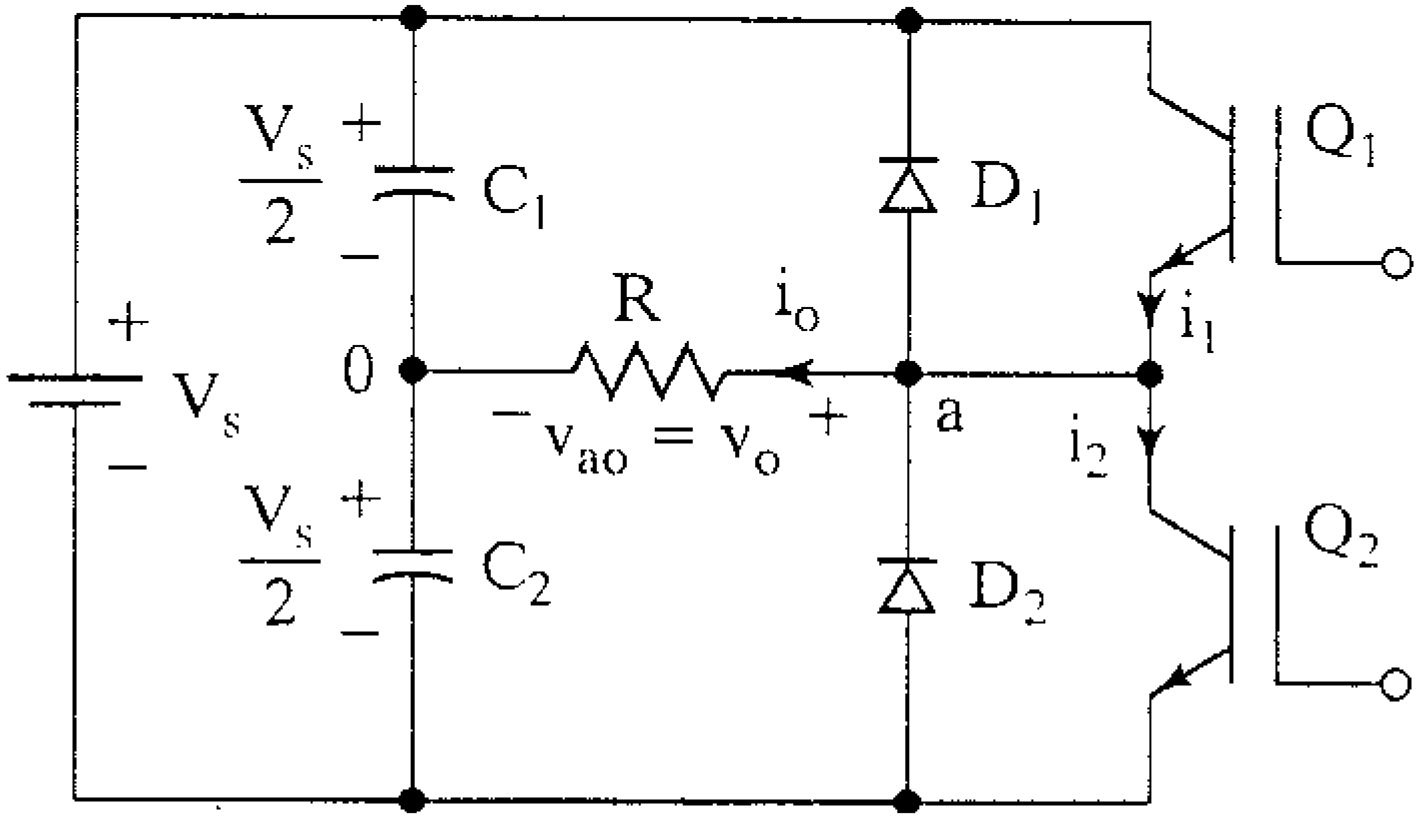
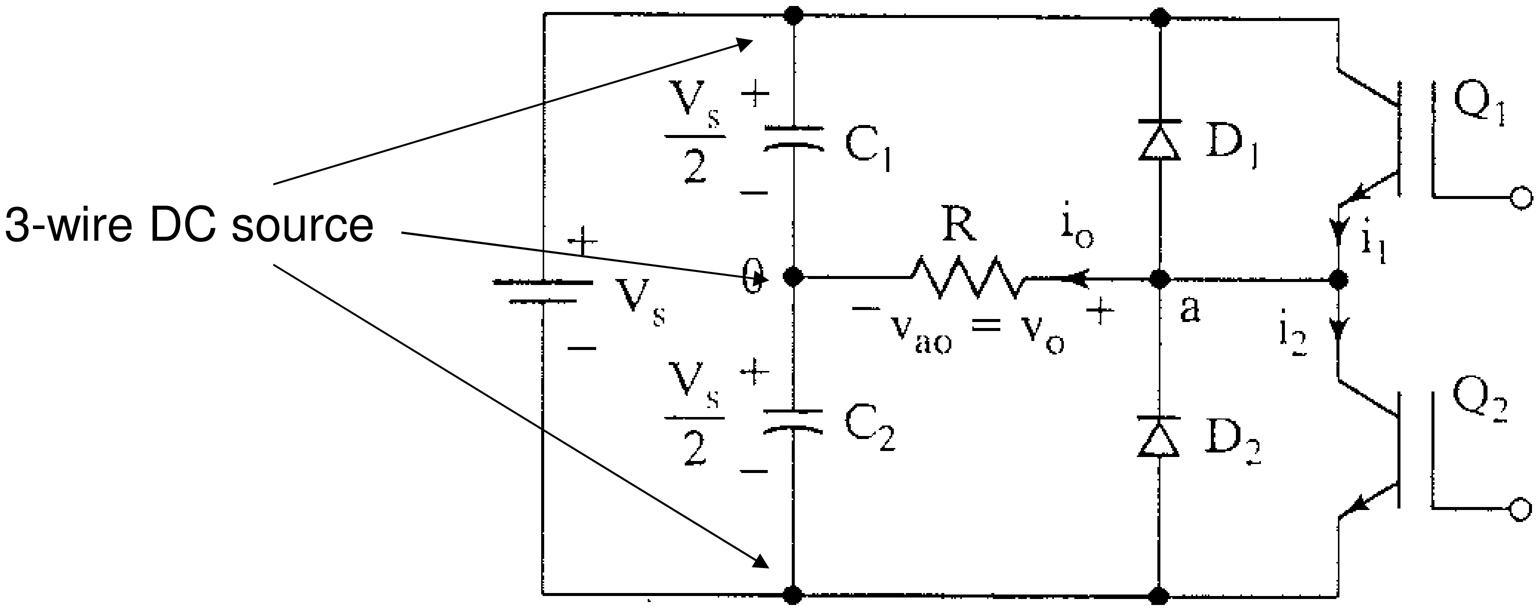
ELGC06
H-Bridge Inverter Basics

Single-phase half-bridge inverter

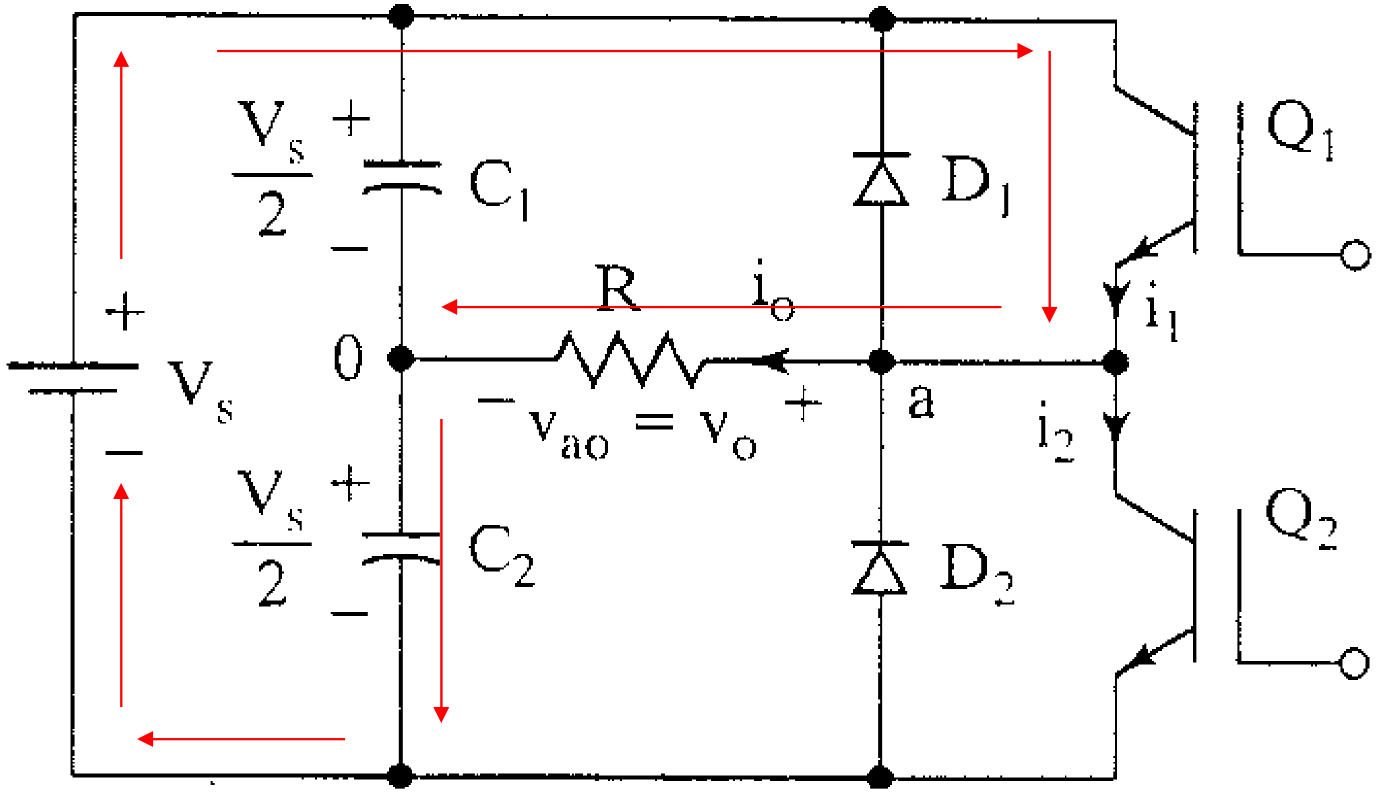


Operational Details



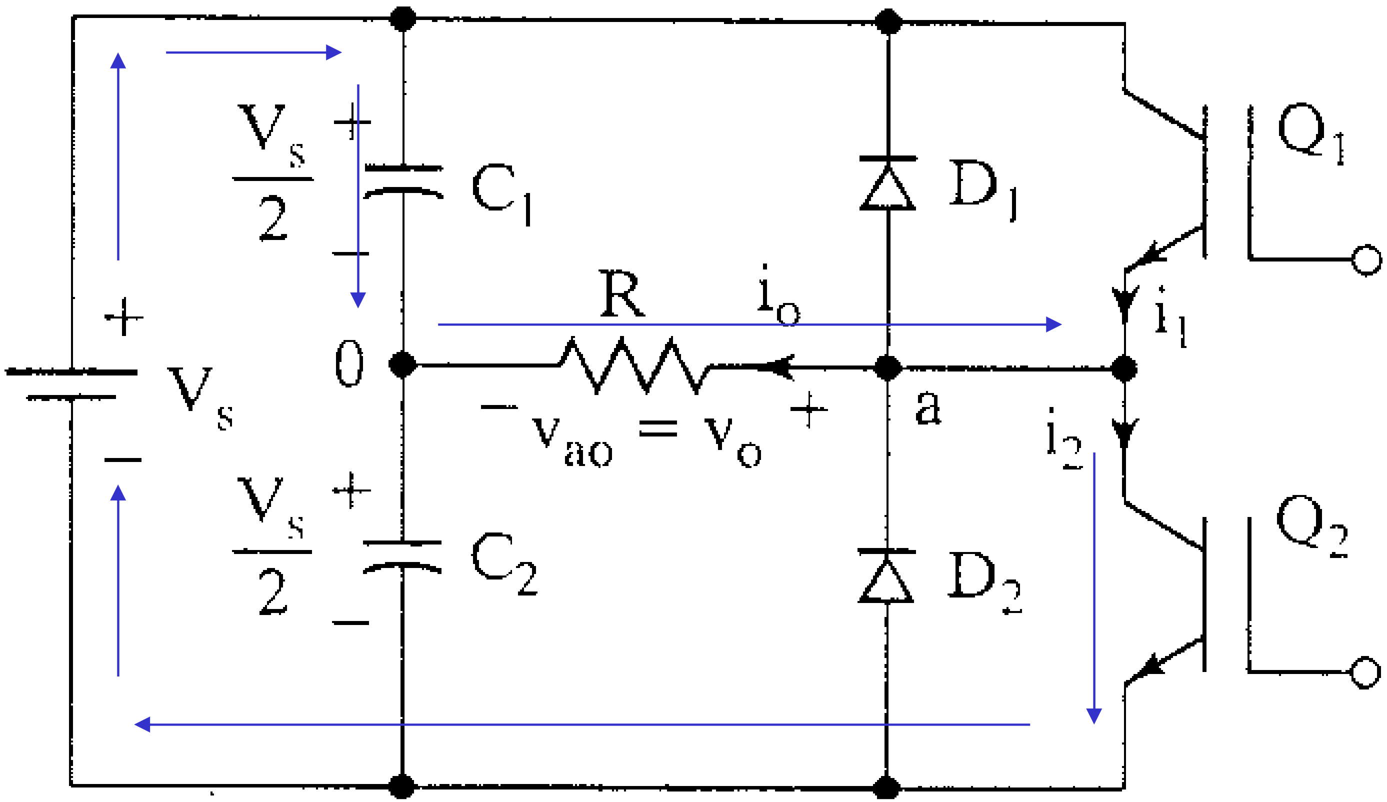
- Consists of 2 choppers, 3-wire DC source
- Transistors switched on and off alternately
- Need to isolate the gate signal for Q_1 (upper device)
- Each provides opposite polarity of $V_s/2$ across the load

