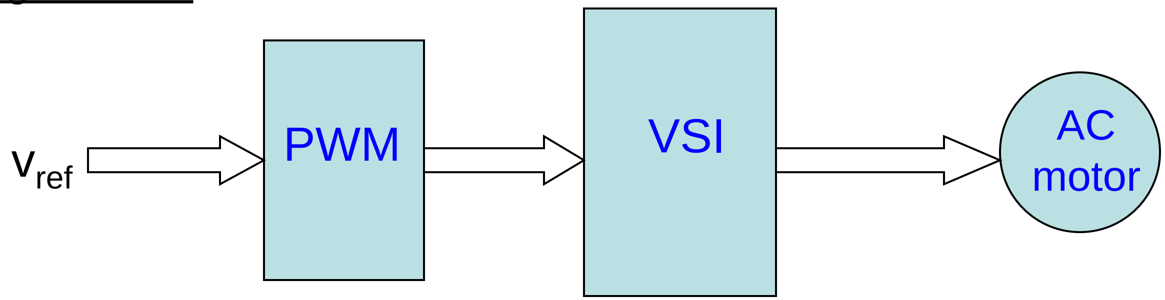
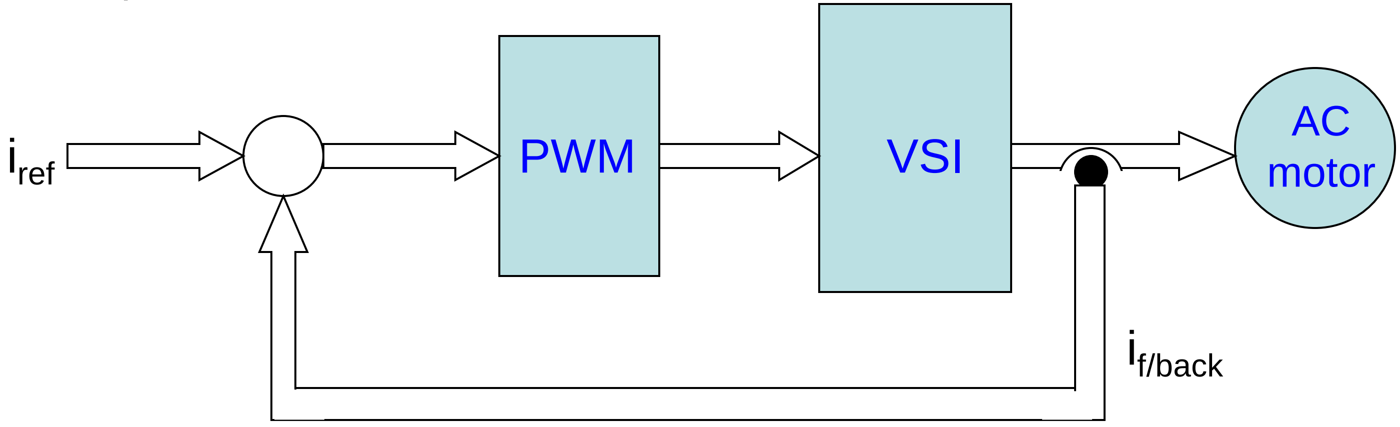
# Space Vector Modulation (SVM)

