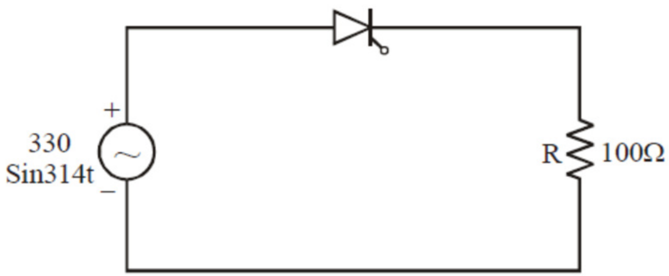
1,2,3-pulse rectifiers problems

Dr.-El. Eng. Nikos Papanikolaou

Associate Professor

Problem 1

What will be the average power in the load for the circuit shown, when $\alpha = \frac{\pi}{4}$. Assume SCR to be ideal. Supply voltage is 330 sin314t. Also calculate the RMS power and the rectification efficiency.



The circuit is that of a single phase half wave controlled rectifier with a resistive load

$$V_{dc} = \frac{V_m}{2\pi}(1+\cos\alpha) \quad ; \quad \alpha = \frac{\pi}{4} \ radians$$

$$V_{dc} = \frac{330}{2\pi}\left(1+\cos\left(\frac{\pi}{4}\right)\right)$$

$$V_{dc} = 89.66 \text{ Volts}$$

$$\text{Average Power} = \frac{V_{dc}^2}{R} = \frac{89.66^2}{100} = 80.38 \text{ Watts}$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{89.66}{100} = 0.8966 \text{ Amps}$$

$$V_{RMS} = \frac{V_m}{2}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$

$$V_{RMS} = \frac{330}{2}\left[\frac{1}{\pi}\left(\pi - \frac{\pi}{4} + \frac{\sin 2\times\frac{\pi}{4}}{2}\right)\right]^{\frac{1}{2}}$$

$$V_{RMS} = 157.32 \ V$$

RMS Power (AC power)

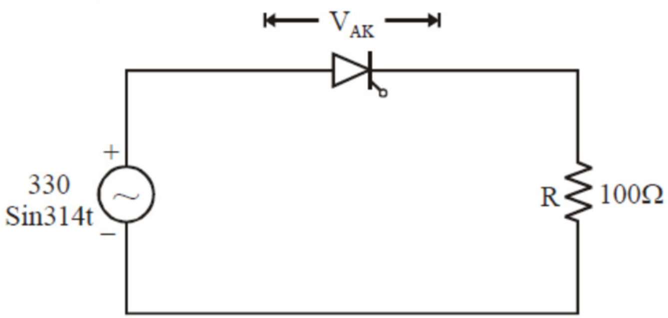
$$= \frac{V_{RMS}^2}{R} = \frac{157.32^2}{100} = 247.50 \text{ Watts}$$

$$\text{Rectification Efficiency} = \frac{\text{Average power}}{\text{RMS power}}$$

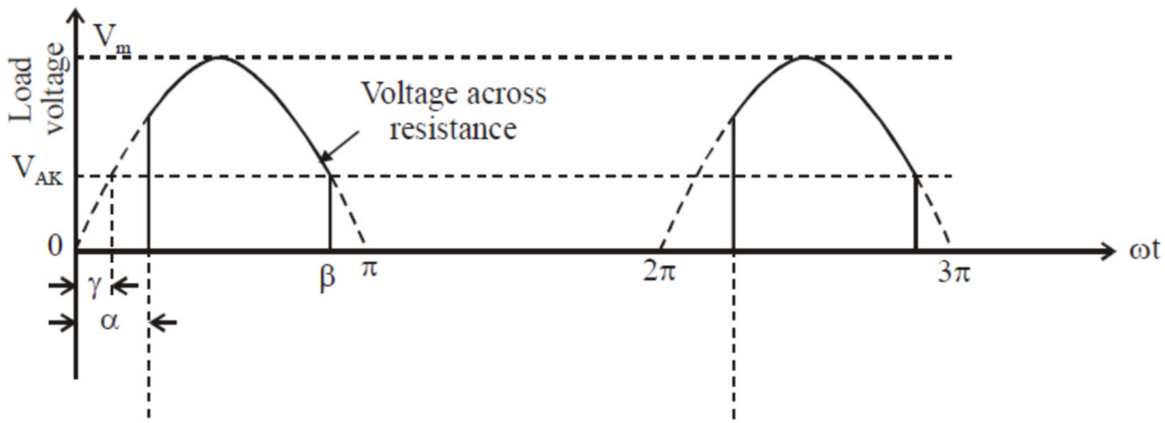
$$= \frac{80.38}{247.47} = 0.3248$$

Problem 2

In the circuit shown find out the average voltage across the load assuming that the conduction drop across the SCR is 1 volt. Take $\alpha = 45^0$.