Q_1 on, Q_2 off, $v_o = V_s/2$

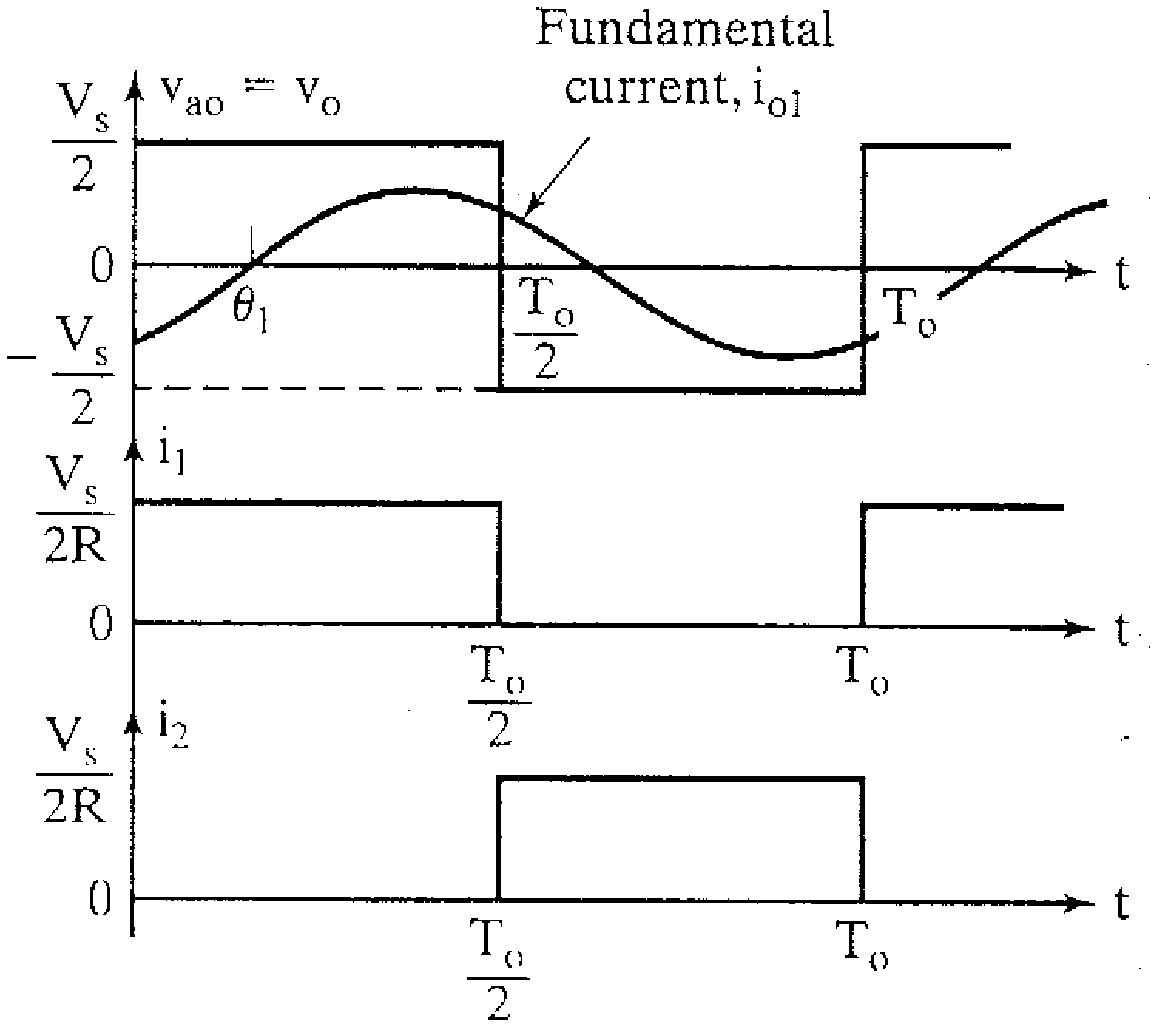


Peak Reverse Voltage of $Q_2 = V_s$

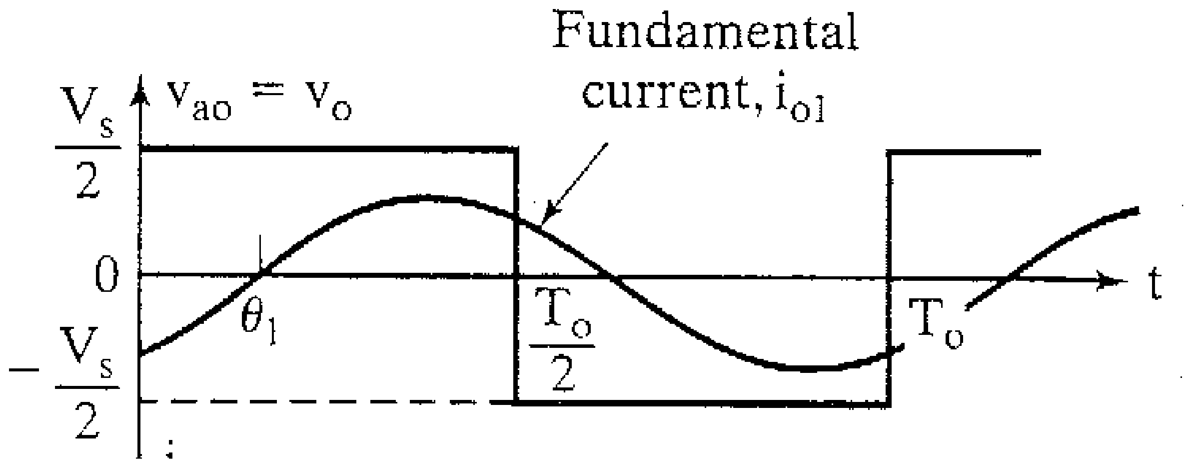
Q_1 off, Q_2 on, $v_o = -V_s/2$



Waveforms with resistive load



Look at the output voltage



rms value of the output voltage, V_o

$$V_o = \left(\frac{2}{T_o} \int_0^{\frac{T_o}{2}} \frac{V_s^2}{4} dt \right)^{\frac{1}{2}} = \frac{V_s}{2}$$

Fourier Series of the instantaneous output voltage

$$v_o = \frac{a_o}{2} + \sum_{n=1}^{\infty}\left(a_n \cos(n\omega t) + b_n \sin(n\omega t)\right)$$

$$a_o, a_n = 0$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} \frac{-V_s}{2}\sin(n\omega t)d(\omega t) + \int_{0}^{\pi} \frac{V_s}{2}\sin(n\omega t)d(\omega t)\right]$$

$$b_n = \frac{2V_s}{n\pi} \to n = 1, 3, 5, \ldots$$

$$v_o = \sum_{n=1,3,5,..}^{\infty} \frac{2V_s}{n\pi}\sin(n\omega t)$$

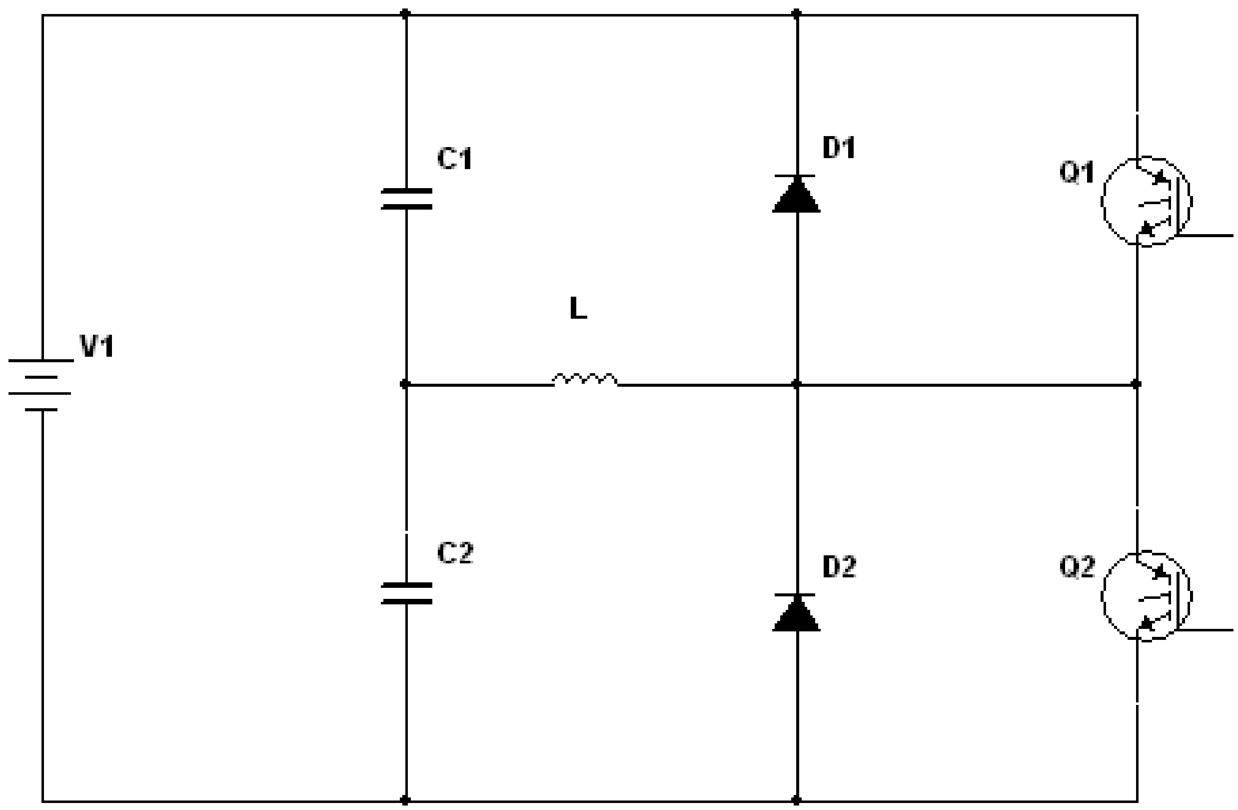
rms value of the fundamental component

$$v_o = \sum_{n=1,3,5,..}^{\infty} \frac{2V_s}{n\pi} \sin n\omega t$$

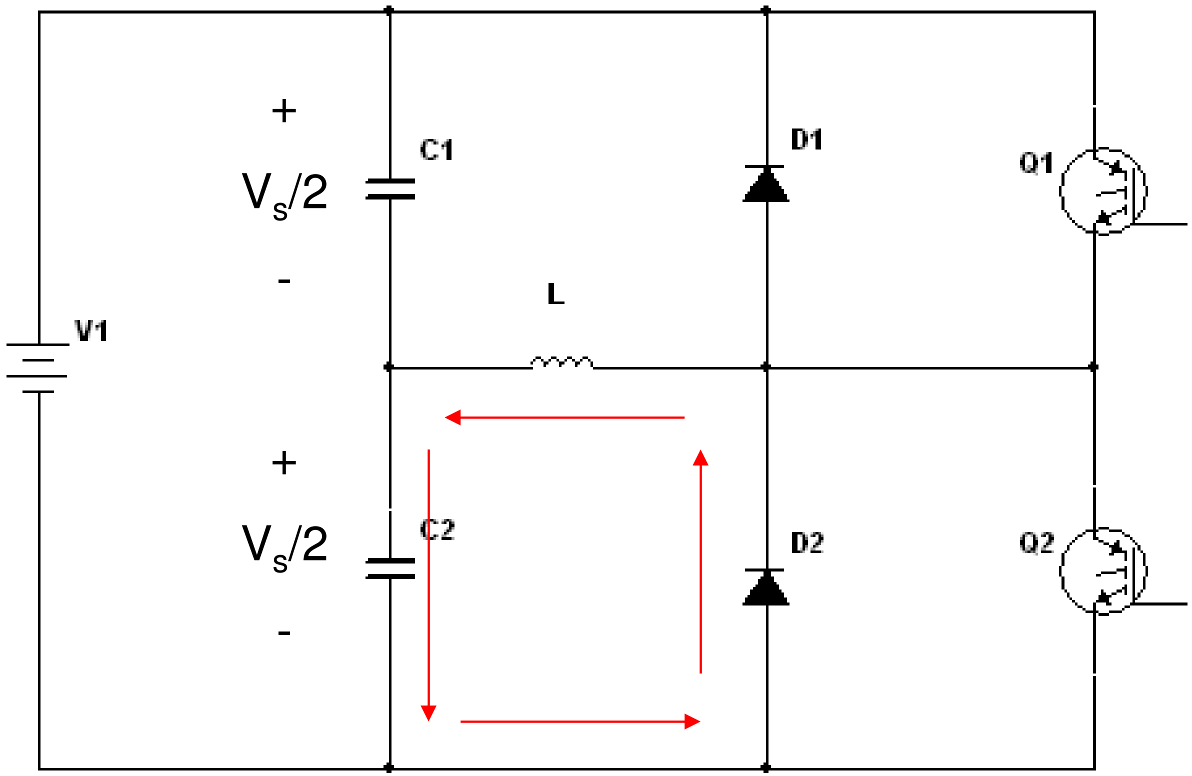
$$V_{o1} = \frac{1}{\sqrt{2}} \frac{2V_s}{\pi}$$