# PWM – Voltage Source Inverter

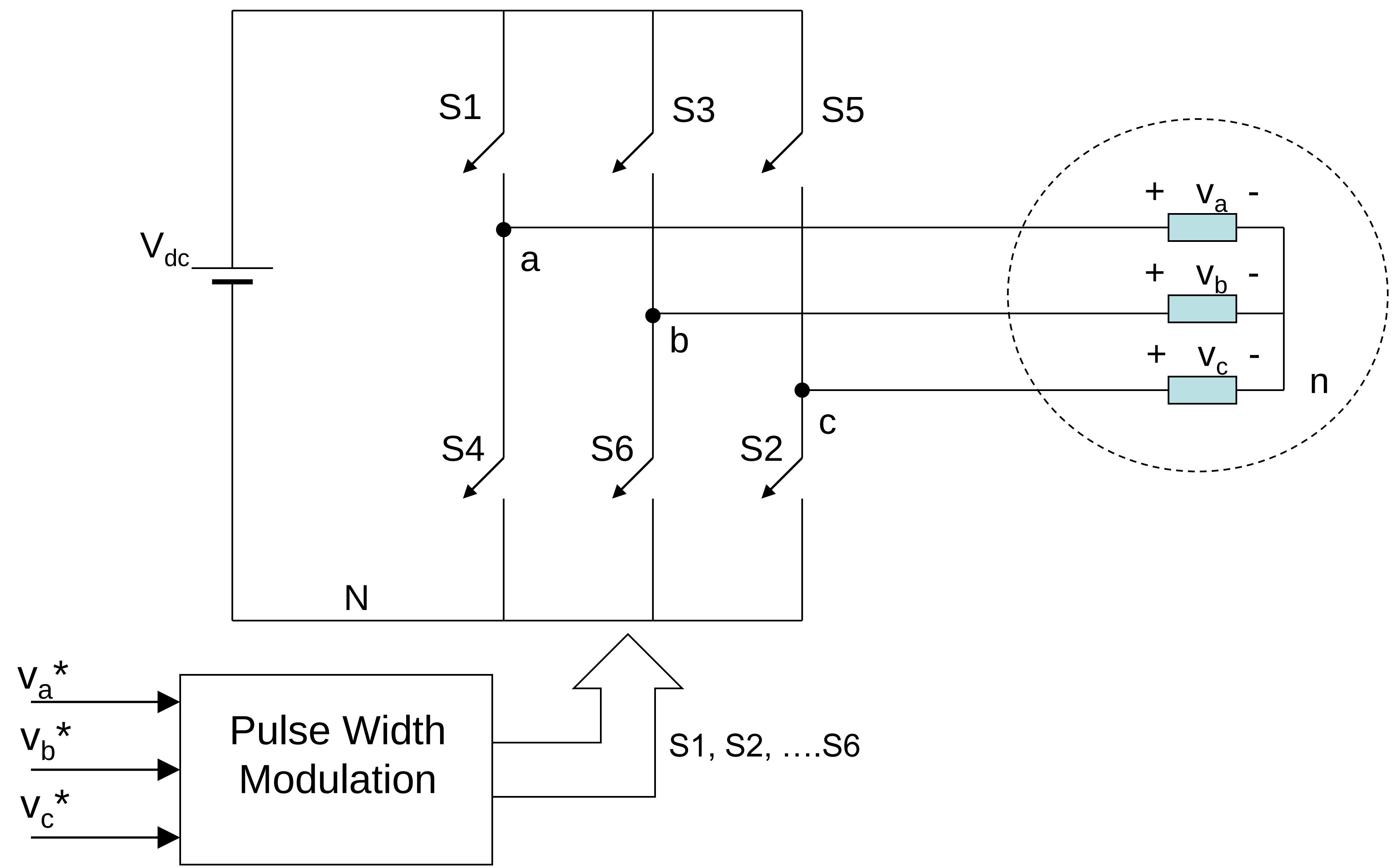
Open loop voltage control




Closed loop current-control

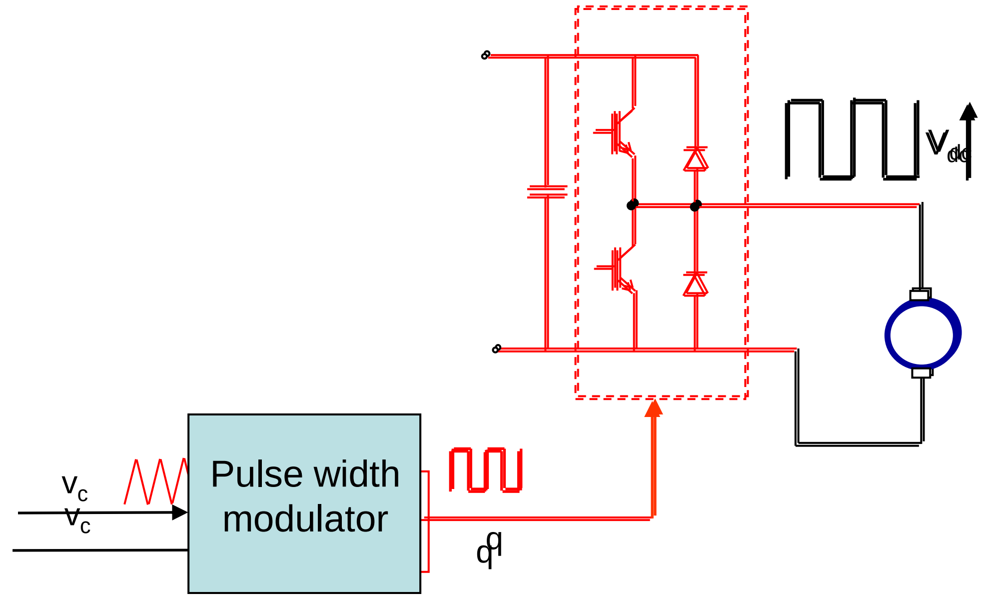


# PWM – Voltage Source Inverter



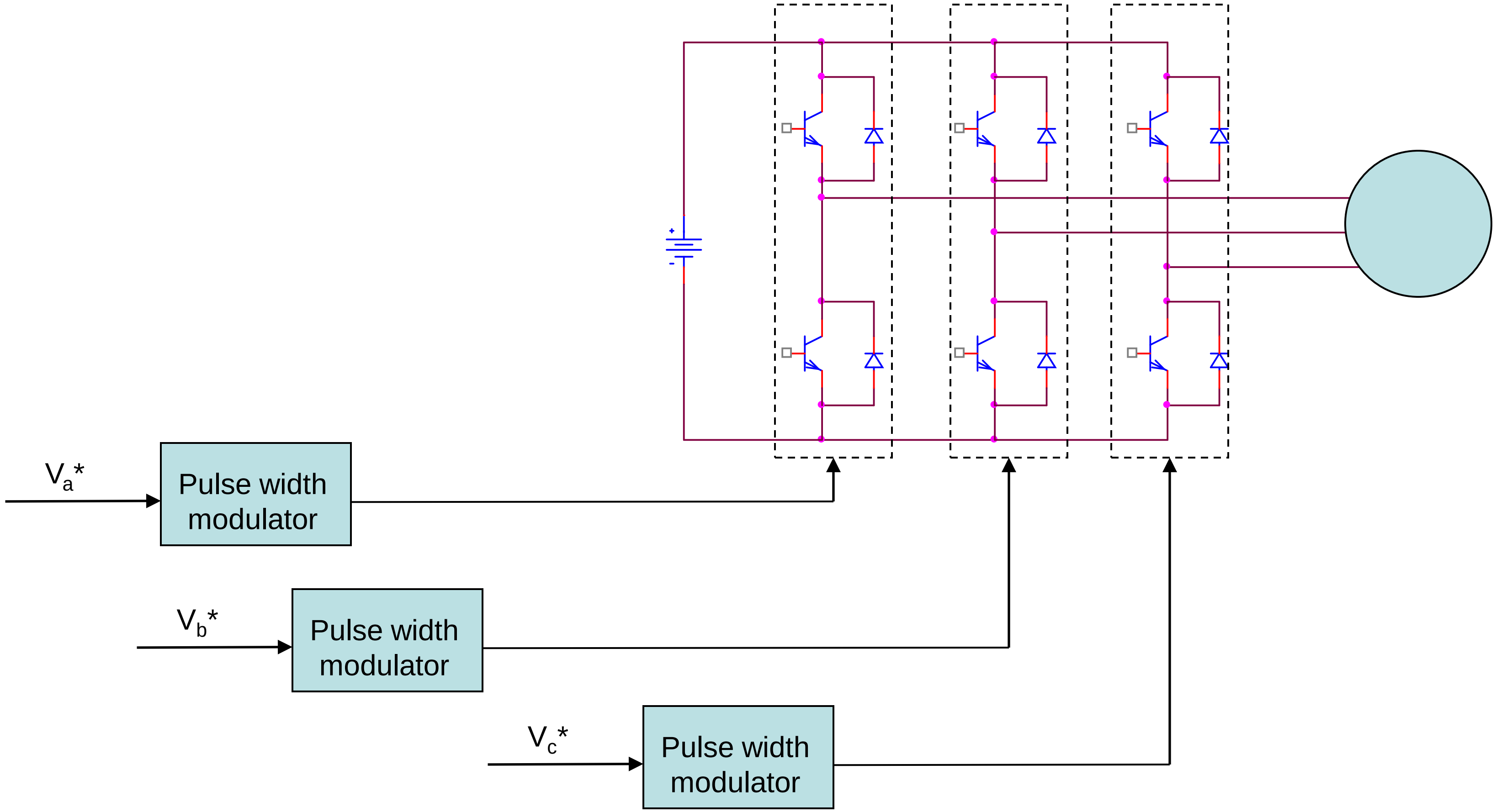
## PWM – Voltage Source Inverter

PWM – single phase



# PWM – Voltage Source Inverter

PWM – extended to 3-phase → Sinusoidal PWM



# Space Vector Modulation

*Definition:*

Space vector representation of a three-phase quantities $x_a(t)$, $x_b(t)$ and $x_c(t)$ with **space distribution** of 120$^o$ apart is given by:

$$\bar{x} = \frac{2}{3}\left(x_a(t) + a x_b(t) + a^2 x_c(t)\right)$$

$a = e^{j2\pi/3} = \cos(2\pi/3) + j\sin(2\pi/3)$
$a^2 = e^{j4\pi/3} = \cos(4\pi/3) + j\sin(4\pi/3)$

*x – can be a voltage, current or flux and does not necessarily has to be sinusoidal*

