ΔΗΜΟΚΡΙΤΕΙΟ
ΠΑΝΕΠΙΣΤΗΜΙΟ
ΘΡΑΚΗΣ

Analysis of Electronic Converters DC / AC

Second Task
"Exercise 28"

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Task

If the output voltage of a single-phase bridge inverter is a rectangular pulse of 120° width and an amplitude of 100 Volts, do the following:

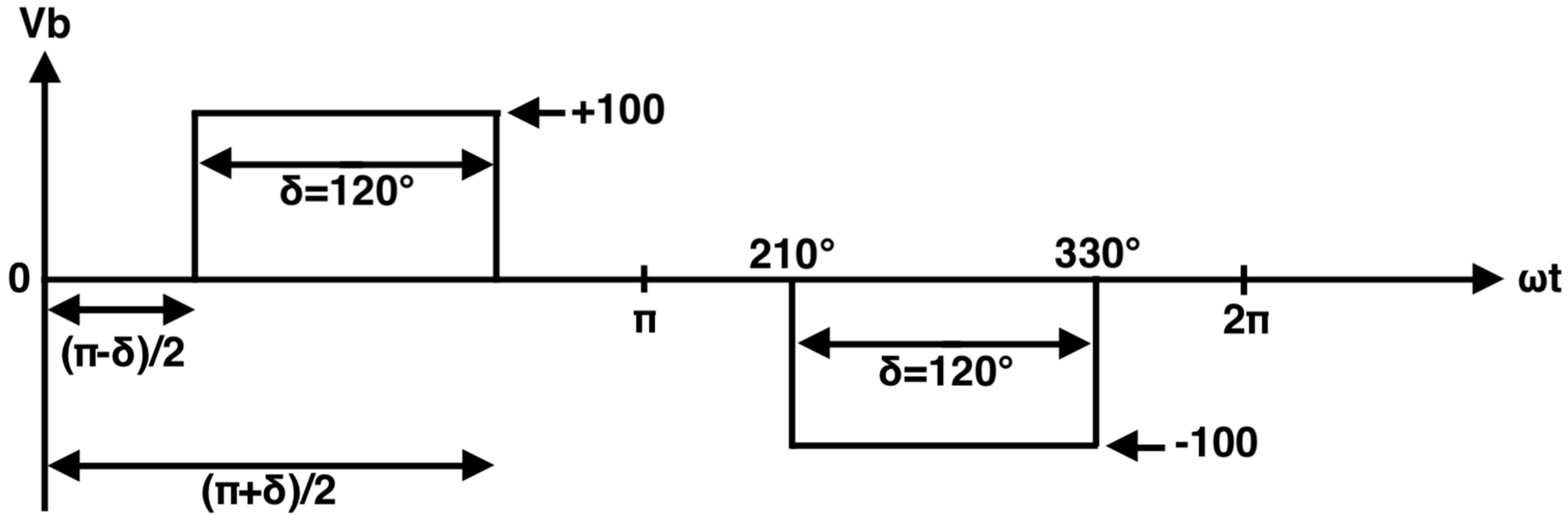
a) Represent the output voltage with Fourier series

b) Calculate the Total Harmonic Distortion (i.e., the THD coefficient) of the output voltage considering only the first 3 upper harmonic components (i.e., 3rd, 5th, 7th)

Solution

a) Represent the output voltage with Fourier series

In this case the ideal waveform of the inverter output voltage will be the following:



The figure shows that since $\delta = 120°$

$(\pi - \delta) / 2 = 30°$

$(\pi + \delta) / 2 = 150°$

Solution

It is generally true that:

$$V_0 = \frac{a_0}{2} + \sum_{n=1,2..}^{\infty} a_n \cdot \cos(n \cdot \omega t) + \sum_{n=1,2..}^{\infty} b_n \cdot \sin(n \cdot \omega t)$$

Where:

$$a_n = \frac{1}{\pi} \int_{\frac{-(\pi+\delta)}{2}}^{\frac{-(\pi-\delta)}{2}} -100 \cdot \cos(n \cdot \omega t) d\omega t + \frac{1}{\pi} \int_{\frac{\pi-\delta}{2}}^{\frac{\pi+\delta}{2}} 100 \cdot \cos(n \cdot \omega t) d\omega t \Rightarrow$$

$$= \frac{100}{n\pi} [-\sin(n\omega t)]_{-150}^{-30} + \frac{100}{n\pi} [\sin(n\omega t)]_{30}^{150} = 0$$