$$V_{o1} = 0.45V_s$$

When the load is highly inductive

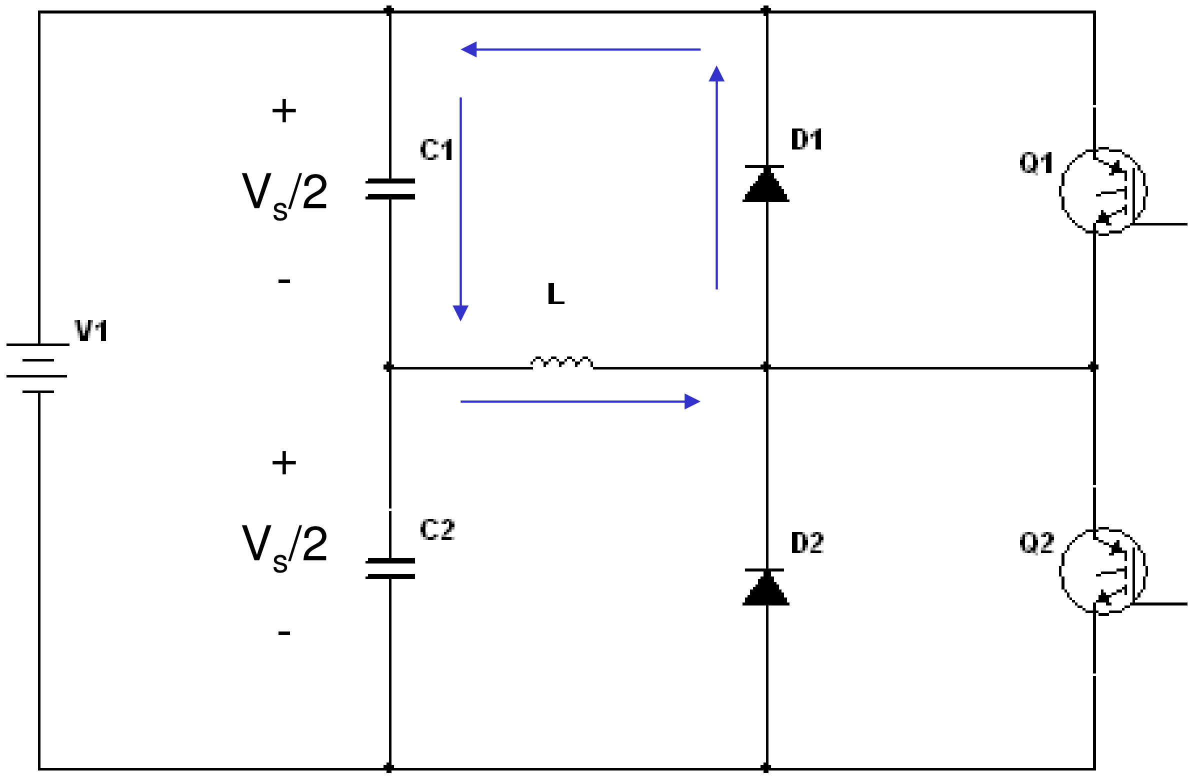


Turn off Q_1 at $t = T_o/2$
Current falls to 0 via D_2, L, $V_s/2$ lower

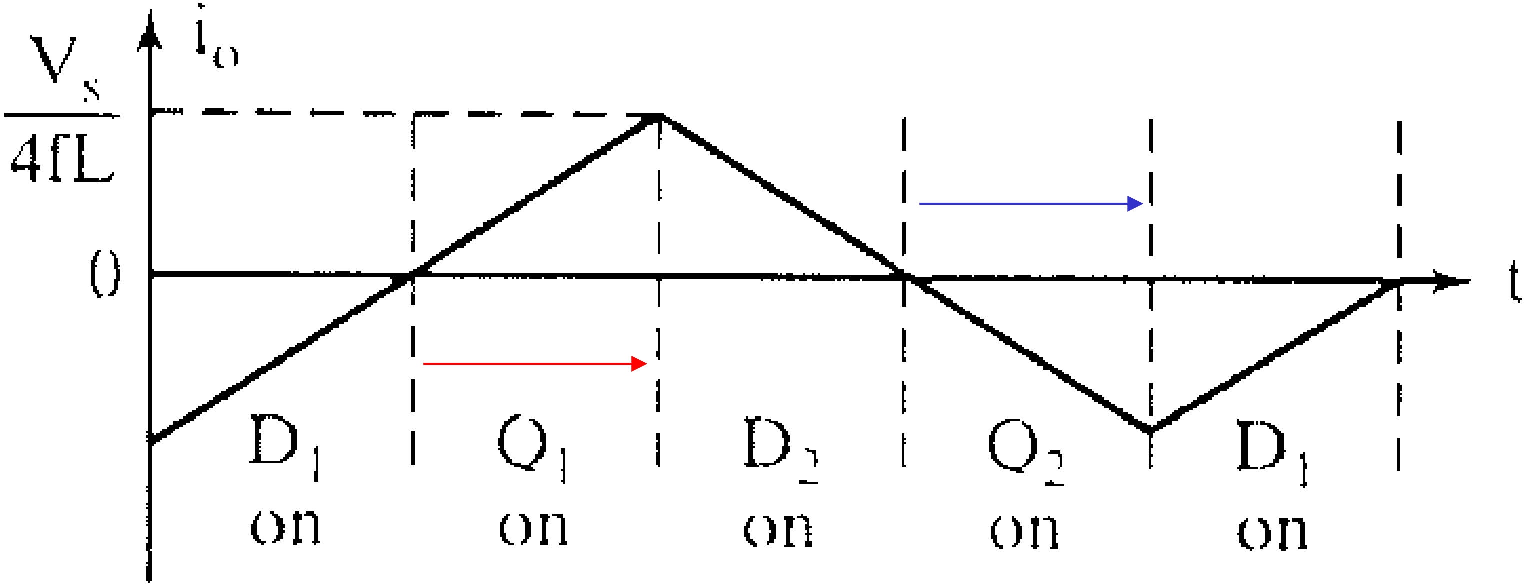


Turn off Q_2 at $t = T_o$

Current falls to 0 via D_1, L, $V_s/2$ upper



Load Current for a highly inductive load



Transistors are only switched on for a quarter-cycle, or 90°

Fourier Series of the output current for an RL load

$$i_o = \frac{v_o}{Z} = \frac{v_o}{R + jn\omega L} = \sum_{n=1,3,5,\ldots}^{\infty} \frac{2V_s}{n\pi\sqrt{R^2 + (n\omega L)^2}} \sin(n\omega t - \theta_n)$$

$$\theta_n = \tan^{-1}\left(\frac{n\omega L}{R}\right)$$

Fundamental Output Power
In most cases, the useful power

$$P_{o1} = V_{o1} I_{o1} \cos\theta_1 = I_{o1}^2 R$$

$$P_{o1} = \left[\frac{2V_s}{\sqrt{2}\pi\sqrt{R^2 + (\omega L)^2}} \right]^2 R$$

DC Supply Current

- If the inverter is lossless, average power absorbed by the load equals the average power supplied by the dc source.

$$\int_0^T v_s(t) i_s(t) dt = \int_0^T v_o(t) i_o(t) dt$$

- For an inductive load, the current is approximately sinusoidal and the fundamental component of the output voltage supplies the power to the load. Also, the dc supply voltage remains essentially at V_s.

DC Supply Current (continued)

$$\int_0^T i_s(t)dt = \frac{1}{V_s}\int_0^T \sqrt{2}V_{o1}\sin(\omega t)\sqrt{2}I_o\sin(\omega t-\theta_1)dt = I_s$$

$$I_s = \frac{V_{o1}}{V_s}I_o\cos(\theta_1)$$

Performance Parameters

- Harmonic factor of the nth harmonic (HF_n)

$$HF_n = \frac{V_{on}}{V_{o1}} \qquad \text{for } n>1$$

V_{on} = rms value of the nth harmonic component

V_{01} = rms value of the fundamental component

Performance Parameters (continued)

- Total Harmonic Distortion (THD)
- Measures the “closeness” in shape between a waveform and its fundamental component

$$THD = \frac{1}{V_{o1}} \left(\sum_{n=2,3,\ldots}^{\infty} V_{on}^{2} \right)^{\frac{1}{2}}$$

Performance Parameters (continued)

- Distortion Factor (DF)
- Indicates the amount of HD that remains in a particular waveform after the harmonics have been subjected to second-order attenuation.

$$DF = \frac{1}{V_{o1}} \left[\sum_{n=2,3,\ldots}^{\infty} \left(\frac{V_{on}}{n^2} \right)^2 \right]^{\frac{1}{2}}$$

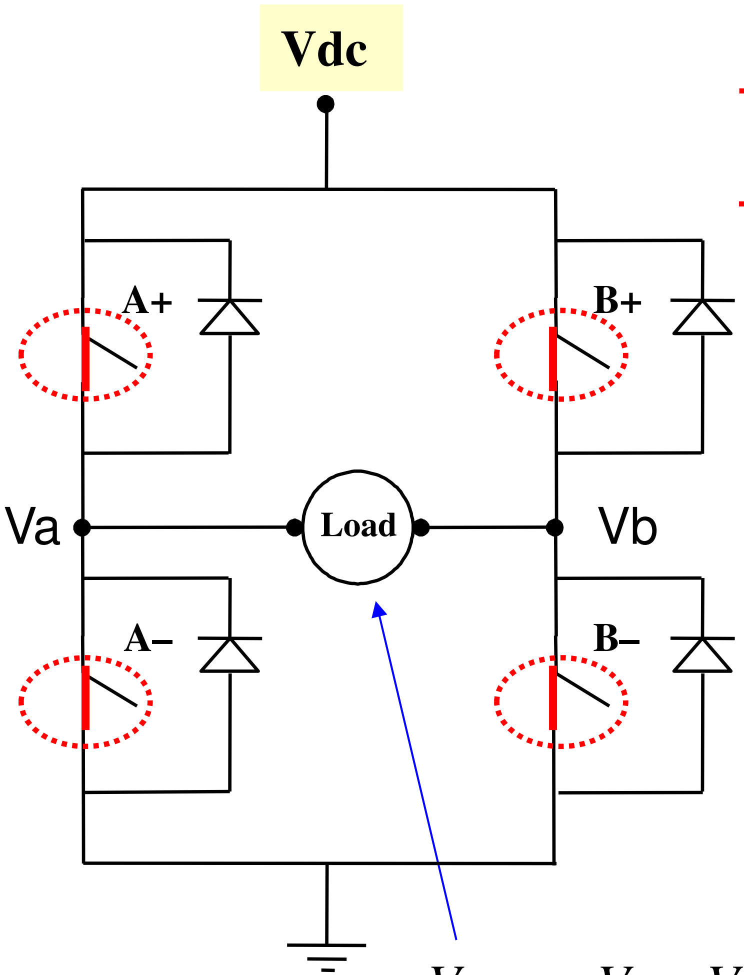
$$DF_n = \frac{V_{on}}{V_{o1} n^2} \qquad \text{for } n>1$$

Performance Parameters (continued)

- Lowest order harmonic (LOH)
- The harmonic component whose frequency is closest to the fundamental, and its amplitude is greater than or equal to 3% of the amplitude of the fundamental component.

H-Bridge Inverter Basics – Creating AC from DC

Single-phase H-bridge (voltage source) inverter topology:



$$V_{load} = V_A - V_B = V_{AB}$$

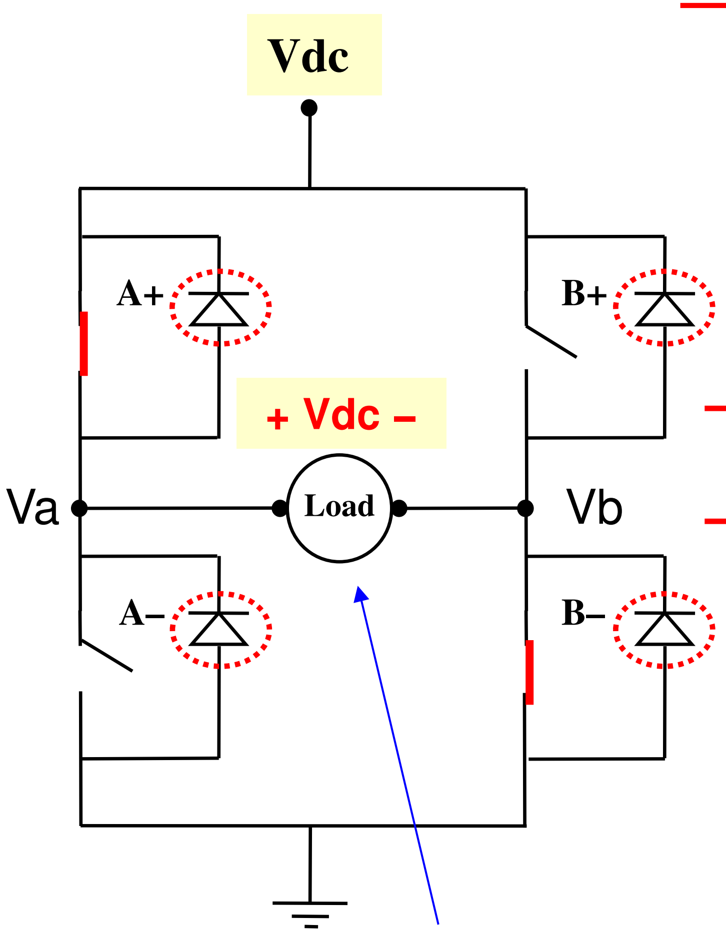
Switching rules
- Either A+ or A– is closed, but **never** at the same time *
- Either B+ or B– is closed, but never at the same time *

*same time closing would cause a short circuit from Vdc to ground (shoot-through)

*To avoid shoot-through when using real switches (i.e. there are turn-on and turn-off delays) a dead-time or blanking time is implemented

Corresponding values of Va and Vb
- A+ closed, Va = Vdc
- A– closed, Va = 0
- B+ closed, Vb = Vdc
- B– closed, Vb = 0

H BRIDGE INVERTER

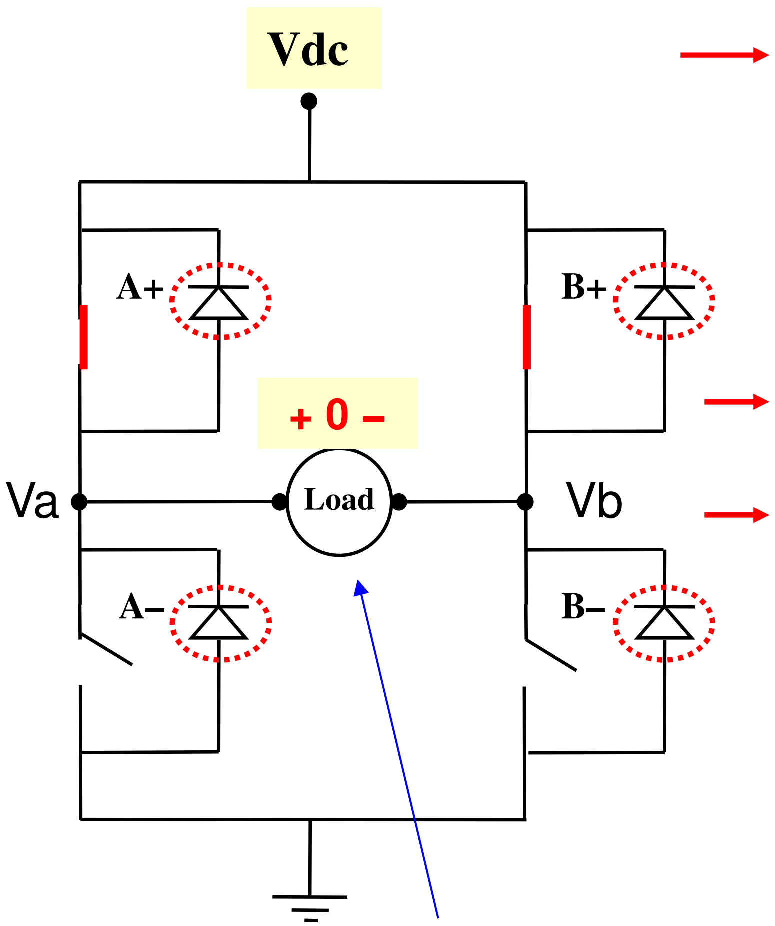


$$V_{load} = V_A - V_B = V_{AB}$$

Corresponding values of Vab
- A+ closed and B– closed, Vab = Vdc
- A+ closed and B+ closed, Vab = 0
- B+ closed and A– closed, Vab = –Vdc
- B– closed and A– closed, Vab = 0