## Space Vector Modulation

$$\bar{x} = \frac{2}{3}\left(x_a(t) + a x_b(t) + a^2 x_c(t)\right)$$

## Space Vector Modulation

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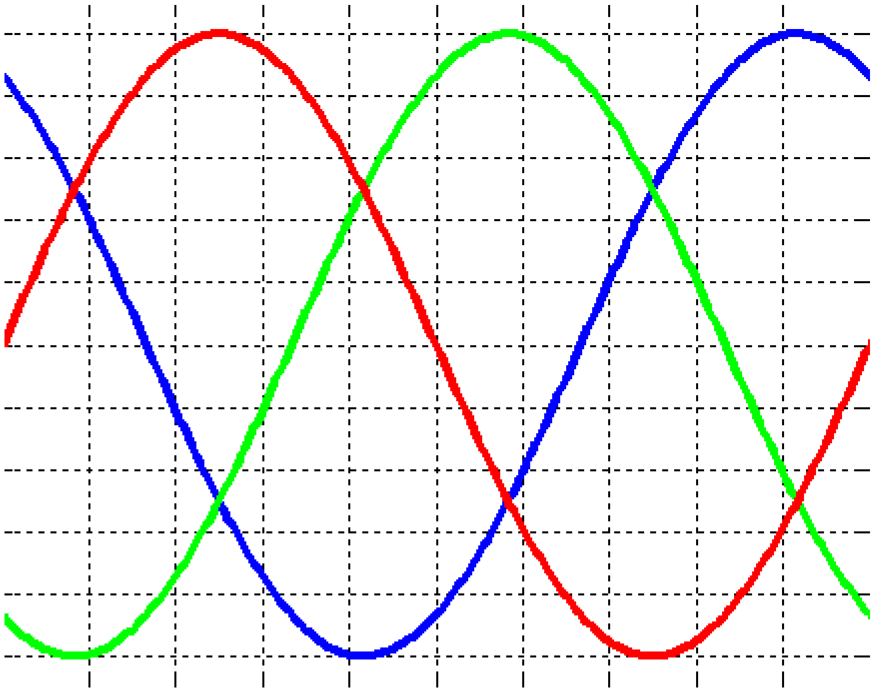
Let's consider 3-phase sinusoidal voltage:

$v_a(t) = V_m \sin(\omega t)$

$v_b(t) = V_m \sin(\omega t - 120^o)$

$v_c(t) = V_m \sin(\omega t + 120^o)$



## Space Vector Modulation

$$\bar{v} = \frac{2}{3}\left(v_a(t) + a v_b(t) + a^2 v_c(t)\right)$$

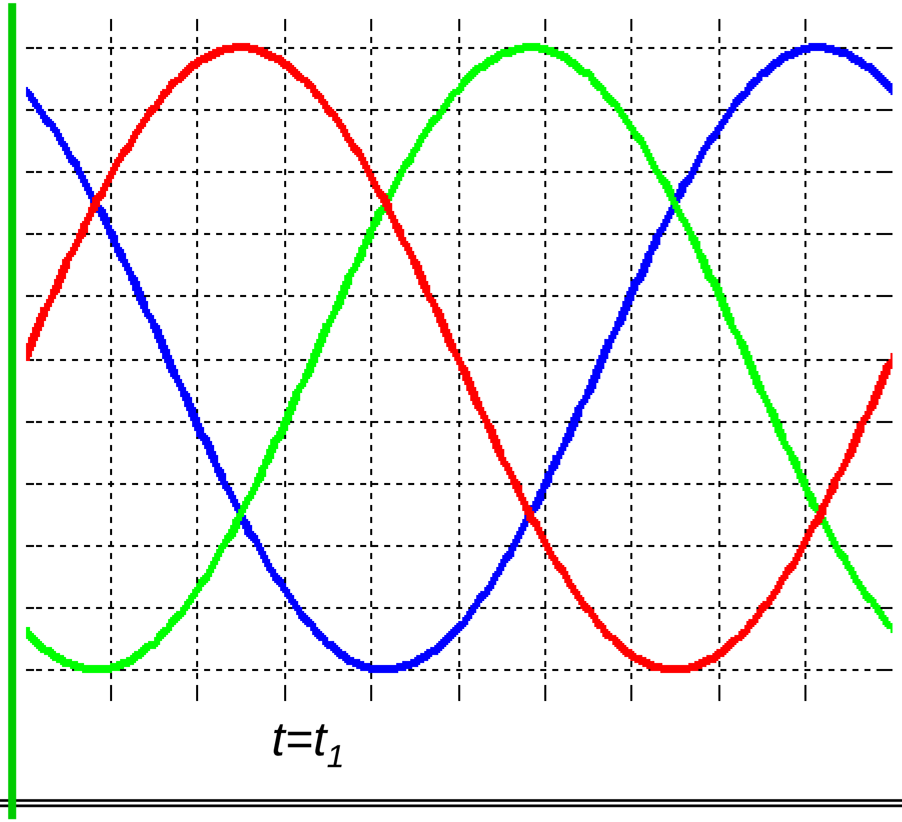
Let's consider 3-phase sinusoidal voltage:

At $t = t_1$, $\omega t = (3/5)\pi$ (= 108°)