The wave form of the load voltage is shown below (not to scale).



It is observed that the SCR turns off when $\omega t = \beta$, where $\beta = (\pi - \gamma)$ because the SCR turns-off for anode supply voltage below 1 Volt.

$$V_{AK} = V_m \sin\gamma = 1 \text{ volt (given)}$$

$$\text{Therefore} \quad \gamma = \sin^{-1}\left(\frac{V_{AK}}{V_m}\right) = \sin^{-1}\left(\frac{1}{330}\right) = 0.17^0\ (0.003 \text{ radians})$$

$$\beta = (180^0 - \gamma) \quad ; \text{ By symmetry of the curve.}$$

$$\beta = 179.83^0 \quad ; \ 3.138 \text{ radians.}$$

Problem 2 (cont.)

$$V_{dc} = \frac{1}{2\pi}\int_{\alpha}^{\beta}(V_m \sin\omega t - V_{AK})\,d(\omega t)$$

$$V_{dc} = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} V_m \sin\omega t.d(\omega t) - V_{AK}\int_{\alpha}^{\beta} d(\omega t)\right]$$

$$V_{dc} = \frac{1}{2\pi}\left[V_m(-\cos\omega t)\Big/_{\alpha}^{\beta} - V_{AK}(\omega t)\Big/_{\alpha}^{\beta}\right]$$

Note: β and α values should be in radians

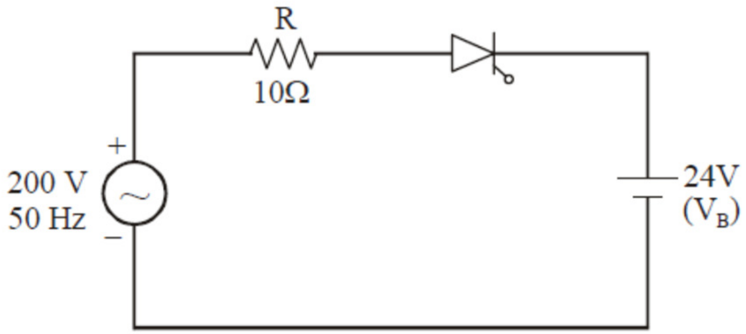
$$V_{dc} = \frac{1}{2\pi}\left[V_m(\cos\alpha - \cos\beta) - V_{AK}(\beta - \alpha)\right]$$

$$V_{dc} = \frac{1}{2\pi}\left[330\left(\cos 45^0 - \cos 179.83^0\right) - 1(3.138 - 0.003)\right]$$

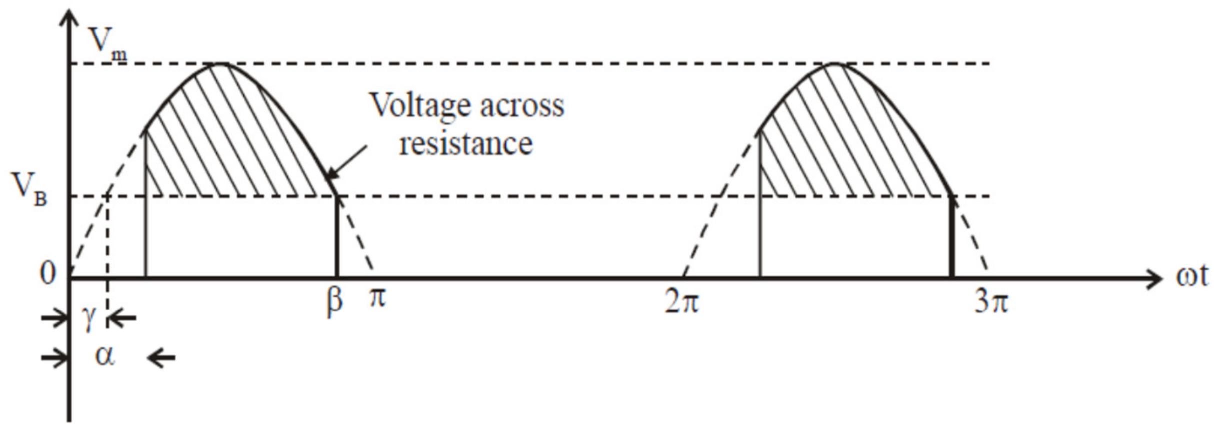
$$V_{dc} = 89.15 \text{ Volts}$$

Problem 3

In the figure find out the battery charging current when $\alpha = \frac{\pi}{4}$. Assume ideal SCR.