- The free wheeling diodes permit current to flow even if all switches are open
- These diodes also permit lagging currents to flow in inductive loads

H BRIDGE INVERTER



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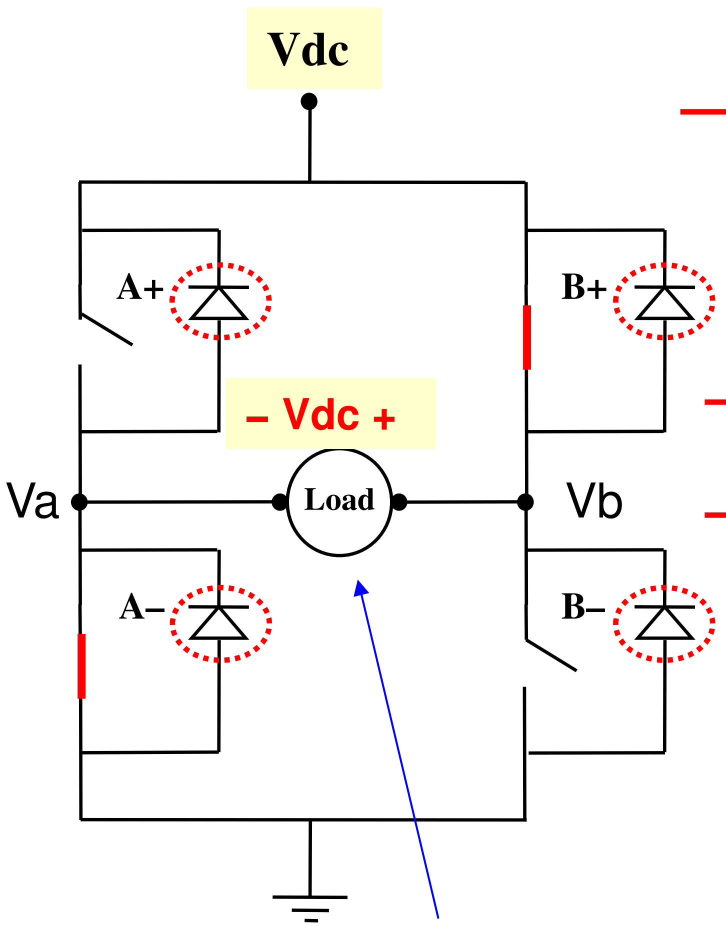
H BRIDGE INVERTER



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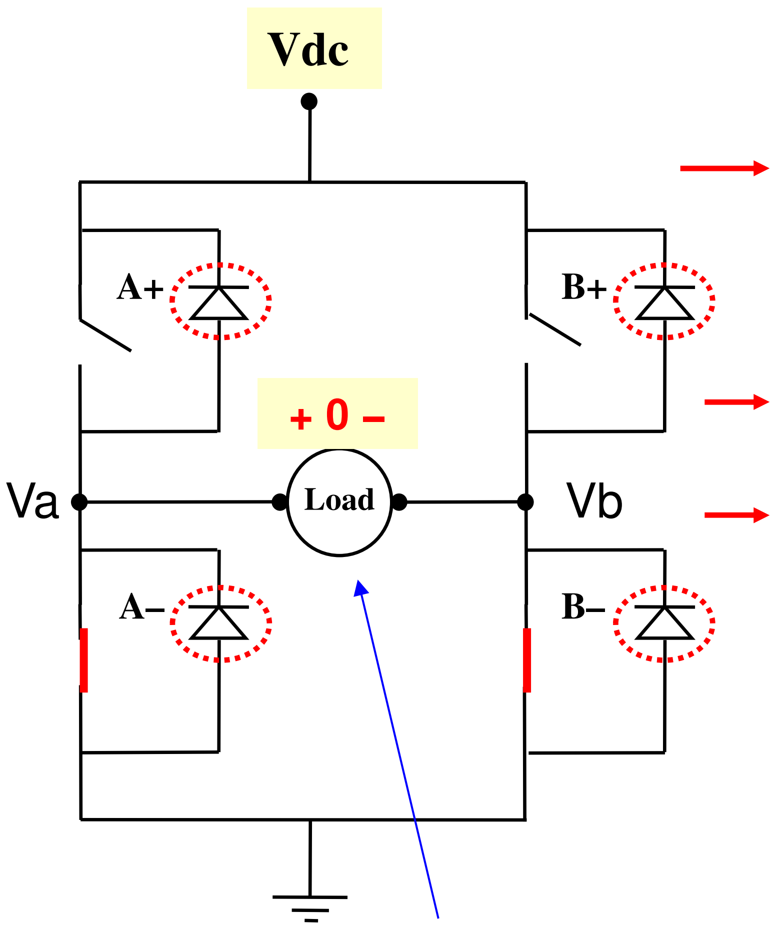

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- The free wheeling diodes permit current to flow even if all switches are open
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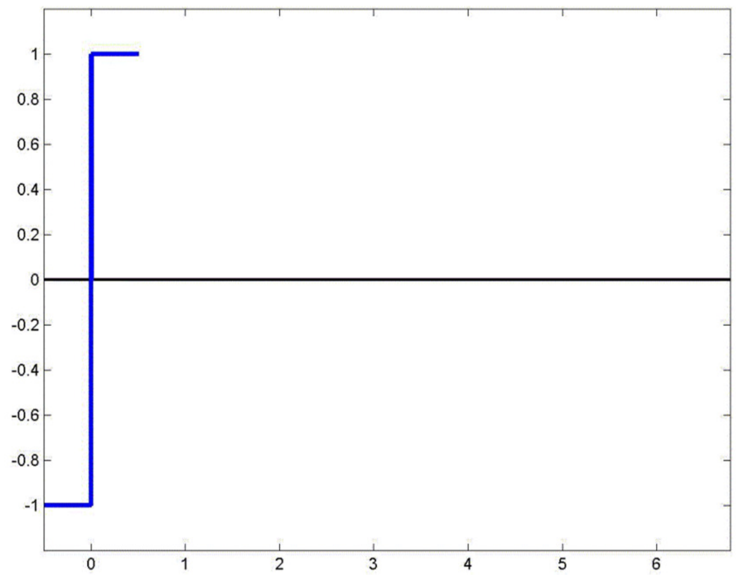
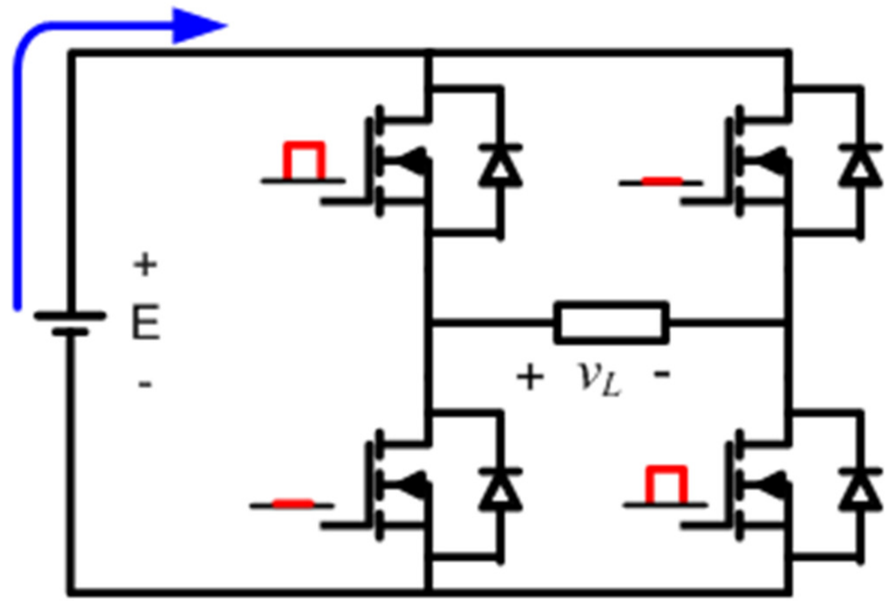
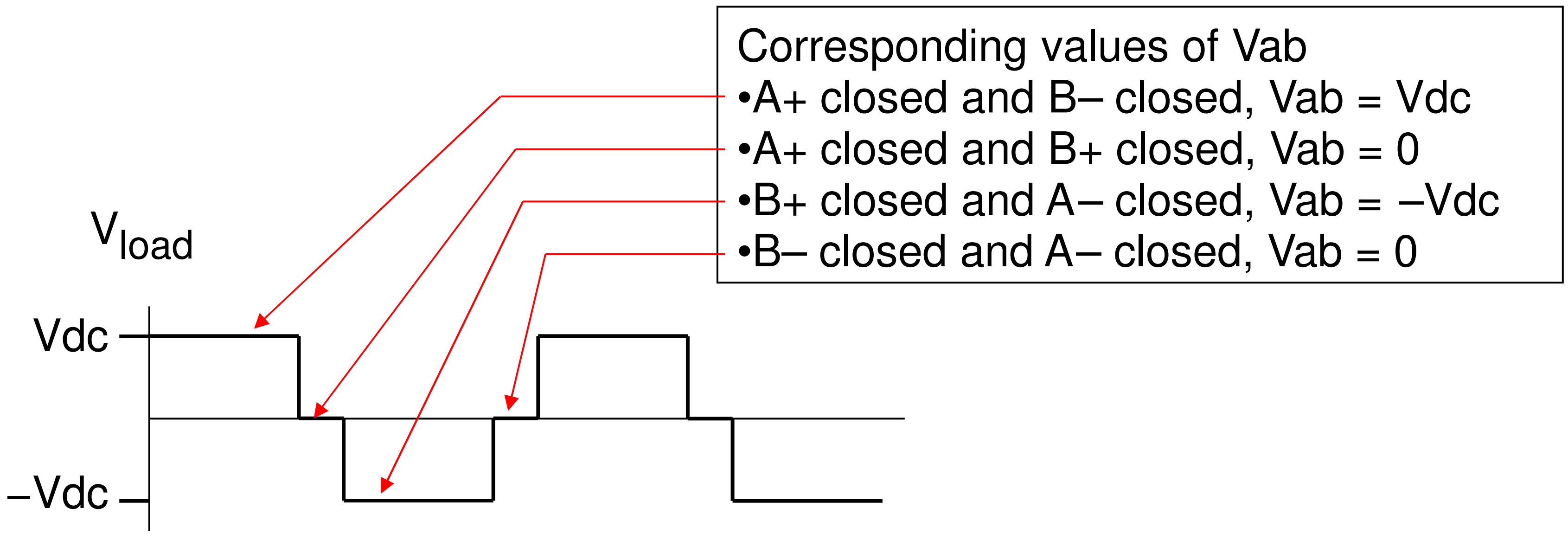
H-Bridge Inverter

• Square wave modulation:





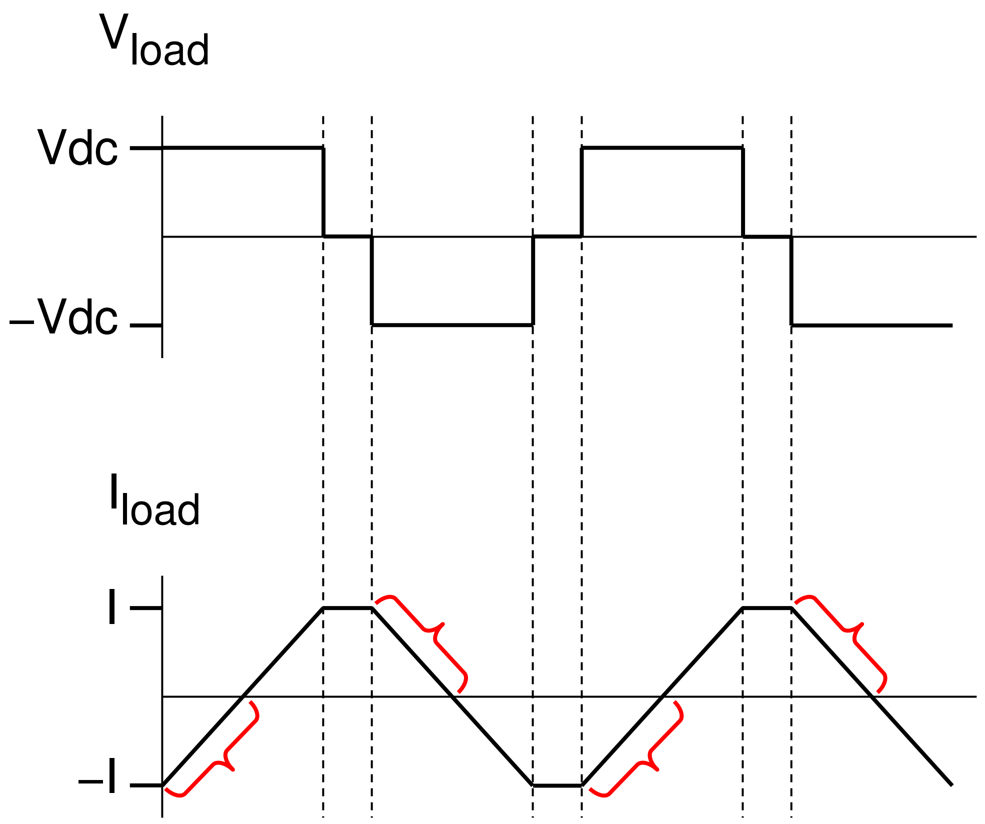
Basic Square Wave Operation
(sometimes used for 50 Hz or 60Hz applications)



The Vab = 0 time is not required but can be used to reduce the rms value of V_{load}