$v_a = 0.9511(V_m)$

$v_b = -0.208(V_m)$

$v_c = -0.743(V_m)$



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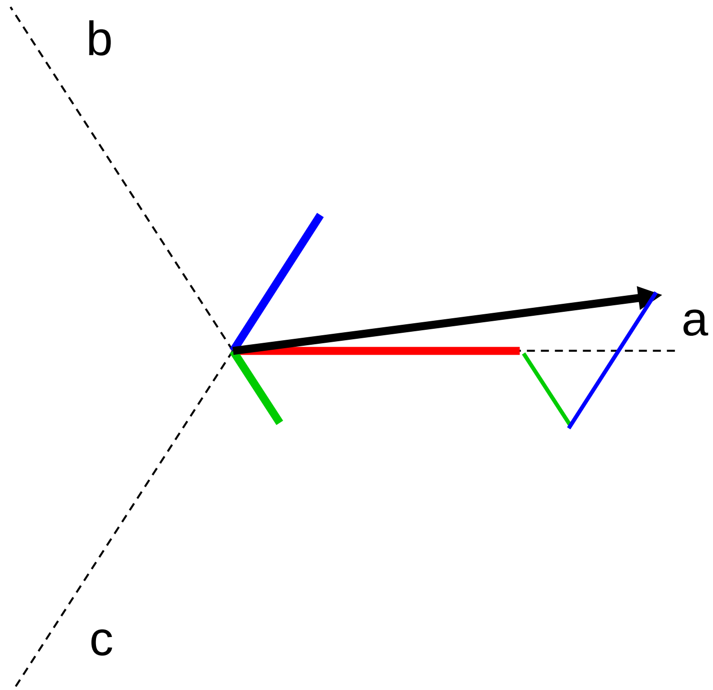
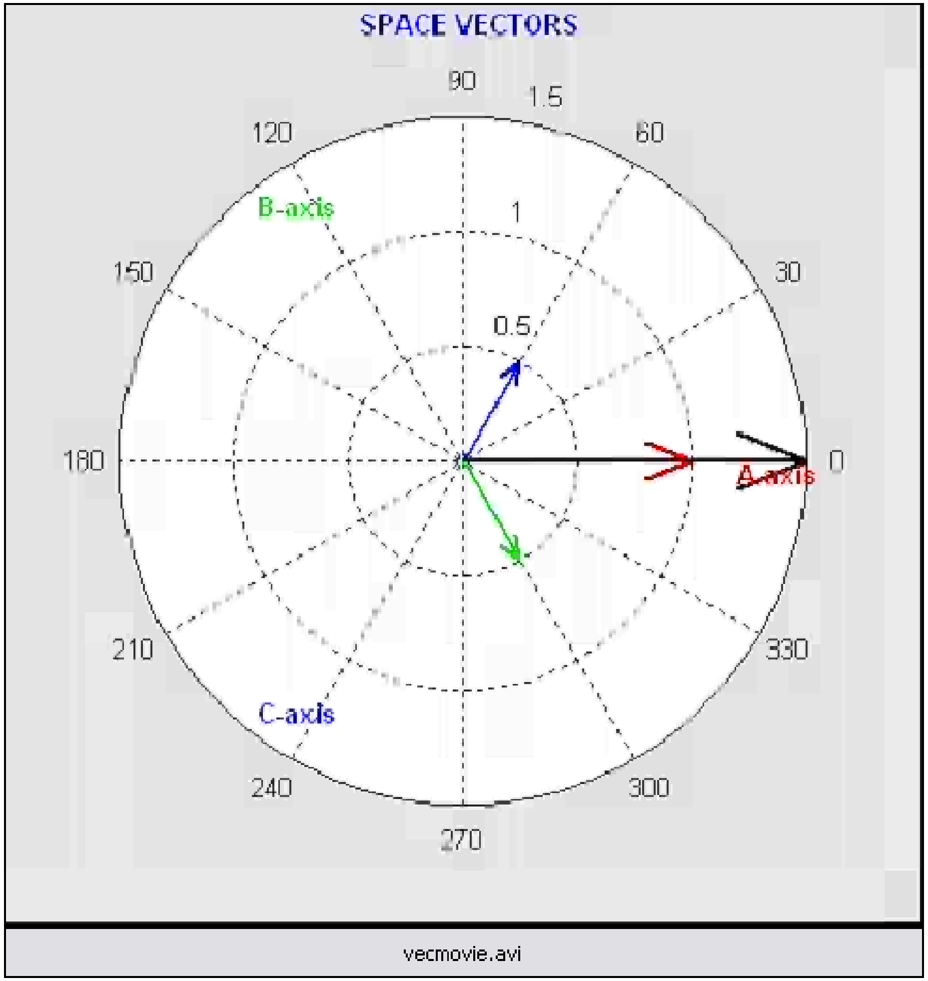




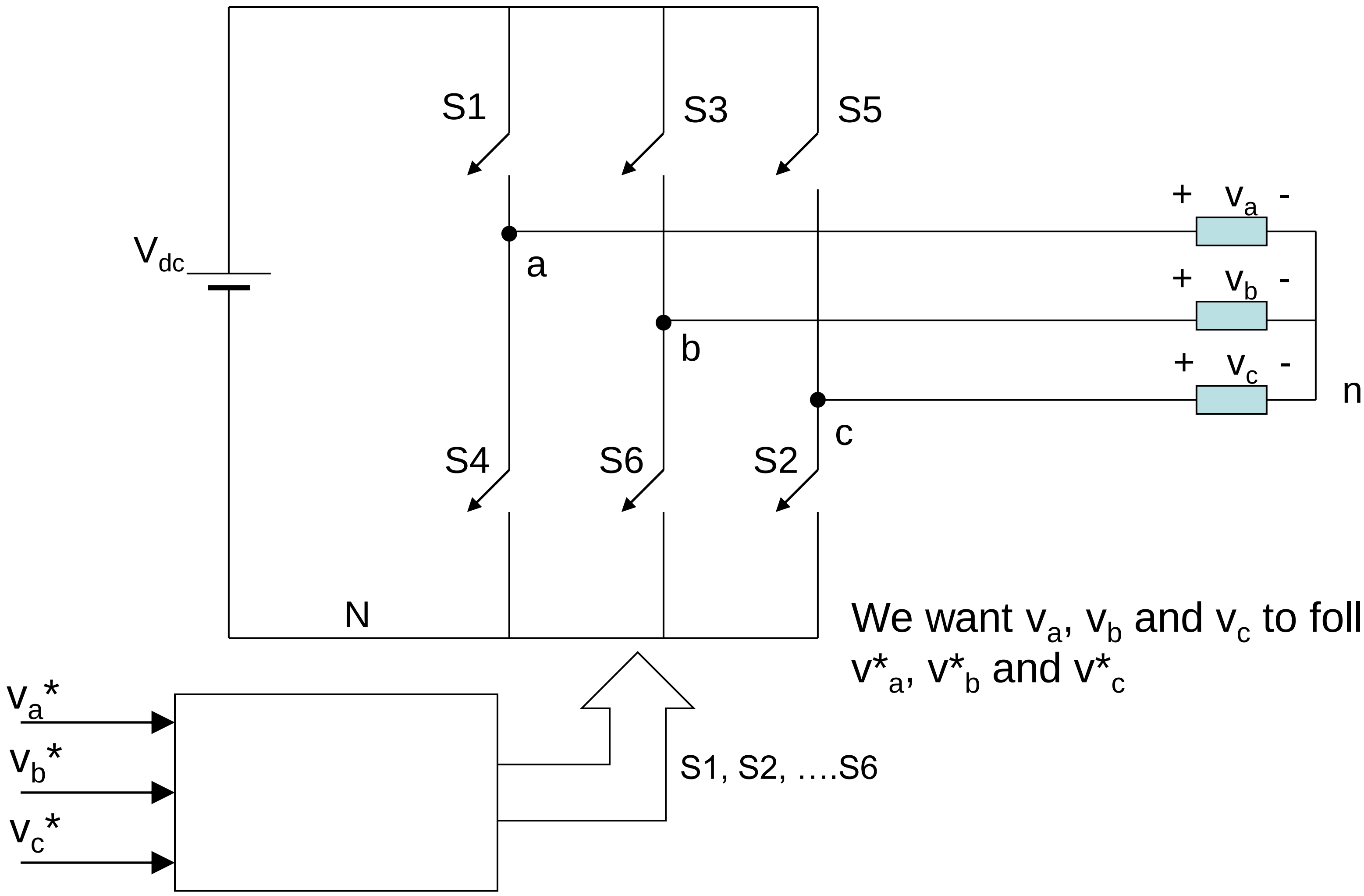

Three phase quantities vary sinusoidally with time (frequency *f*)