It is obvious that the SCR cannot conduct when the instantaneous value of the supply voltage is less than 24 V, the battery voltage. The load voltage waveform is as shown (voltage across ion).



$$V_B = V_m \sin\gamma$$

$$24 = 200\sqrt{2}\sin\gamma$$

Problem 3 (cont.)

$$\gamma = \sin^{-1}\left(\frac{24}{200\times\sqrt{2}}\right) = 4.8675^0 = 0.085 \text{ radians}$$

$$\beta = \pi - \gamma = 3.056 \text{ radians}$$

Average value of voltage across 10Ω

$$= \frac{1}{2\pi}\left[\int_{\alpha}^{\beta}(V_m \sin\omega t - V_B).d(\omega t)\right]$$

(The integral gives the shaded area)

$$= \frac{1}{2\pi}\left[\int_{\frac{\pi}{4}}^{3.056}\left(200\times\sqrt{2}\sin\omega t - 24\right).d(\omega t)\right]$$

$$= \frac{1}{2\pi}\left[200\sqrt{2}\left(\cos\frac{\pi}{4} - \cos 3.056\right) - 24\left(3.056 - \frac{\pi}{4}\right)\right]$$

$$= 68 \text{ Vots}$$

Therefore charging current

$$= \frac{\text{Average voltage across R}}{R}$$

$$= \frac{68}{10} = 6.8 \text{ Amps}$$

Note: If value of γ is more than α, then the SCR will trigger only at $\omega t = \gamma$, (assuming that the gate signal persists till then), when it becomes forward biased.

Therefore $$V_{dc} = \frac{1}{2\pi}\left[\int_{\gamma}^{\beta}(V_m \sin\omega t - V_B).d(\omega t)\right]$$

Problem 4

In a single phase full wave rectifier supply is 200 V AC. The load resistance is 10Ω, $\alpha = 60^0$. Find the average voltage across the load and the power consumed in the load.

In a single phase full wave rectifier

$$V_{dc} = \frac{V_m}{\pi}(1 + \cos\alpha)$$

$$V_{dc} = \frac{200 \times \sqrt{2}}{\pi}(1 + \cos 60^0)$$

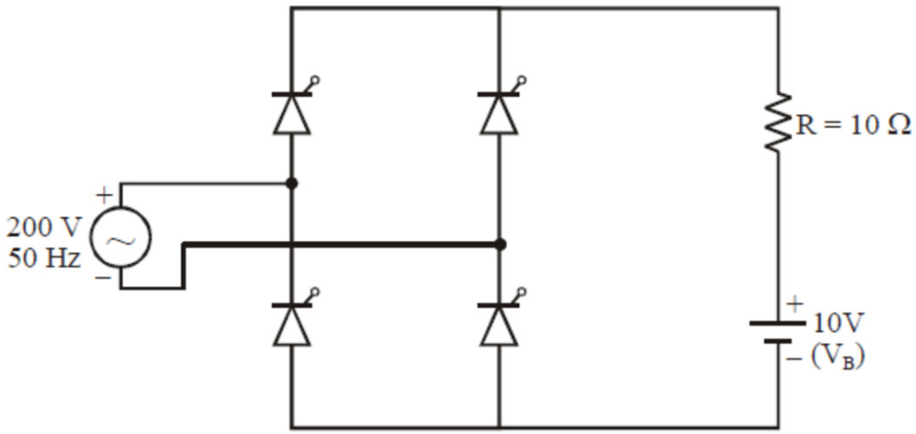
$$V_{dc} = 135 \text{ Volts}$$

Average Power

$$= \frac{V_{dc}^2}{R} = \frac{135^2}{10} = 1.823\ kW$$

Problem 5

In the circuit shown find the charging current if the trigger angle $\alpha = 90^0$.



