n\pi} \quad k = 0,1,2, \dots$$

$$V_{ab} = \sum_{n=6k\pm 1}^{\infty} \frac{3V_d}{n\pi} \sin(n\omega_o t)$$

$$V_{bc} = \sum_{n=6k\pm 1}^{\infty} \frac{3V_d}{n\pi} \sin\left(n\omega_o t + \frac{2\pi}{3}\right)$$

$$V_{ca} = \sum_{n=6k\pm 1}^{\infty} \frac{3V_d}{n\pi} \sin\left(n\omega_o t - \frac{2\pi}{3}\right)$$

- ✓ Because of the 90° symmetry the even components will be equal to zero. 
- ✓ In addition, there is no neutral connection, so the ac line currents contain no dc or triplen harmonics

# HARMONIC ANALYSIS

➤ Delta 120° from the end

$$V_{ab} = \sum_{n=6k \pm 1}^{\infty} \frac{3V_d}{n\pi} \sin\left(n\omega_0 t + \frac{n\pi}{3}\right)$$

➤ RMS Value for Vab with 120° pulses

$$\begin{aligned} V_{ab(rms)} &= \sqrt{\frac{1}{T} \int_0^T V_{ab}^2 dt} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} V_{ab}^2 d\theta} = \sqrt{\frac{2}{2\pi} \int_0^{\pi} V_{ab}^2 d\theta} = \sqrt{\frac{1}{\pi} \int_0^{\pi} V_{ab}^2 d\theta} \\ &= \sqrt{\frac{1}{\pi} \left[ \int_0^{\frac{\pi}{3}} \frac{V_d^2}{4} d\theta + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} V_d^2 d\theta + \int_{\frac{2\pi}{3}}^{\pi} \frac{V_d^2}{4} d\theta \right]} = \sqrt{\frac{1}{\pi} \left( \frac{V_d^2}{4} \left[ \frac{\pi}{3} - 0 \right] + V_d^2 \left[ \frac{2\pi}{3} - \frac{\pi}{3} \right] + \frac{V_d^2}{4} \left[ \pi - \frac{2\pi}{3} \right] \right)} \\ &= \sqrt{\frac{1}{\pi} \left( \frac{V_d^2}{4} \frac{\pi}{3} + V_d^2 \frac{\pi}{3} + \frac{V_d^2}{4} \frac{\pi}{3} \right)} = \sqrt{\frac{1}{\pi} \left( \frac{6\pi V_d^2}{12} \right)} = \frac{V_d}{\sqrt{2}} \end{aligned}$$

1.	S1	S1	0	0	0	0
2.	0	S2	S2	0	0	0
3.	0	0	S3	S3	0	0
4.	0	0	0	S4	S4	0
5.	0	0	0	0	S5	S5
6.	S6	0	0	0	0	S6
	0-60°	60°-120°	120°-180°	180°-240°	240°-300°	300°-360°
$V_{ab}$	$V_d$	$V_d/2$	$-V_d/2$	$-V_d$	$-V_d/2$	$V_d/2$
$V_{bc}$	$-V_d/2$	$V_d/2$	$V_d$	$V_d/2$	$-V_d/2$	$-V_d$
$V_{ca}$	$-V_d/2$	$-V_d$	$-V_d/2$	$V_d/2$	$V_d$	$V_d/2$

# HARMONIC ANALYSIS

➤ THD<sub>v</sub>

$$V_{ab,1,rms} = \frac{3V_{dc}}{\sqrt{2}\pi}$$

$$THD_v = \frac{\sqrt{V_{ab,rms}^2 - V_{ab,1,rms}^2}}{V_{ab,1,rms}} = \frac{\sqrt{\frac{V_{dc}^2}{2} - \frac{9V_{dc}^2}{2\pi^2}}}{\frac{3V_{dc}}{\sqrt{2}\pi}} = \frac{\sqrt{\frac{V_{dc}^2}{2} \left(1 - \frac{9}{\pi^2}\right)}}{\frac{3V_{dc}}{\sqrt{2}\pi}} = \frac{\sqrt{\left(1 - \frac{9}{\pi^2}\right)}}{\frac{3}{\pi}} = 0.31 \text{ ή } 31\%$$



**Thank you!**