$\Rightarrow$ space vector rotates at $2\pi f$, magnitude $V_m$

## Space Vector Modulation

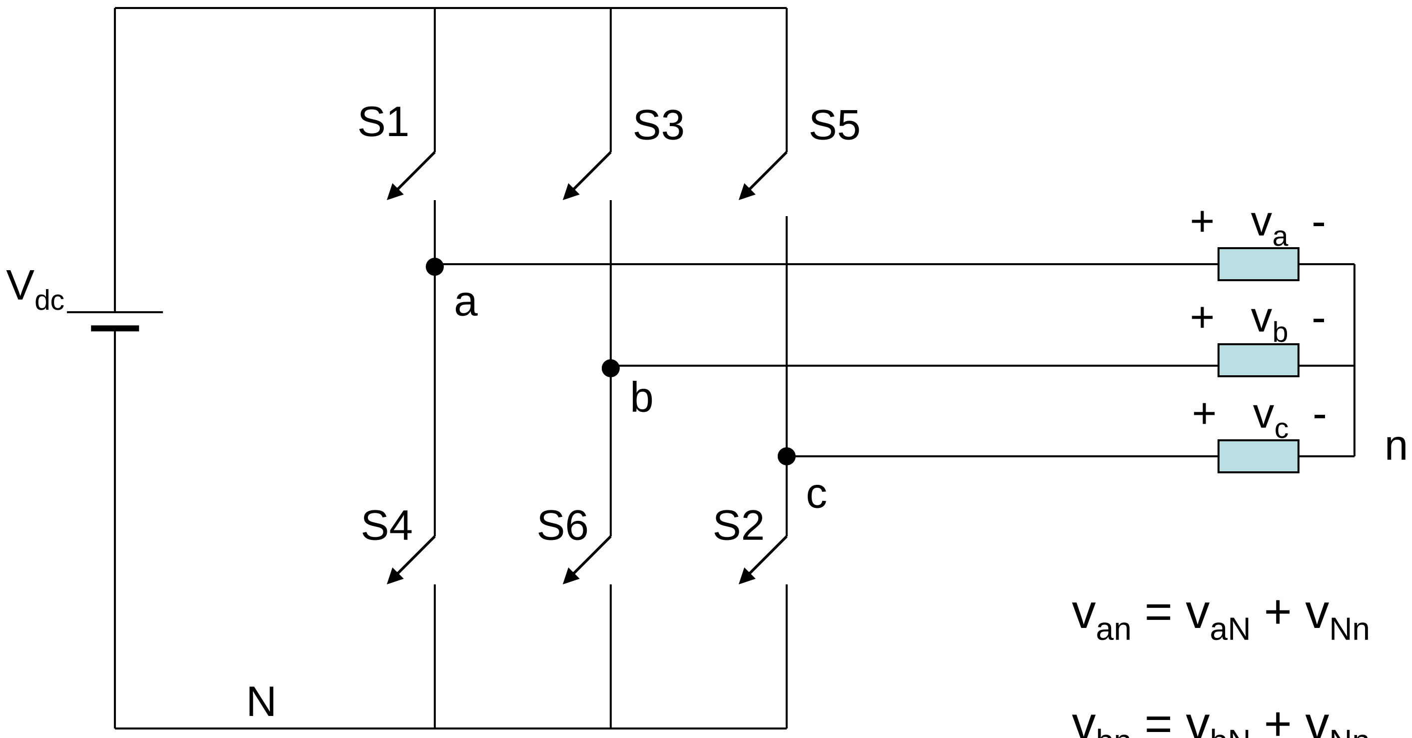
How could we synthesize sinusoidal voltage using VSI ?

# Space Vector Modulation




We want $v_a$, $v_b$ and $v_c$ to follow $v^*_a$, $v^*_b$ and $v^*_c$

## Space Vector Modulation




$$v_{an} = v_{aN} + v_{Nn}$$

$$v_{bn} = v_{bN} + v_{Nn}$$

$$v_{cn} = v_{cN} + v_{Nn}$$

From the definition of space vector:

$$\bar{v} = \frac{2}{3}\left(v_a(t) + a v_b(t) + a^2 v_c(t)\right)$$

# Space Vector Modulation

$= 0$

$$\bar{v} = \frac{2}{3}\left(v_{aN} + a v_{bN} + a^2 v_{cN} + v_{Nn}\left(1 + a + a^2\right)\right)$$

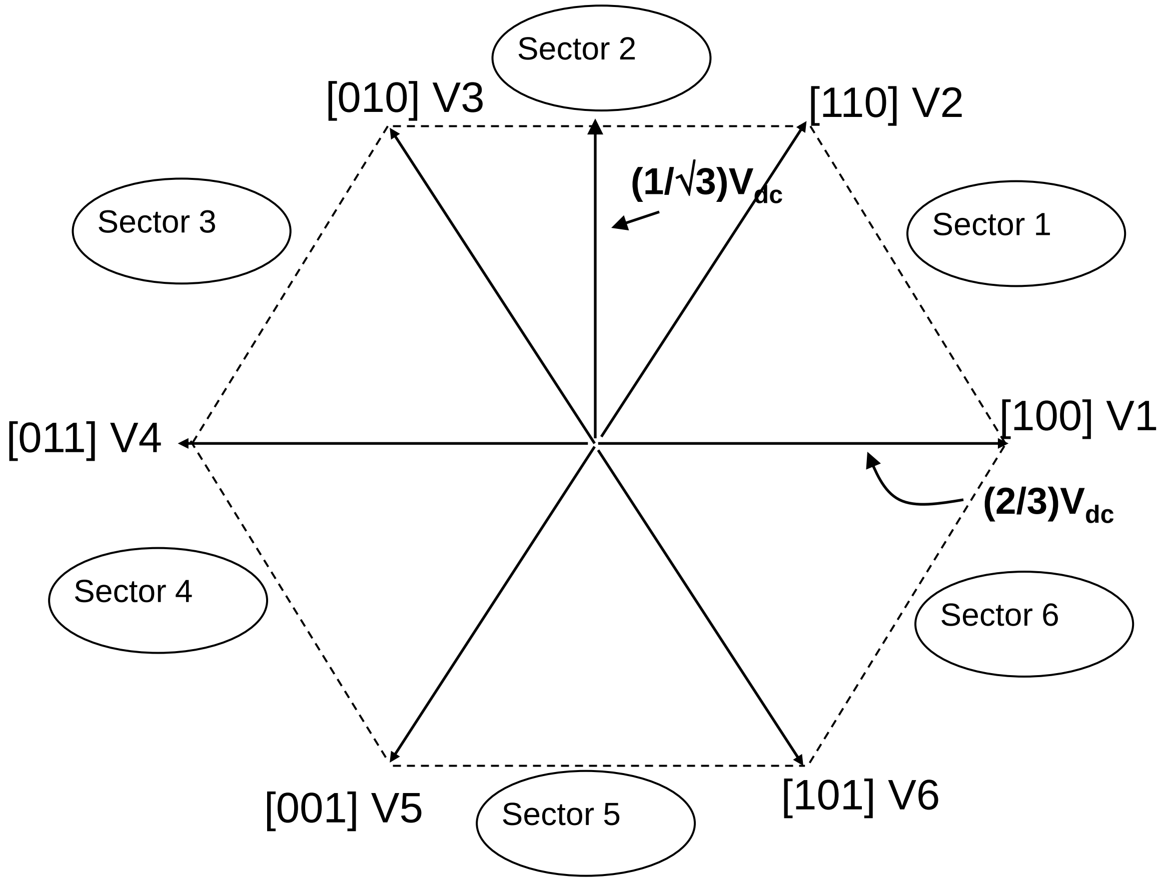
$v_{aN} = V_{dc}S_a$, $v_{bN} = V_{dc}S_b$, $v_{cN} = V_{dc}S_c$, $S_a, S_b, S_c = 1 \text{ or } 0$