With the usual notation

$$V_B = V_m \sin\gamma$$

$$10 = 200\sqrt{2}\sin\gamma$$

Therefore $$\gamma = \sin^{-1}\left(\frac{10}{200\times\sqrt{2}}\right) = 0.035 \text{ radians}$$

$$\alpha = 90^0 = \frac{\pi}{2} \text{ radians} \quad ; \ \beta = (\pi - \gamma) = 3.10659$$

$$\text{Average voltage across } 10\Omega = \frac{2}{2\pi}\left[\int_\alpha^\beta (V_m \sin\omega t - V_B).d(\omega t)\right]$$

$$= \frac{1}{\pi}\left[-V_m \cos\omega t - V_B(\omega t)\right]_\alpha^\beta$$

$$= \frac{1}{\pi}\left[V_m(\cos\alpha - \cos\beta) - V_B(\beta - \alpha)\right]$$

$$= \frac{1}{\pi}\left[200\times\sqrt{2}\left(\cos\frac{\pi}{2} - \cos 3.106\right) - 10\left(3.106 - \frac{\pi}{2}\right)\right]$$

$$= 85 \text{ V}$$

Note that the values of α & β are in radians.

$$\text{Charging current} = \frac{\text{dc voltage across resistance}}{\text{resistance}}$$

$$= \frac{85}{10} = 8.5 \text{ Amps}$$

Problem 6

A single phase full wave controlled rectifier is used to supply a resistive load of 10 Ω from a 230 V, 50 Hz, supply and firing angle of 90^0. What is its mean load voltage? If a large inductance is added in series with the load resistance, what will be the new output load voltage?

For a single phase full wave controlled rectifier with resistive load,

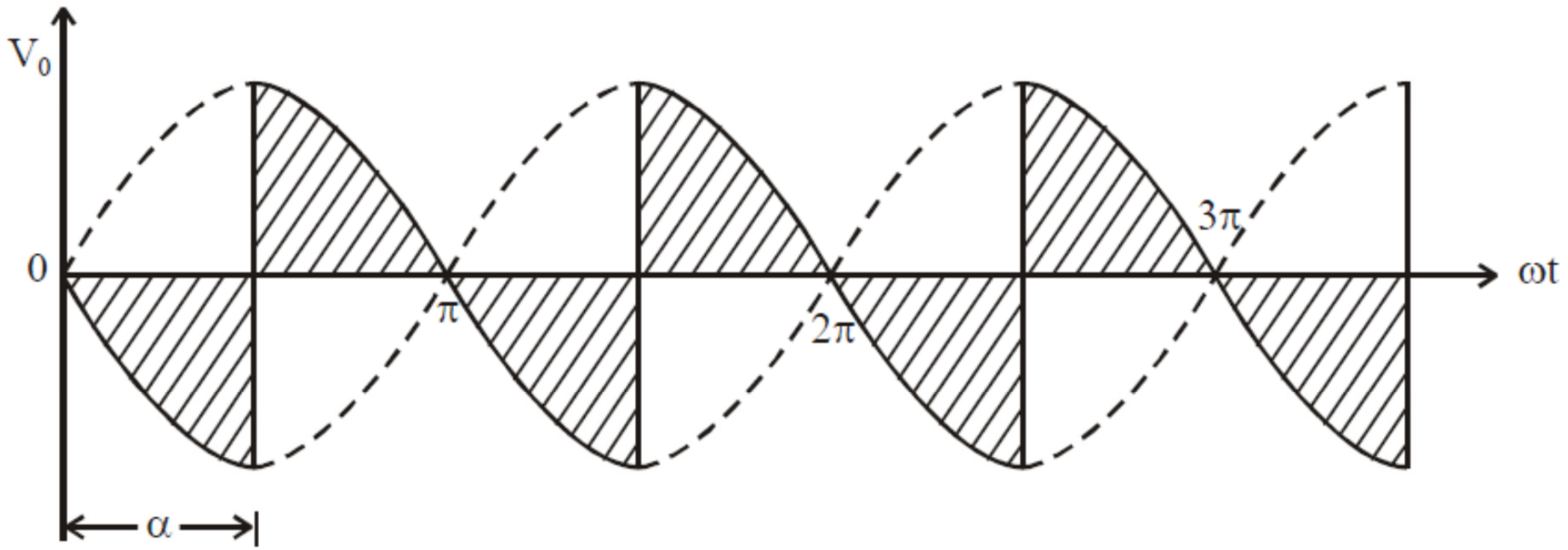
$$V_{dc} = \frac{V_m}{\pi}(1+\cos\alpha)$$

$$V_{dc} = \frac{230\times\sqrt{2}}{\pi}\left(1+\cos\frac{\pi}{2}\right)$$

$$V_{dc} = 103.5 \text{ Volts}$$

Problem 6 (cont.)