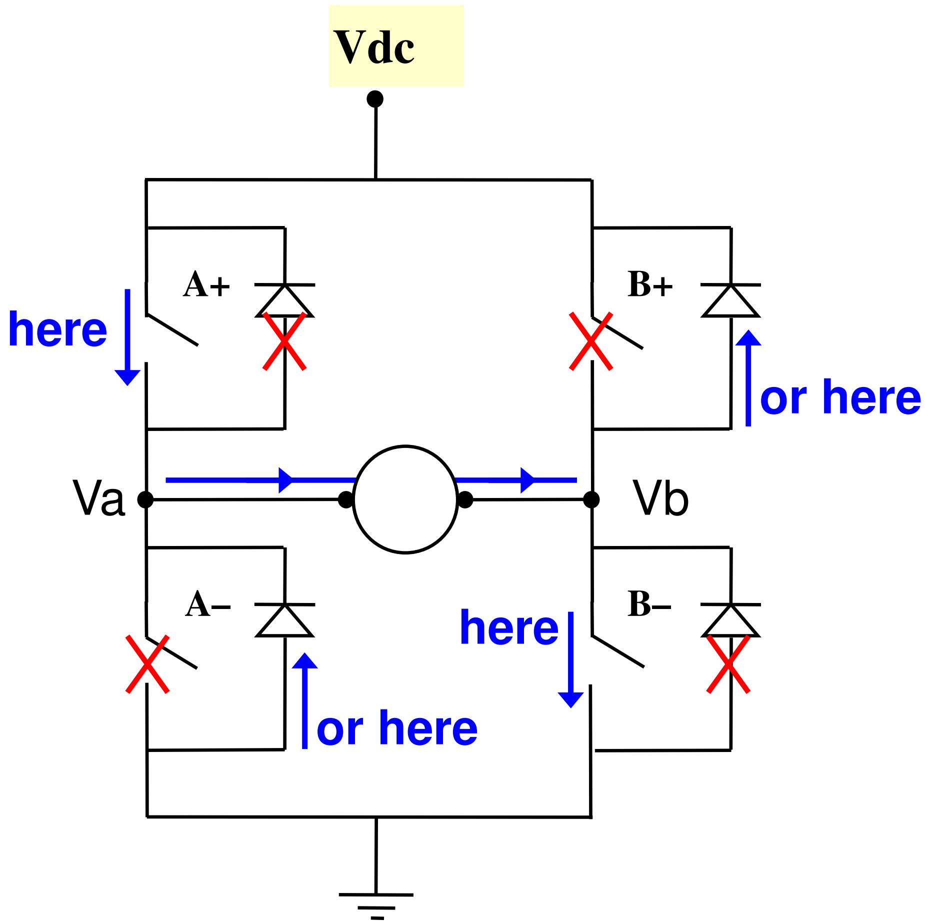
Many Loads Have Lagging Current – Consider an Inductor

There must be a provision for voltage and current to have opposite signs
with respect to each other



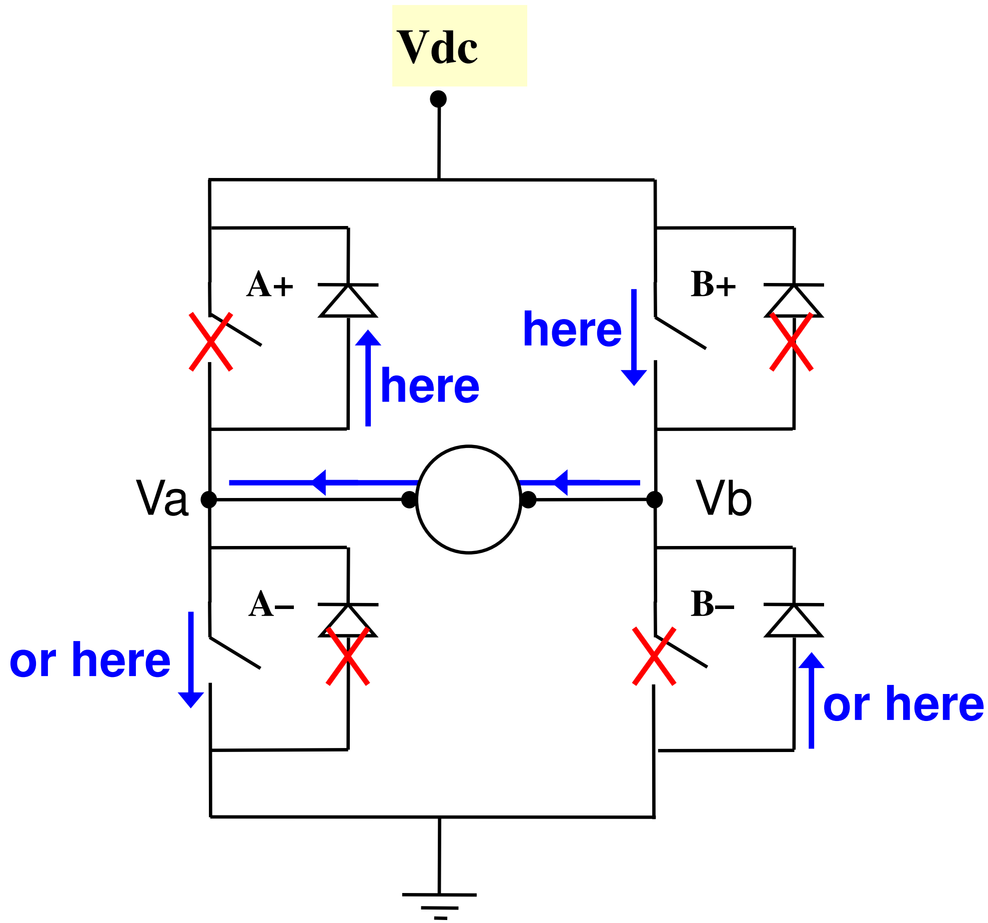
Load Current Can Always Flow, Regardless of Switching State

Example - when current flows left to right through the load



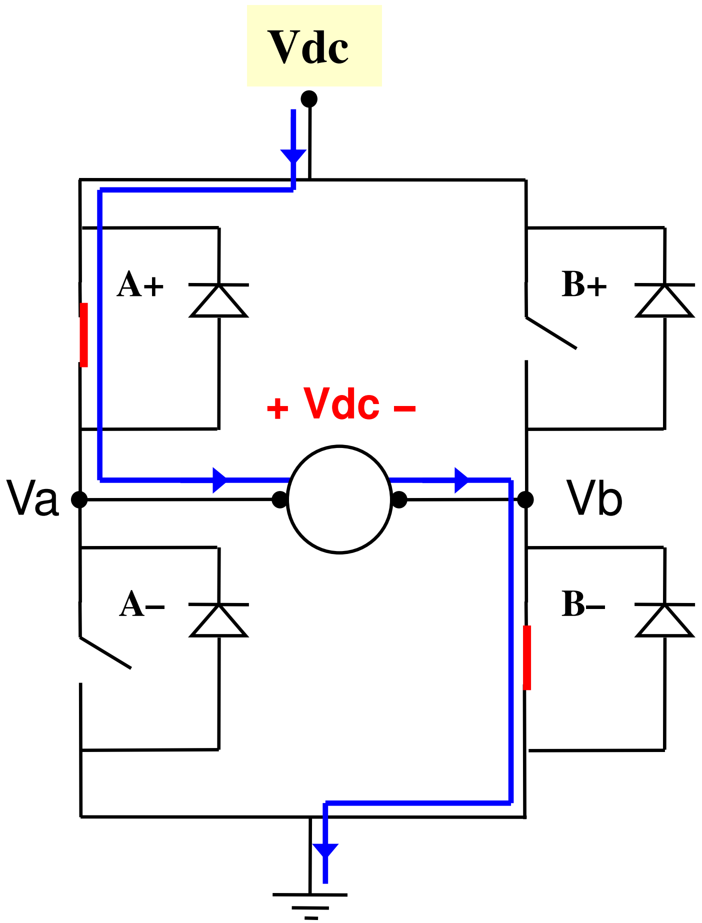
Load Current Can Always Flow, cont.

Example - when current flows right to left through the load



Load Current Can Always Flow, cont.

H BRIDGE INVERTER

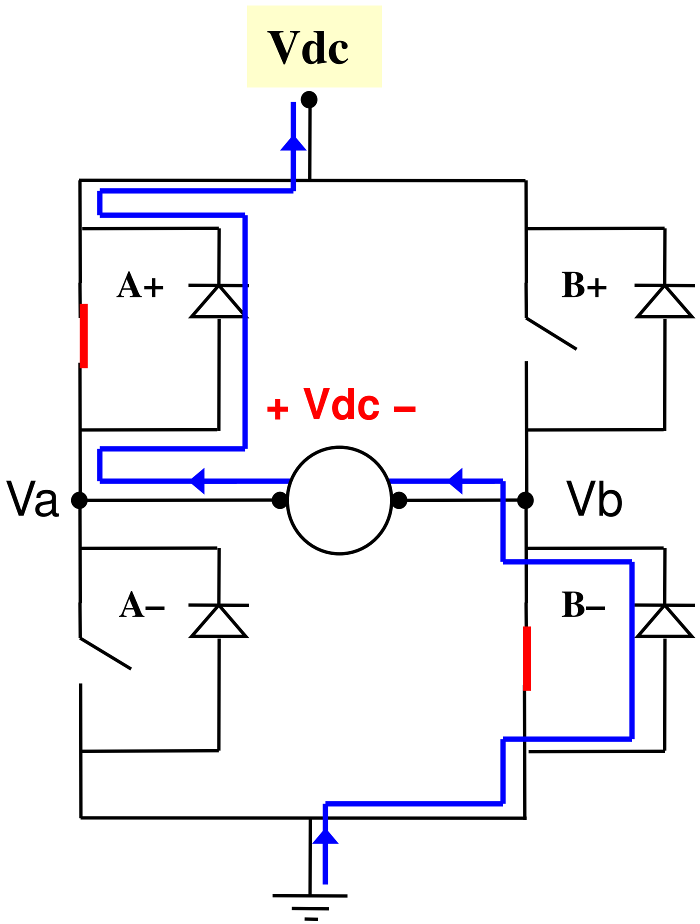


Corresponding values of Vab
- A+ closed and B– closed, Vab = Vdc
- A+ closed and B+ closed, Vab = 0
- B+ closed and A– closed, Vab = –Vdc
- B– closed and A– closed, Vab = 0

- Load consuming power
- Load generating power

Load Current Can Always Flow, cont.

H BRIDGE INVERTER



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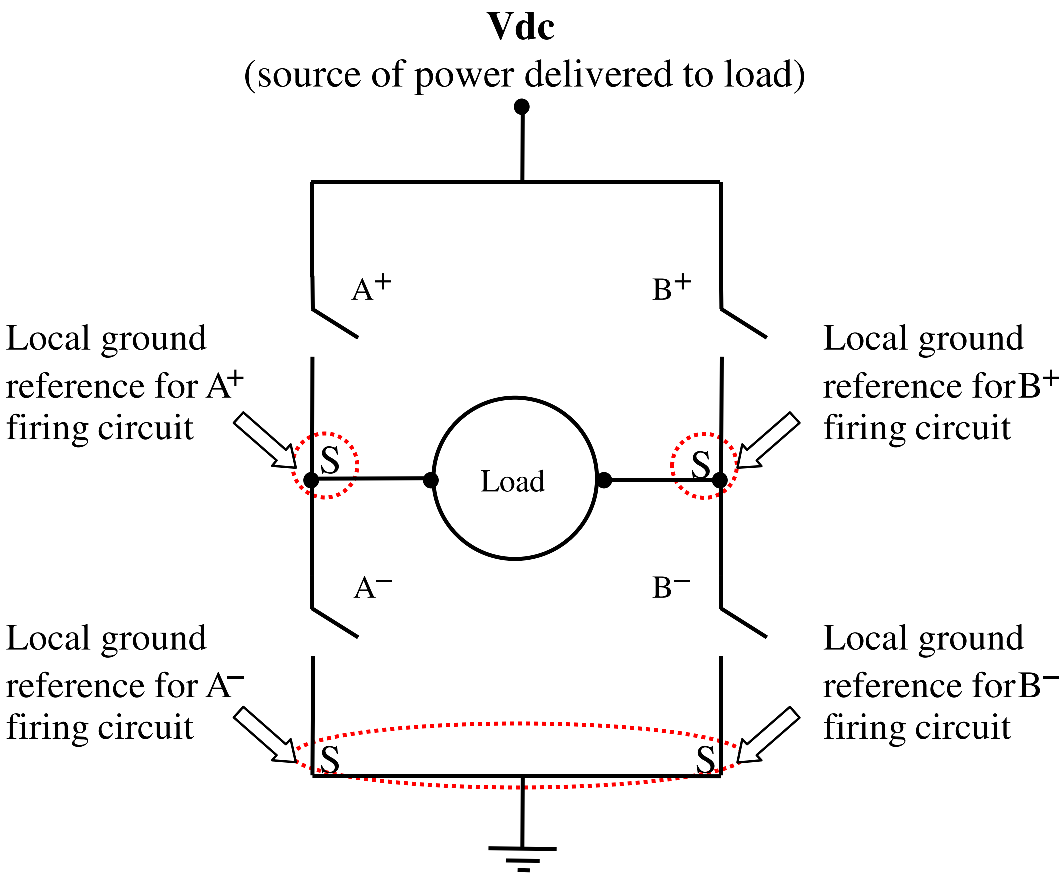
- Load consuming power
- Load generating power

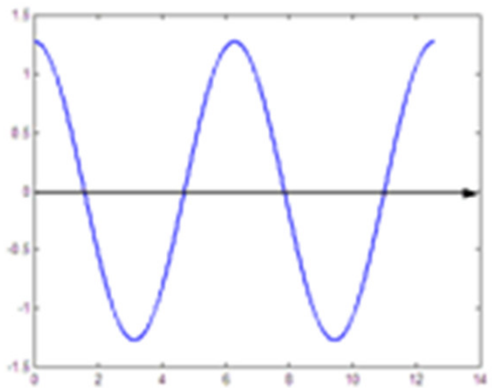
The four firing circuits do not have the same ground reference. Thus, the firing circuits require isolation.