$$\boxed{\bar{v} = \frac{2}{3}V_{dc}\left(S_a + aS_b + a^2 S_c\right)}$$

$$\bar{v} = \frac{2}{3}\left(v_a(t) + a v_b(t) + a^2 v_c(t)\right)$$

# Space Vector Modulation

$$\bar{v} = \frac{2}{3} V_{dc} \left( S_a + a S_b + a^2 S_c \right)$$



# Space Vector Modulation

Reference voltage is sampled at regular interval, T

Within sampling period, $v_{ref}$ is synthesized using adjacent vectors and zero vectors

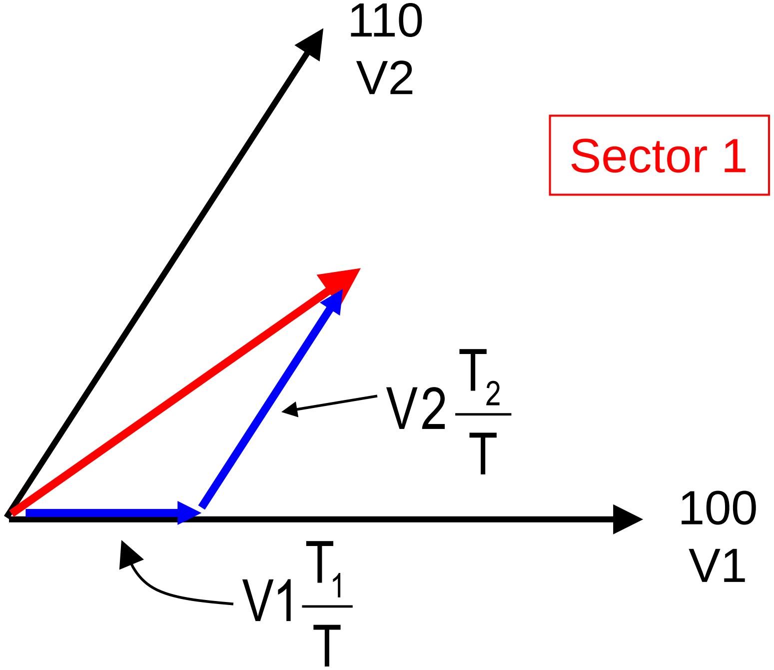
If T is sampling period,

V1 is applied for $T_1$,

V2 is applied for $T_2$

Zero voltage is applied for the rest of the sampling period,

$$T_0 = T - T_1 - T_2$$



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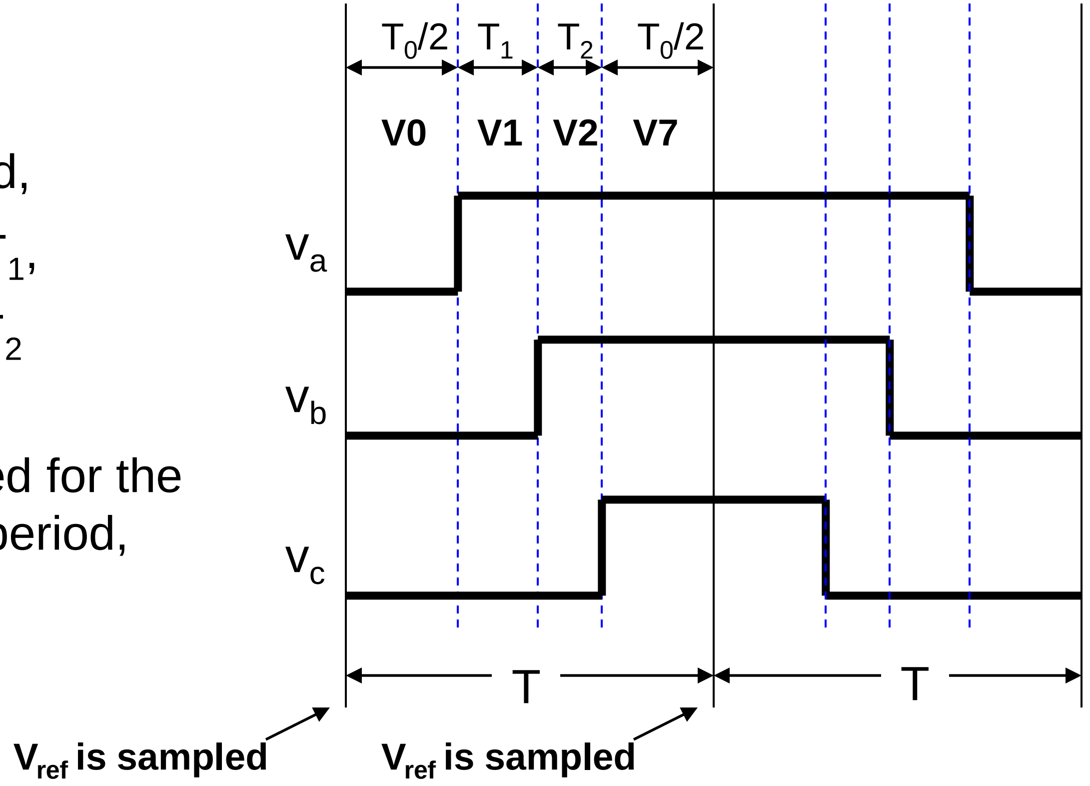


## Space Vector Modulation

How do we calculate $T_1$, $T_2$, $T_0$ and $T_7$?

They are calculated based on volt-second integral of $v_{ref}$

$$\frac{1}{T}\int_0^T \bar{v}_{ref}dt = \frac{1}{T}\left[\int_0^{T_o} v_0 dt + \int_0^{T_1} v_1 dt + \int_0^{T_2} v_2 dt + \int_0^{T_7} v_7 dt\right]$$