So, the results for n = 1,2,3,4,5,6,7…

Is that $a_n = 0$

Additionally, $a_0 = 0$

Solution

And for the bn factor:

$$b_n = \frac{1}{\pi} \int_{\frac{-(\pi+\delta)}{2}}^{\frac{-(\pi-\delta)}{2}} -100 \cdot \sin(n \cdot \omega t) d\omega t + \frac{1}{\pi} \int_{\frac{\pi-\delta}{2}}^{\frac{\pi+\delta}{2}} 100 \cdot \sin(n \cdot \omega t) d\omega t \Rightarrow$$

$$= \frac{100}{n\pi} [\cos(n\omega t)]_{-150}^{-30} + \frac{100}{n\pi} [-\cos(n\omega t)]_{30}^{150} \Rightarrow$$

$$= \frac{100}{n\pi} [\cos(-30n) - \cos(150n) - \cos(150n) + \cos(30n)] \Rightarrow$$

$$= \frac{200}{n\pi} [\cos(30n) - \cos(150n)]$$

Solution

So, the results for n=1,2,3,4,5,6,7…

That the $b_1 = 1,732$

That the $b_2 = 0$

That the $b_3 = 0$

That the $b_4 = 0$

That the $b_5 = -1,732$

That the $b_6 = 0$

That the $b_7 = -1,732$

That the $b_8 = 0$

That the $b_9 = 0$

That the $b_{10} = 0$

Solution

Therefore, the output voltage will be:

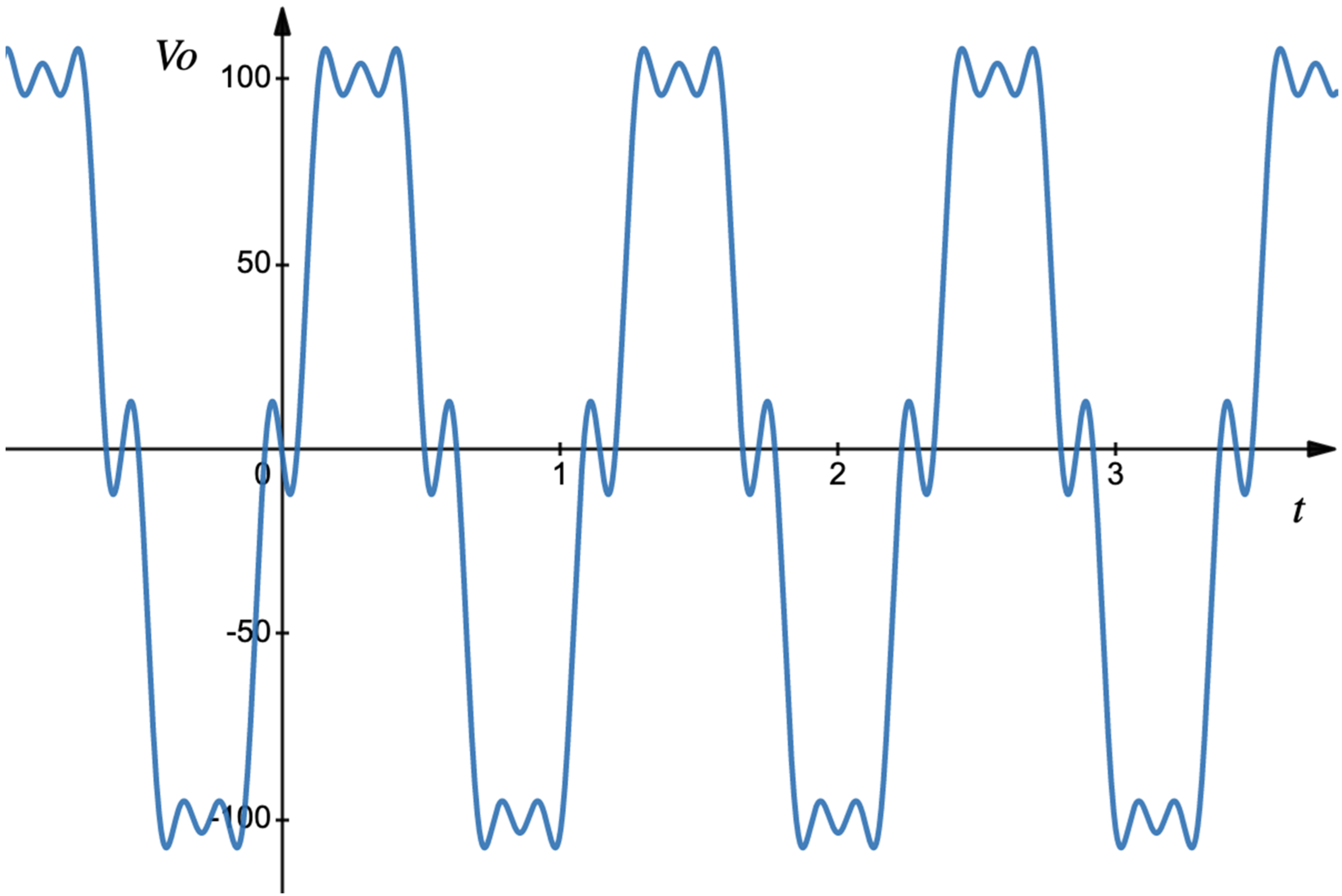
$$V_0 = \frac{a_0}{2} + \sum_{n=1,2..}^{\infty} a_n \cdot \cos(n \cdot \omega t) + \sum_{n=1,2..}^{\infty} b_n \cdot \sin(n \cdot \omega t) \Rightarrow$$