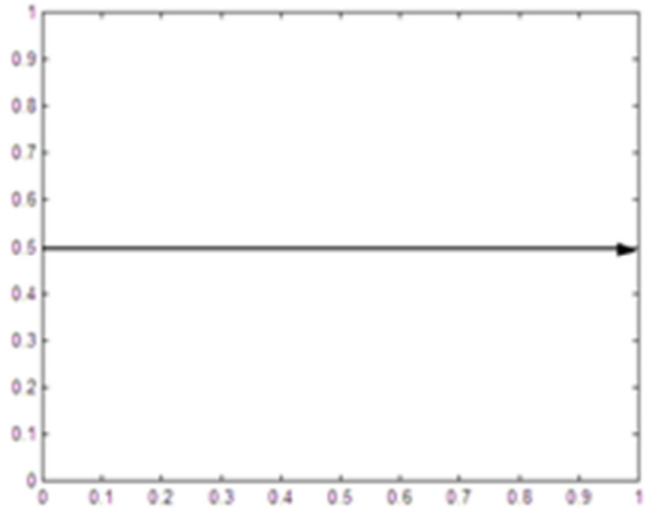


H-Bridge Inverter

- Harmonics with square wave modulation

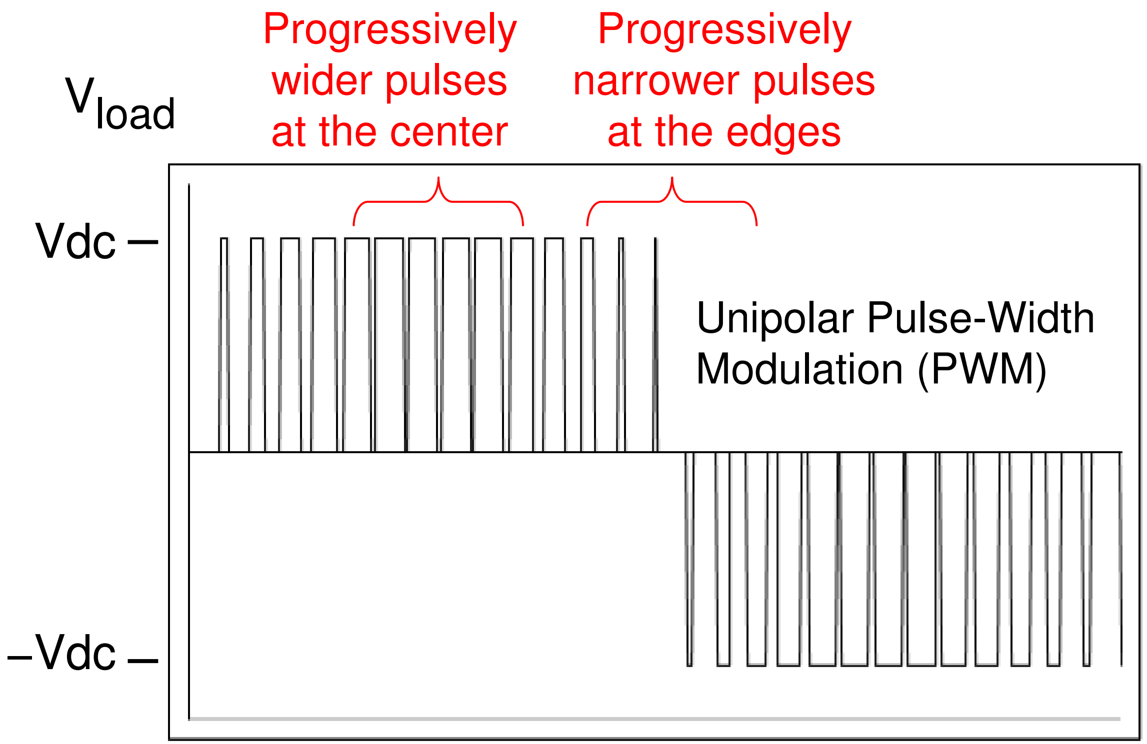
Harmonic #1 (fundamental)





Question - How can a sinusoidal (or other) input signal be amplified with low distortion?

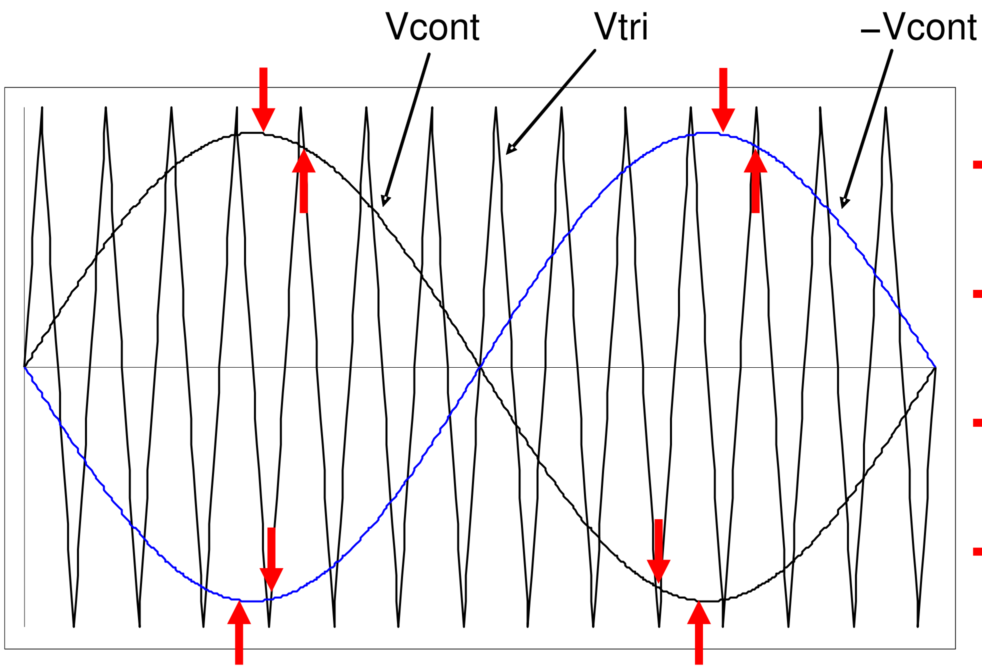
Answer – the switching can be controlled in a smart way so that the FFT of V_{load} has a strong fundamental component, plus high-frequency switching harmonics that can be easily filtered out and "thrown into the trash"



Implementation of Unipolar Pulse Width Modulation (PWM)

Vcont is the input signal we want to amplify at the output of the inverter.

Vcont is usually a sinewave, but it can also be a music signal.



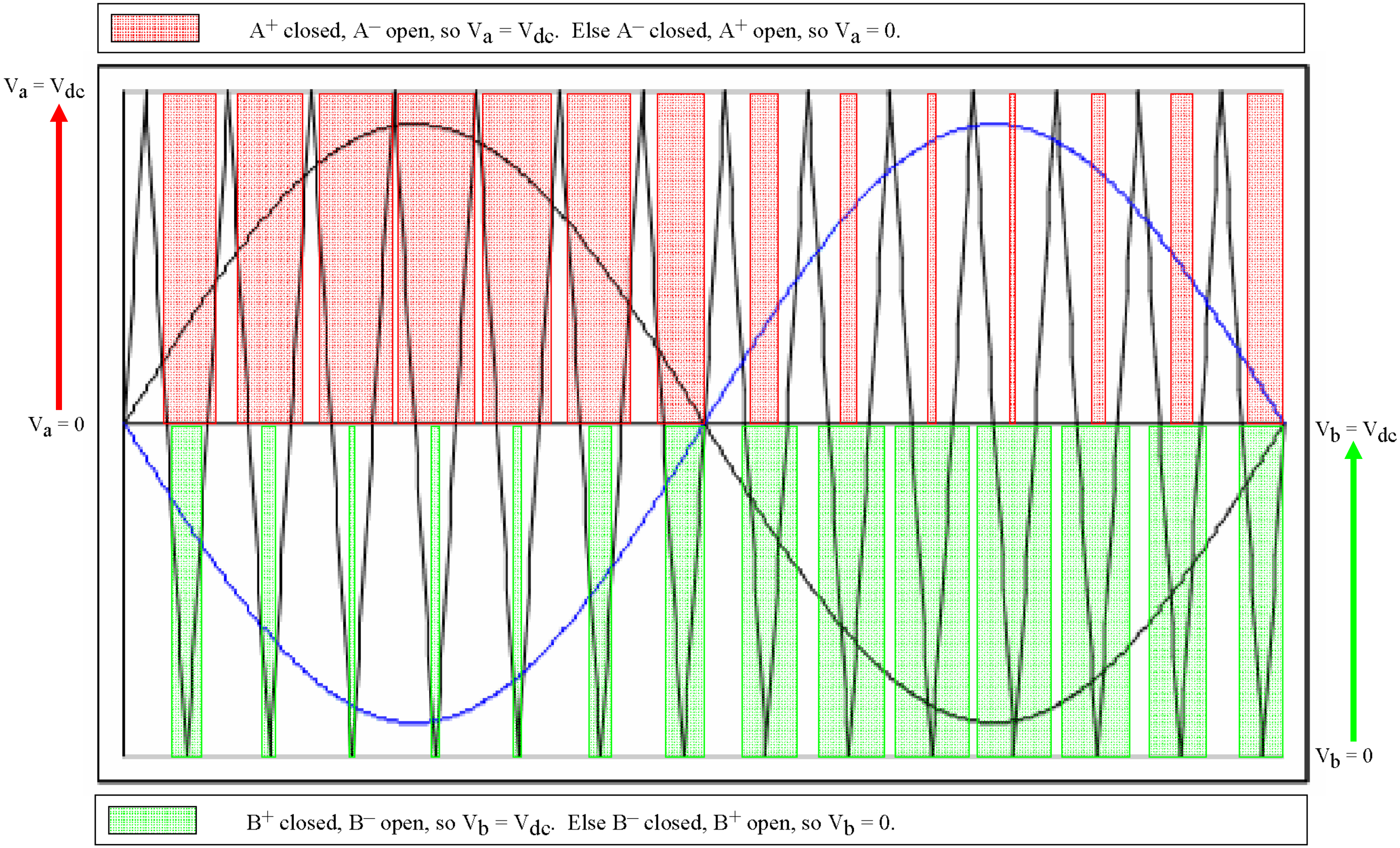
The implementation rules are:

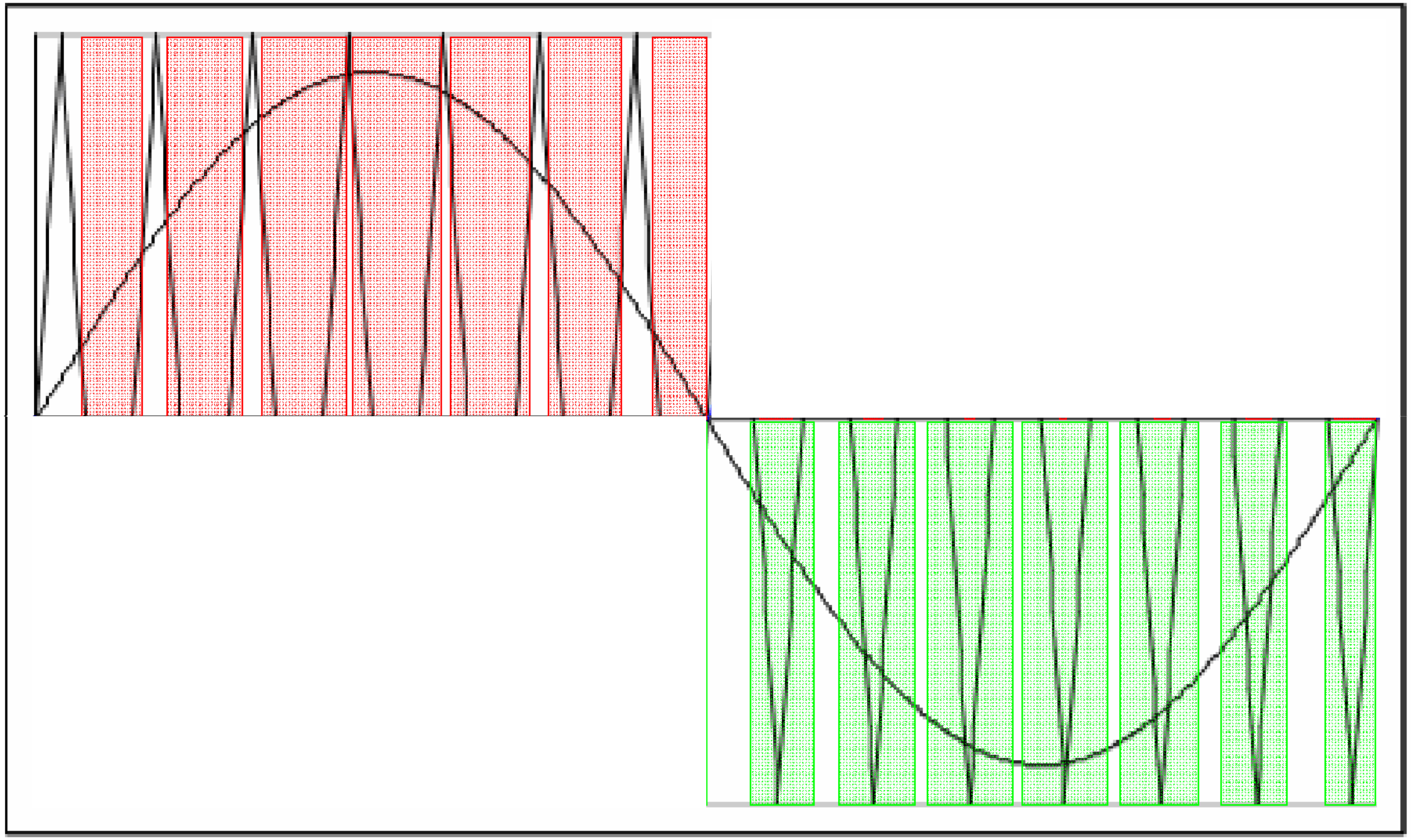
- Vcont > Vtri , close switch A+, open switch A– , so voltage Va = Vdc
- Vcont < Vtri , open switch A+, close switch A– , so voltage Va = 0
- –Vcont > Vtri , close switch B+, open switch B– , so voltage Vb = Vdc
- –Vcont < Vtri , open switch B+, close switch B– , so voltage Vb = 0