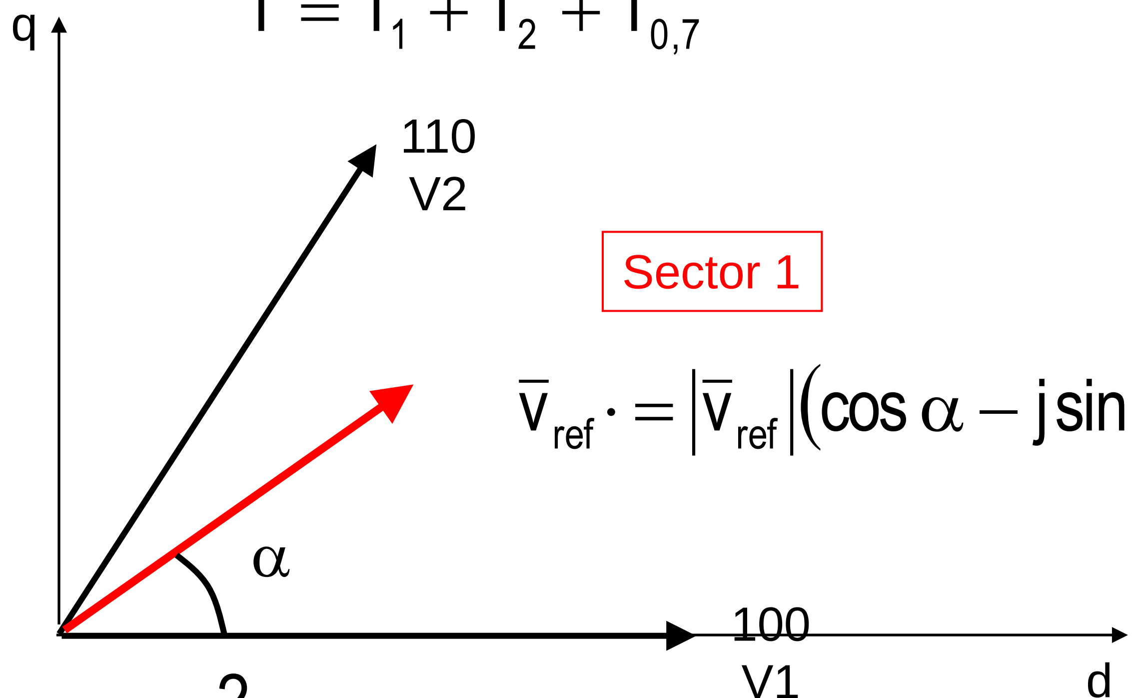
$$\bar{v}_{ref} \cdot T = v_o \cdot T_o + v_1 \cdot T_1 + v_2 \cdot T_2 + v_7 \cdot T_7$$

$$\bar{v}_{ref} \cdot T = T_o \cdot 0 + \frac{2}{3}V_d \cdot T_1 + \frac{2}{3}V_d(\cos 60^o + j\sin 60^o)T_2 + T_7 \cdot 0$$

$$\bar{v}_{ref} \cdot T = \frac{2}{3}V_d \cdot T_1 + \frac{2}{3}V_d(\cos 60^o + j\sin 60^o)T_2$$

# Space Vector Modulation

$$T = T_1 + T_2 + T_{0,7}$$




$$\bar{v}_{ref} \cdot = \left|\bar{v}_{ref}\right|\left(\cos\alpha - j\sin\alpha\right)$$

$$\bar{v}_{ref} \cdot T = \frac{2}{3} V_d \cdot T_1 + \frac{2}{3} V_d (\cos 60^o + j\sin 60^o) T_2$$

## Space Vector Modulation

$$\bar{v}_{ref} \cdot T = \frac{2}{3} V_d \cdot T_1 + \frac{2}{3} V_d (\cos 60^o + j \sin 60^o) T_2$$

$$T \left| \bar{v}_{ref} \right| \cos\alpha = \frac{2}{3} V_d T_1 + \frac{1}{3} V_d T_2 \qquad T \left| \bar{v}_{ref} \right| \sin\alpha = \frac{1}{\sqrt{3}} V_d T_2$$

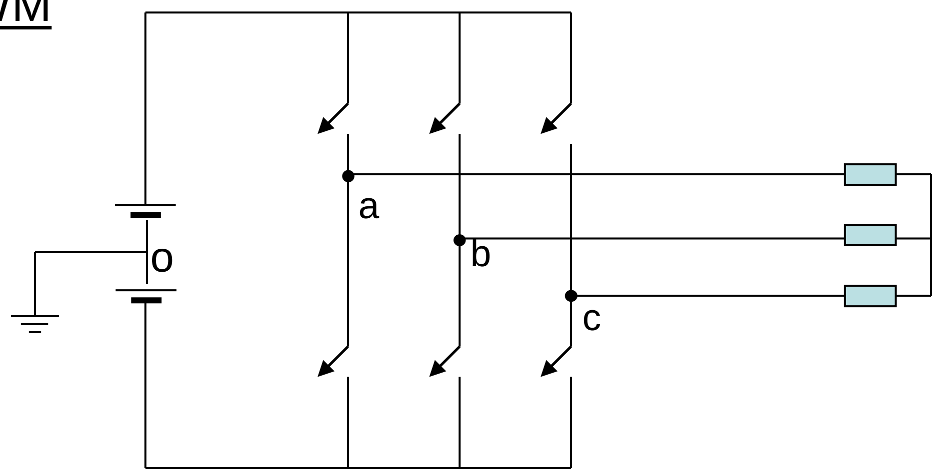
Solving for $T_1$, $T_2$ and $T_{0,7}$ gives:

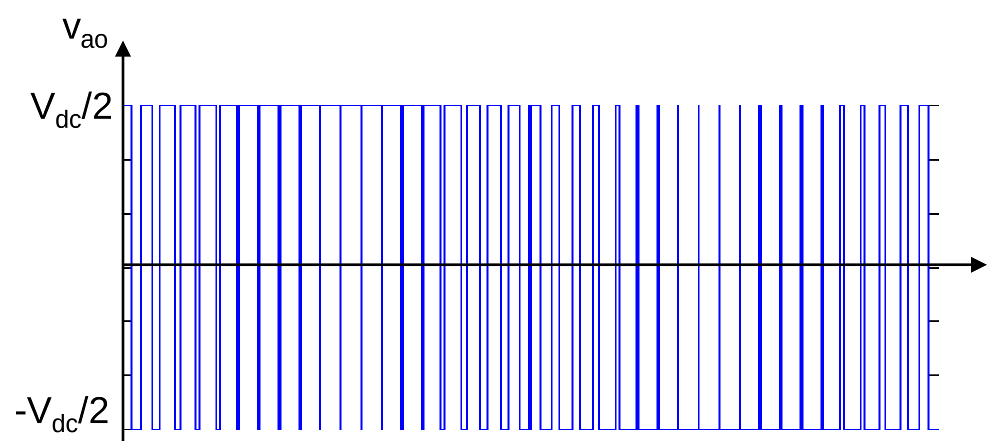
$$T_1 = \frac{3}{2} m \left[ \frac{T}{\sqrt{3}} \cos\alpha - \frac{1}{3} T \sin\alpha \right] \qquad T_2 = m T \sin\alpha \qquad \text{where} \quad m = \frac{v_{ref}}{V_d / \sqrt{3}}$$

## Space Vector Modulation

Comparison between SVM and SPWM

SPWM




For m = 1, amplitude of fundamental for $v_{ao}$ is $V_{dc}/2$

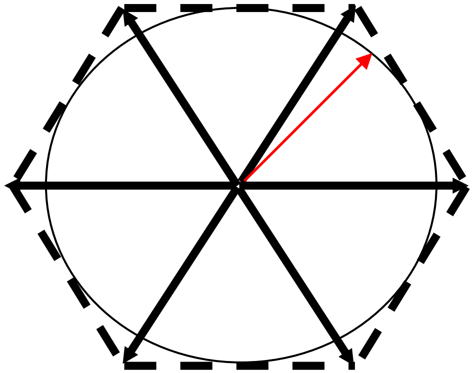
$$\therefore \text{amplitude of line-line} = \frac{\sqrt{3}}{2} V_{dc}$$

## Space Vector Modulation

Comparison between SVM and SPWM

SVM

We know max possible phase voltage without overmodulation is $\frac{1}{\sqrt{3}}V_{dc}$



$\therefore$ amplitude of line-line = $V_{dc}$

Line-line voltage increased by: $\dfrac{V_{dc} - \frac{\sqrt{3}}{2}V_{dc}}{\frac{\sqrt{3}}{2}V_{dc}} x100 \approx 15\%$

### 2.1.1 Questions

What parameters are required for the implementation of SVM method?