When a large inductance is added in series with the load, the output voltage wave form will be as shown below, for trigger angle $\alpha = 90^0$.



$$V_{dc} = \frac{2V_m}{\pi}\cos\alpha$$

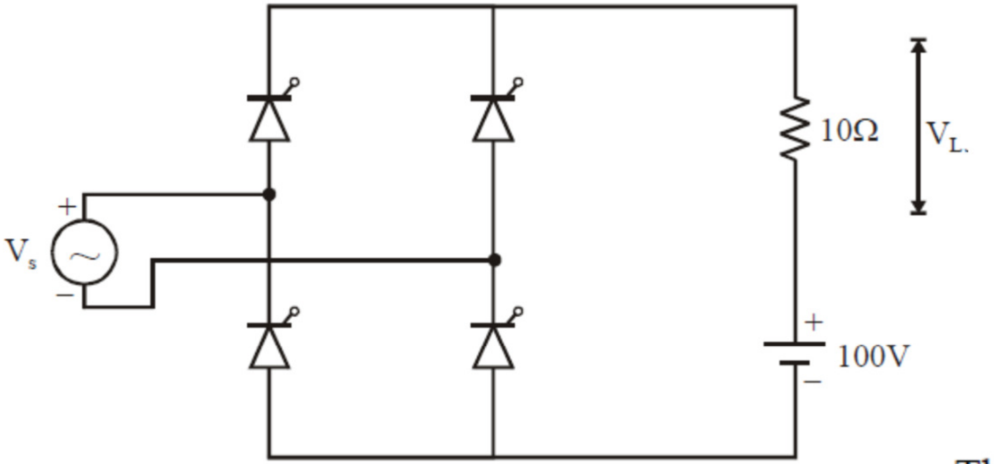
Since $\alpha = \frac{\pi}{2}$; $cos\alpha = \cos\left(\frac{\pi}{2}\right) = 0$

Therefore $V_{dc} = 0$ and this is evident from the waveform also.

Problem 7

The figure shows a battery charging circuit using SCRs. The input voltage to the circuit is 230 V RMS. Find the charging current for a firing angle of 45^0. If any one of the SCR is open circuited, what is the charging current?

Solution



With the usual notations

$$V_S = V_m \sin \omega t$$

$$V_S = \sqrt{2} \times 230 \sin \omega t$$

$V_m \sin \gamma = V_B$, the battery voltage

$$\sqrt{2} \times 230 \sin \gamma = 100$$

Therefore $$\gamma = \sin^{-1}\left(\frac{100}{\sqrt{2} \times 230}\right)$$

$\gamma = 17.9^0$ or 0.312 radians

$$\beta = (\pi - \gamma) = (\pi - 0.312)$$

$\beta = 2.829$ radians

Problem 7 (cont.)

Average value of voltage across load resistance

$$=\frac{2}{2\pi}\left[\int_{\alpha}^{\beta}\left(V_m\sin\omega t-V_B\right)d\left(\omega t\right)\right]$$

$$=\frac{1}{\pi}\left[-V_m\cos\omega t-V_B\left(\omega t\right)\right]_{\alpha}^{\beta}$$

$$=\frac{1}{\pi}\left[V_m\left(\cos\alpha-\cos\beta\right)-V_B\left(\beta-\alpha\right)\right]$$

$$=\frac{1}{\pi}\left[230\times\sqrt{2}\left(\cos\frac{\pi}{4}-\cos 2.829\right)-100\left(2.829-\frac{\pi}{4}\right)\right]$$

$$=\frac{1}{\pi}\left[230\times\sqrt{2}\left(0.707+0.9517\right)-204.36\right]$$

$$=106.68\text{ Volts}$$

$$\text{Charging current }=\frac{\text{Voltage across resistance}}{R}$$

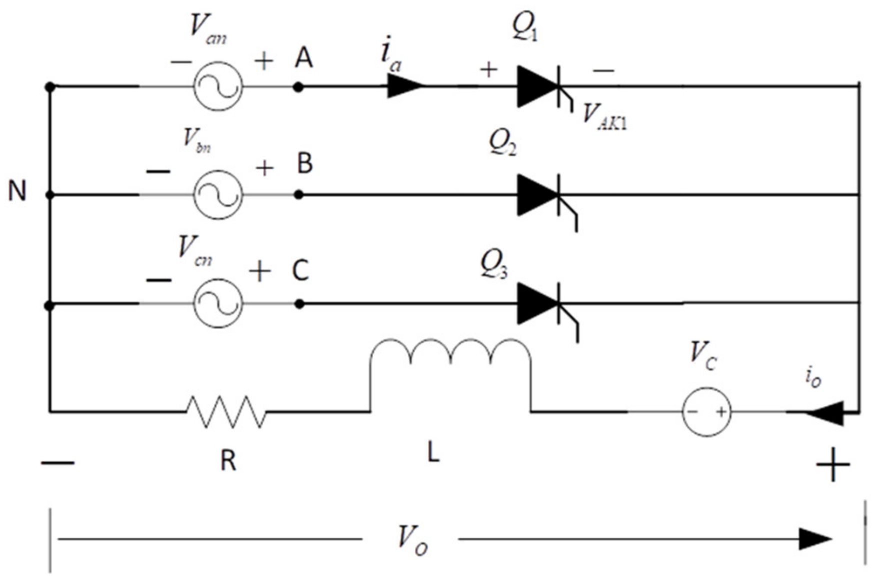
$$=\frac{106.68}{10}=10.668\text{ Amps}$$

If one of the SCRs is open circuited, the circuit behaves like a half wave rectifier. The average voltage across the resistance and the charging current will be half of that of a full wave rectifier.

$$\text{Therefore Charging Current }=\frac{10.668}{2}=5.334\text{ Amps}$$

Problem 8

A 3 phase half controlled bridge rectifier is feeding a RL load. If input voltage is 400 sin314t and SCR is fired at $\alpha = \frac{\pi}{4}$. Find average load voltage. If any one supply line is disconnected what is the average load voltage.



$\alpha = \frac{\pi}{4}$ radians which is less than $\frac{\pi}{3}$

Therefore $$V_{dc} = \frac{3V_m}{2\pi}[1+\cos\alpha]$$

$$V_{dc} = \frac{3\times 400}{2\pi}\left[1+\cos 45^0\right]$$

$$V_{dc} = 326.18 \text{ Volts}$$

If any one supply line is disconnected, the circuit behaves like a single phase half controlled rectifies with RL load.

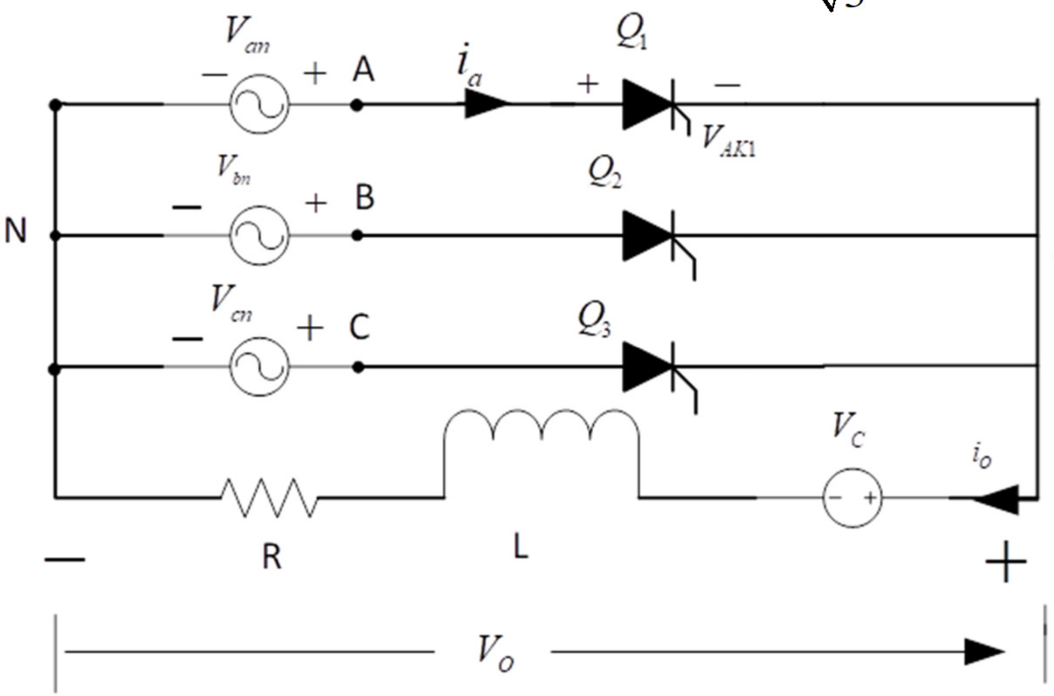
$$V_{dc} = \frac{V_m}{\pi}[1+\cos\alpha]$$

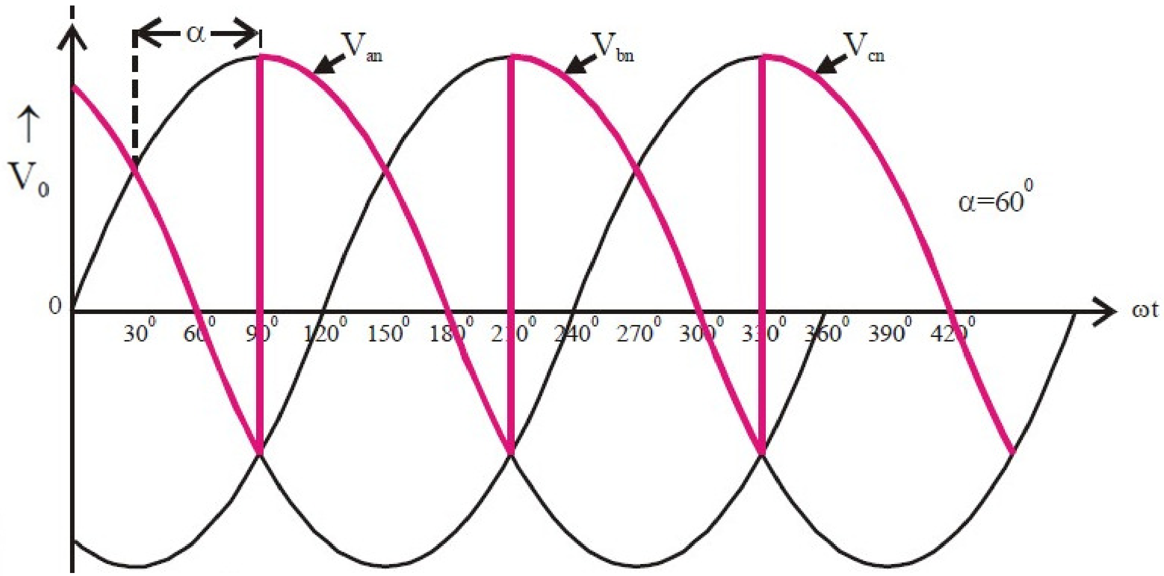
$$V_{dc} = \frac{400}{\pi}\left[1+\cos 45^0\right]$$

$$V_{dc} = 217.45 \text{ Volts}$$

Problem 9

$$V_{AN} = \frac{400\sqrt{2}}{\sqrt{3}} sin120\pi t \ , \ V_C = 40V \ , \ R = 15\Omega \ , \text{ constant load current}$$





Calculate average and RMS load current values, α = 60°

Problem 9 (cont.)

$$V_o = \frac{3V_{Lm}}{2\pi}\cos\alpha = \frac{3\cdot 400\sqrt{2}}{2\pi}\cos 60^o = \frac{3\cdot 400\sqrt{2}}{2\pi}\frac{1}{2} = 135V$$

$$V_{o,rms} = \sqrt{3}V_m\left[\frac{1}{6}+\frac{\sqrt{3}}{8\pi}\cos 2\alpha\right]^{\frac{1}{2}} = \sqrt{3}\,\frac{400\sqrt{2}}{\sqrt{3}}\left[\frac{1}{6}-\frac{\sqrt{3}}{8\pi}\frac{1}{2}\right]^{\frac{1}{2}} = 204V$$

$$I_o = \frac{V_o - V_C}{R} = \frac{135V - 40V}{15\Omega} = 6.3A$$

$$V_{o,rms} = \sqrt{\left(RI_{o,rms}\right)^2 + V_C^2} \Rightarrow I_{o,rms} = \frac{\sqrt{V_{o,rms}^2 - V_C^2}}{R} = \frac{\sqrt{204^2 - 40^2}}{15} = 13.3A$$