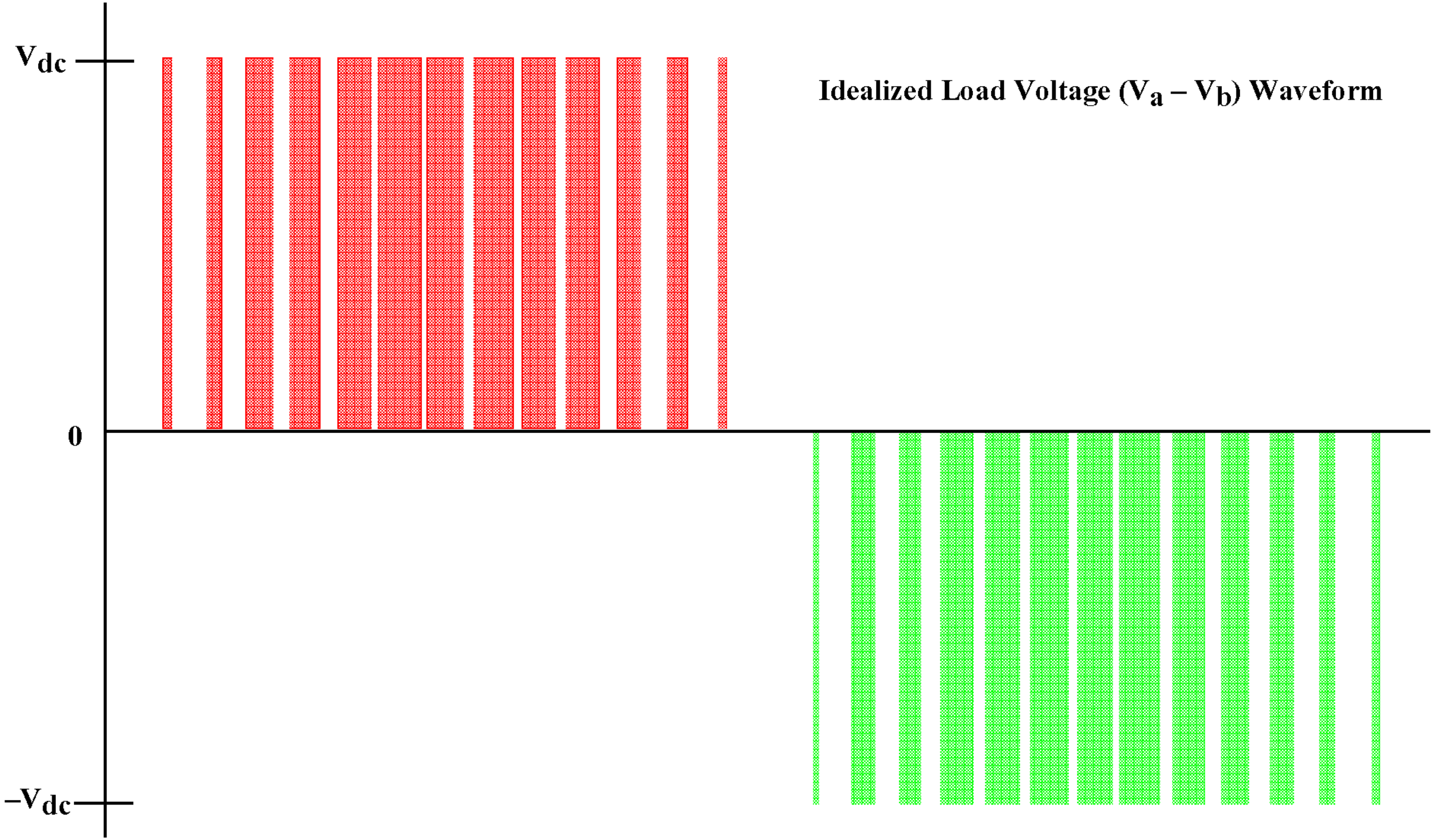
V_{tri} is a triangle wave whose frequency is at least 30 times greater than Vcont.

Ratio m_a = peak of control signal divided by peak of triangle wave

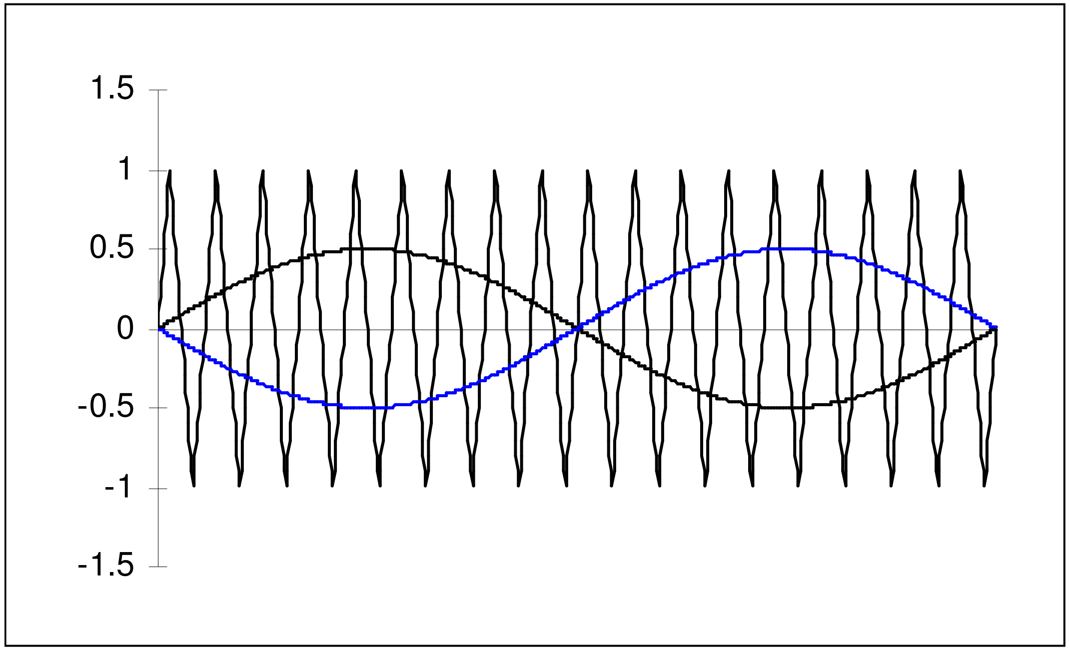
Ratio m_f = frequency of triangle wave divided by frequency of control signal


A^+ closed, A^- open, so $V_a = V_{dc}$. Else A^- closed, A^+ open, so $V_a = 0$.
$V_a = V_{dc}$
$V_a = 0$
$V_b = V_{dc}$
$V_b = 0$
B^+ closed, B^- open, so $V_b = V_{dc}$. Else B^- closed, B^+ open, so $V_b = 0$.





V_{dc}
Idealized Load Voltage ($V_a - V_b$) Waveform
0
$-V_{dc}$



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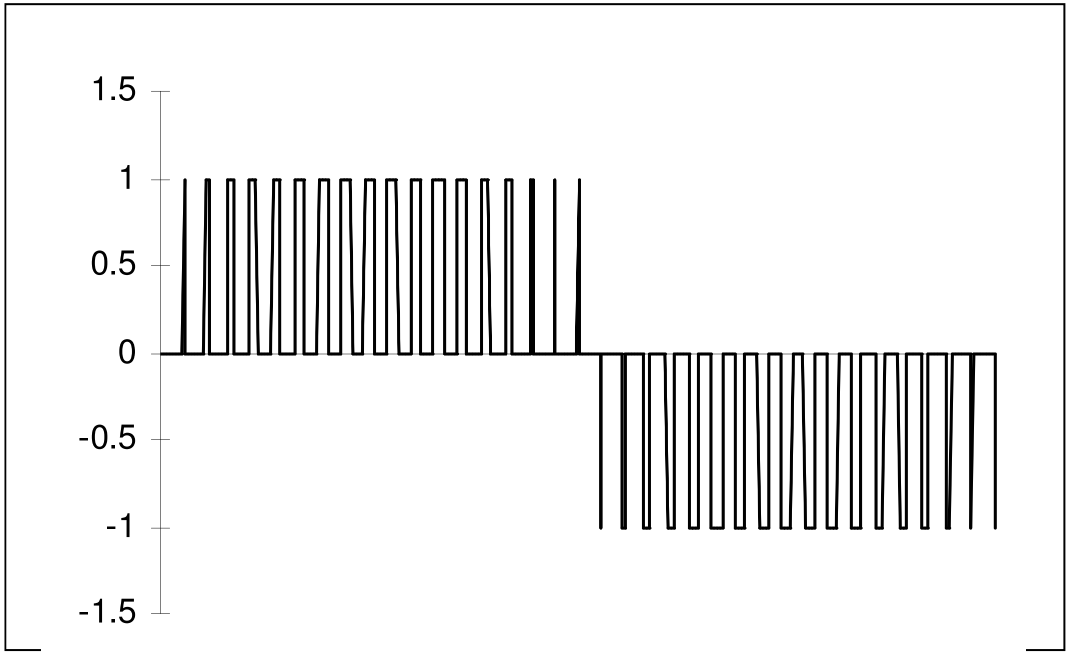
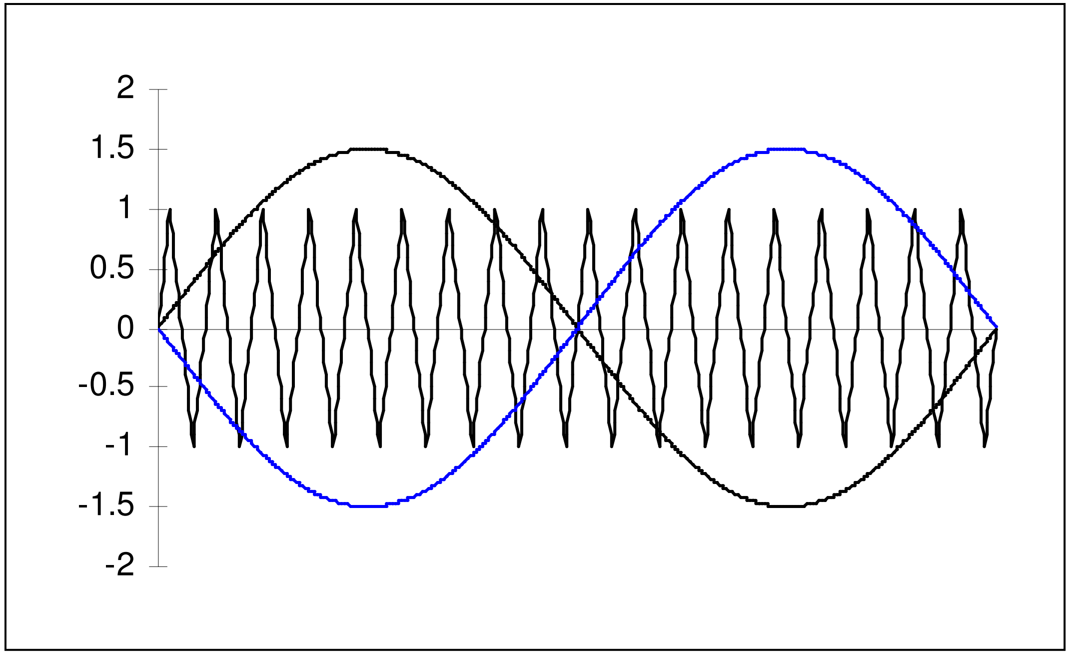
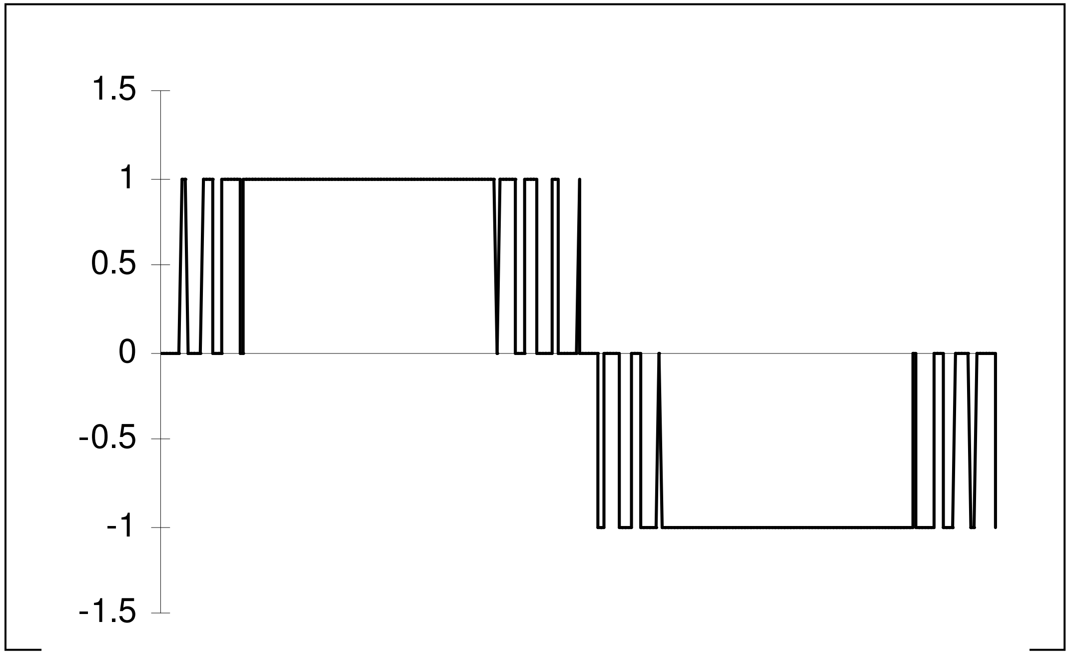


Load voltage with

$m_a = 0.5$

(i.e., in the linear region)





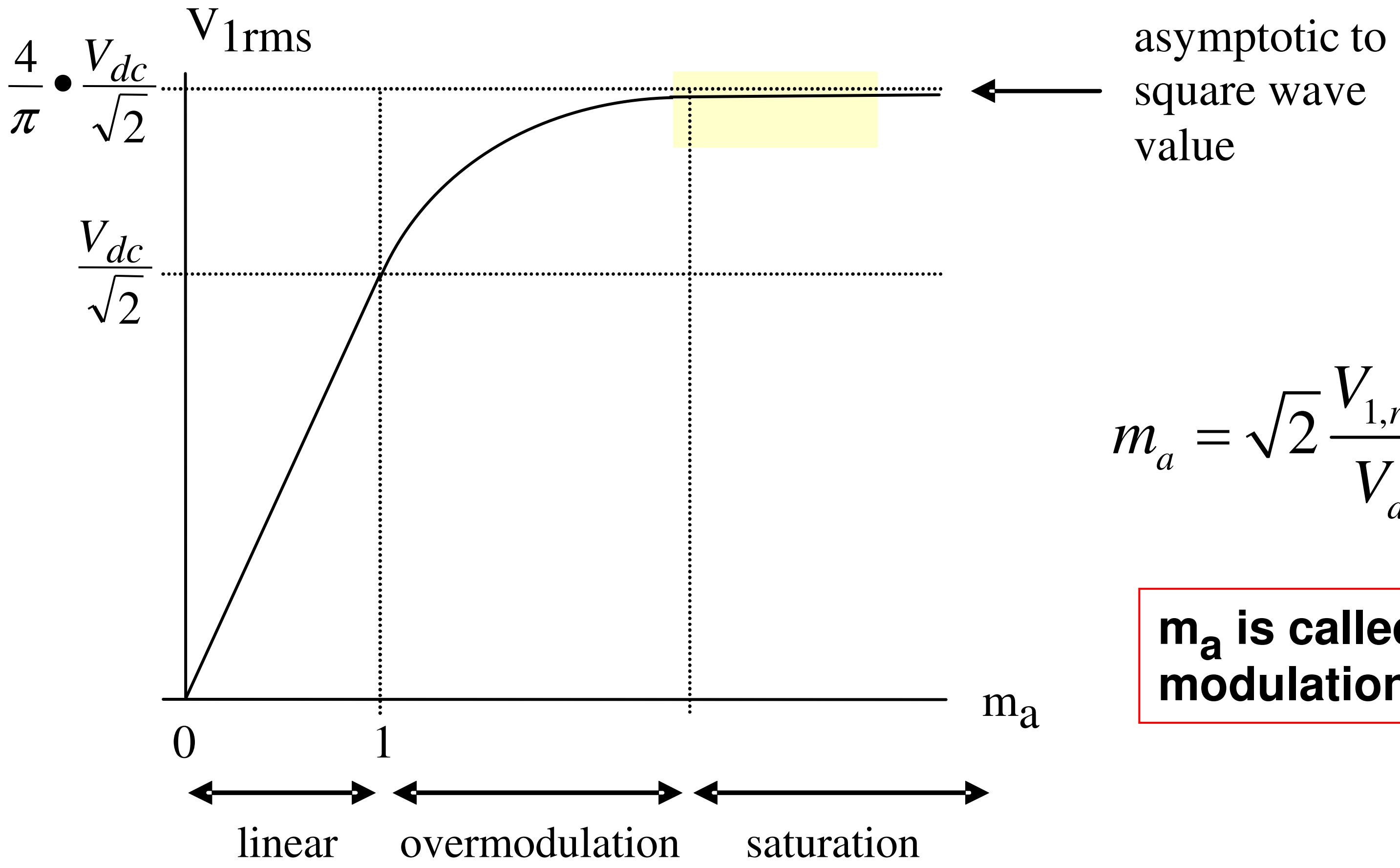
Load voltage with

m_a = 1.5

(i.e., overmodulation)

Variation of RMS value of no-load fundamental inverter output voltage (V_{1rms}) with m_a

For single-phase inverters m_a also equals the ratio between the peak output voltage and the input V_{dc} voltage.



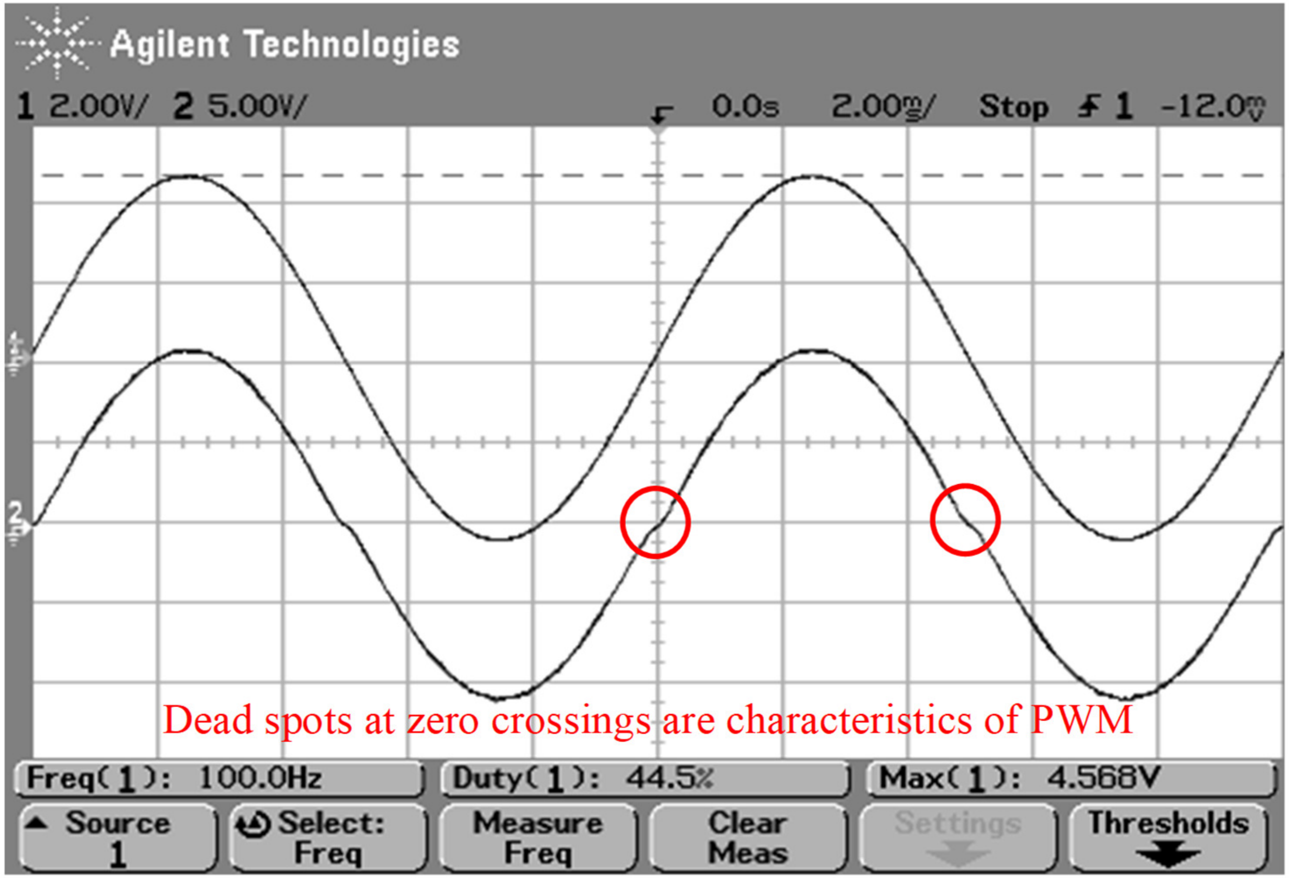
$$m_a = \sqrt{2}\frac{V_{1,rms}}{V_{dc}}$$

m_a is called the modulation index

RMS magnitudes of load voltage frequency components with respect to $\frac{V_{dc}}{\sqrt{2}}$ for $f_{tri} >> f_{cont}$

Frequency	$m_a = 0.2$	$m_a = 0.4$	$m_a = 0.6$	$m_a = 0.8$	$m_a = 1.0$	
f_{cont}	0.200	0.400	0.600	0.800	1.000	
$2f_{tri} \pm f_{cont}$	0.190	0.326	0.370	0.314	0.181	$2f_{tri}$ cluster
$2f_{tri} \pm 3f_{cont}$		0.024	0.071	0.139	0.212	
$2f_{tri} \pm 5f_{cont}$				0.013	0.033	
$4f_{tri} \pm f_{cont}$	0.163	0.157	0.008	0.105	0.068	$4f_{tri}$ cluster
$4f_{tri} \pm 3f_{cont}$	0.012	0.070	0.132	0.115	0.009	
$4f_{tri} \pm 5f_{cont}$			0.034	0.084	0.119	
$4f_{tri} \pm 7f_{cont}$				0.017	0.050	

100Hz Signal as Input, Inverter Output

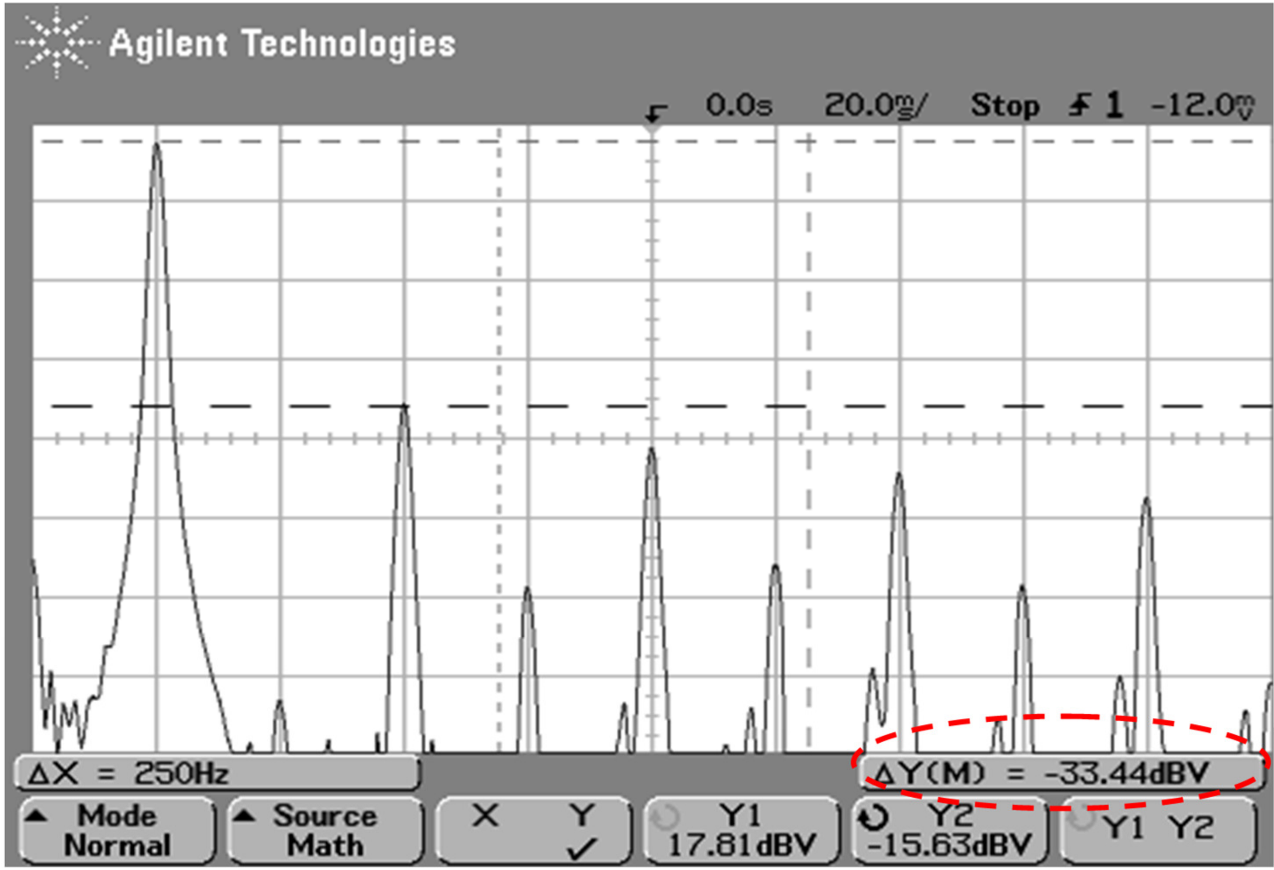


Waveform generator output

Inverter output

Top curve: 100Hz waveform generator output,
Bottom curve: Output of inverter powering 5Ω power load resistor
(scope set to average over one cycle)

FFT of 100Hz Inverter Output

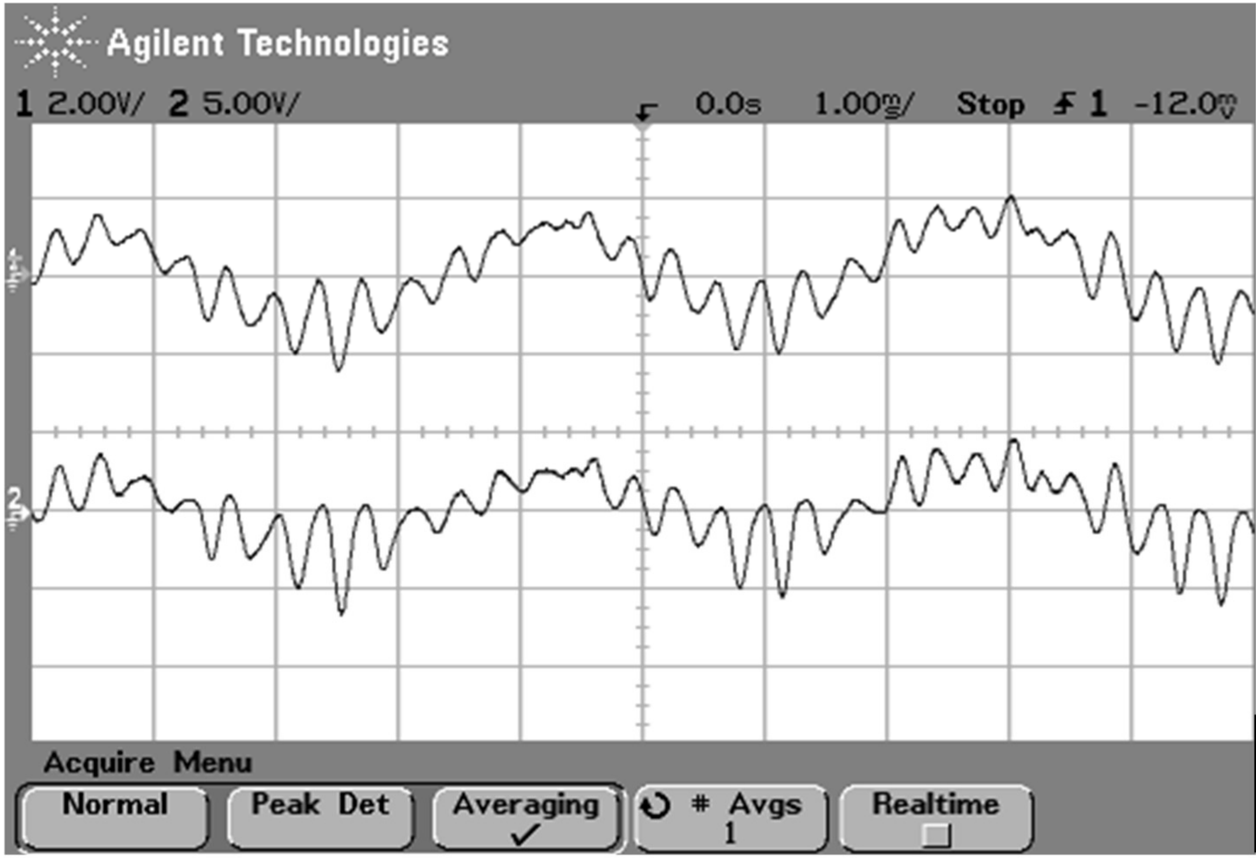


1kHz span, 500Hz center

Save screen snapshot #1

FFT of inverter output with 100Hz input signal

Inverter Performance with Music Input



CD player output

Inverter output

Save screen snapshot #3

Top curve: Audio output of CD player to inverter,
Bottom curve: Output of inverter to speakers
(scope set to average over one cycle)

PWM controlled H-Bridge Inverter

- **Very efficient**
- **Distortion higher than linear amplifier, but a linear amplifier has, at best, 50% efficiency**
- **Perfectly suited for motor drives where voltage and frequency control are needed**
- **Well suited for bass music amplification, such as automotive applications, or where high power is more important than a little loss in quality**