$$V_0 = \sum_{n=1,3,5..}^{\infty} b_n \sin(n\omega t) \Rightarrow$$

$$V_0 = \sum_{n=1,3,5..}^{\infty} \frac{200}{n\pi} \{\cos(30n) - \cos(150n)\} \sin n\omega t$$

Solution

The figure of Vo using the first 3 higher harmonics:



Solution

b) Calculate the Total Harmonic Distortion (i.e., the THD coefficient) of the output voltage considering only the first 3 upper harmonic components (i.e., 3rd, 5th, 7th)

The Coefficient of Total Harmonic Distortion (THD) is a quality factor of the output voltage. If this coefficient is zero it means that the output voltage is purely sinusoidal.

Solution

This factor is given by the following formula:

$$THD_V = \frac{1}{\hat{V}_{o,1}} \times \Big(\sum_{n=3,5,7...}^{\infty} (\hat{V}_{o,n})^2 \Big)^{1/2} \times 100$$

Where:

$V_{o,1}$ = The voltage amplitude in the basic harmonic component

$V_{o,n}$ = The voltage amplitude in the n^{th} harmonic component

Solution

Considering only the first three higher harmonic components:

$$THD_V = \frac{1}{\hat{V}_{o,1}} \times (\hat{V}_{o,3}{}^2 + \hat{V}_{o,5}{}^2 + \hat{V}_{o,7}{}^2)^{1/2} \times 100$$

Where:

$$\hat{V}_{o,1} = b_1 = \frac{200}{\pi}(-\cos 150^o + \cos 30^0) = 110,2658 \text{ Volts}$$

$$\hat{V}_{o,3} = b_3 = \frac{200}{3 \times \pi}\{-\cos(3 \times 150^o) + \cos(3 \times 30^o)\} = 0 \text{ Volts}$$

$$\hat{V}_{o,5} = b_5 = \frac{200}{5 \times \pi}\{-\cos(5 \times 150^o) + \cos(5 \times 30^o)\} = -22,0532 \text{ Volts}$$

$$\hat{V}_{o,7} = b_7 = \frac{200}{7 \times \pi}\{-\cos(7 \times 150^o) + \cos(7 \times 30^o)\} = -15,7523 \text{ Volts}$$

Solution

So, the result for the THD factor is:

$$THD_V = \frac{1}{110,2658} \times (0^2 + 22,0532^2 + 15,7523^2)^{1/2} \times 100 \Rightarrow$$

$$THD_V = \frac{27,1013}{110,2658} \times 100 = 24,5782\%$$

Thank you for your attention!