What values are obtained from the calculation of SVM method?

### 2.1.2 Answer

The required parameters according to equation ( 4 ) are:

- The reference voltage represented with a space vector (angle and magnitude).
- The voltage value on the DC bus.
- The PWM period.

After the calculation we get:

- Two base vectors.
- The time durations of these two base vectors.
- The time duration of the zero vectors.

### 2.2.1 Question

If SVM method is applied to an inverter for motor control, is it necessary to know the values of the three phase voltages for driving the motor? Why?

### 2.2.2 Answer

To use SVM method, it is not necessary to know the three phase voltages. The reason is that the input parameter for SVM method is the voltage space vector for driving the motor. The switching timing for the inverter bridge arms is directly obtained from the angle and magnitude of this vector.

### 2.3.1 Problem

It is expected to output a three-phase voltage that is represented with a space vector $\underline{u}_s = 100e^{j165°}$ (V) using SVM method. The known parameters are:

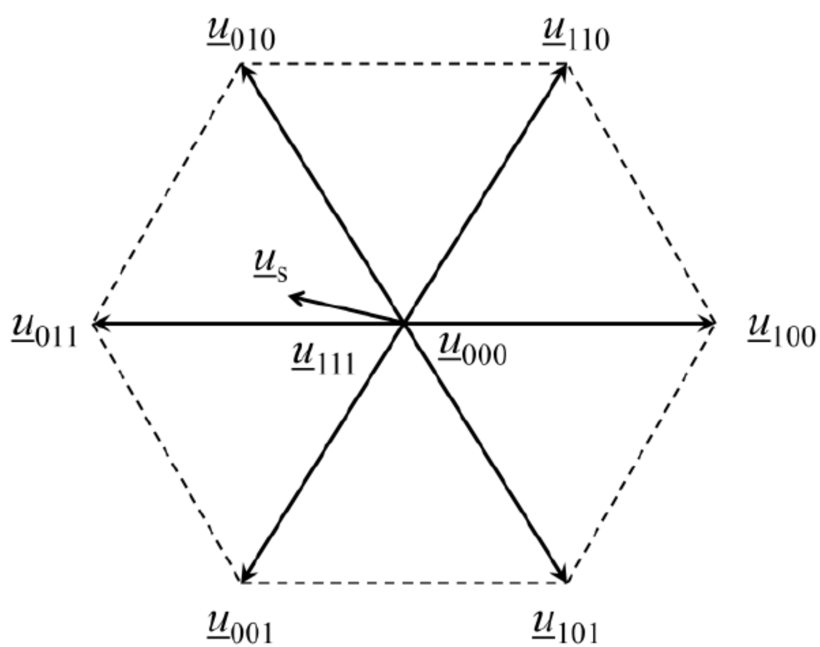
- DC link voltage, $U_{dc} = 600$ V.
- PWM frequency, $f = 8$ kHz

Questions:

1. Please determine which base vectors are needed for generating this voltage.
2. Please calculate the time durations of these base vectors.

### 2.3.2 Solution to Question 1

The expected space vector in the base vector hexagon is shown in the following figure. It is obvious that the expected vector $\underline{u}_s$ should be represented using $\underline{u}_{010}$, $\underline{u}_{011}$ and the zero vectors.



### 2.3.3 Solution to Question 2

$$\begin{cases} r_1 = \sqrt{3}\dfrac{u_s}{U_{dc}}\sin(60° - \theta) \\ r_2 = \sqrt{3}\dfrac{u_s}{U_{dc}}\sin\theta \end{cases} . \qquad \begin{cases} T_0 = (1 - r_1 - r_2)T \\ T_1 = r_1 T \\ T_2 = r_2 T \end{cases}$$

$$\begin{cases} r_1 = \sqrt{3} \times \dfrac{100}{600} \times \sin(60° - (165° - 120°)) \approx 0.0747 \\ r_2 = \sqrt{3} \times \dfrac{100}{600} \times \sin(165° - 120°) \approx 0.204 \end{cases} .$$

PWM period,

$$T = \frac{1}{f} = 125\ (\mu s)$$

Therefore, the durations for zero vectors is

$$T_0 = (1 - 0.0747 - 0.204) \times 125 = 90.2\ (\mu s),$$

for vector $\underline{u}_{010}$ is

$$T_1 = 0.0747 \times 125 = 9.3\ (\mu s),$$

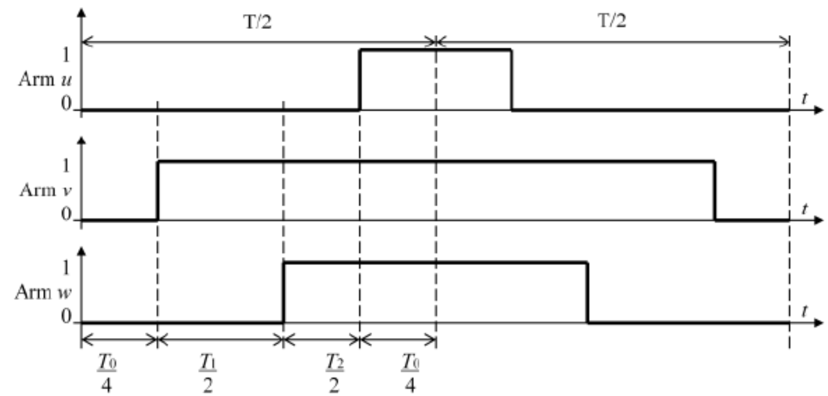
and for vector $\underline{u}_{011}$ is

$$T_2 = 0.204 \times 125 = 25.5\ (\mu s).$$

3. Please draw one period of the PWM switching sequence to generate the expected voltage using symmetric pulsation and both zero vectors, and indicate the duration of every switching state in the diagram.
4. Please draw one period of the PWM switching sequence to generate the expected voltage using flattop method, and indicate the duration of every switching state.

### 2.3.4 Solution to Question 3

SVM using both zero vectors:




### 2.3.5 Solution to Question 4

SVM flattop pulsation using zero vector, $\underline{u}_{111}$: