Additional Problems on Power Electronics

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Professor

EXERCISE 1

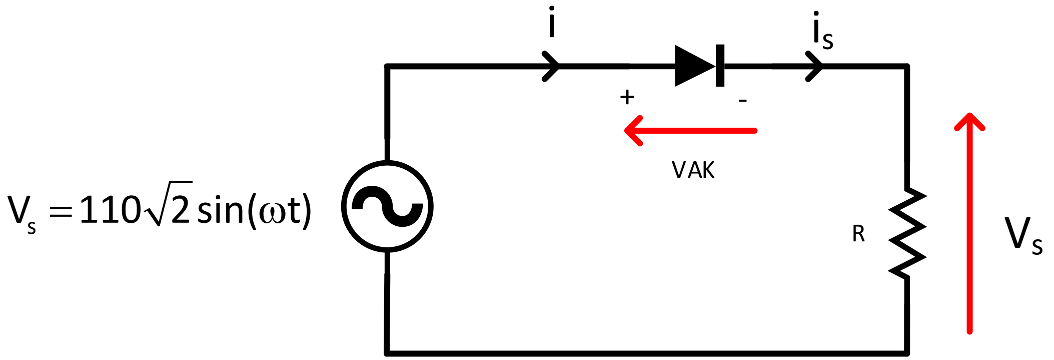
For the following circuit $V_s = 110\sqrt{2}\sin(\omega t)$. The waveforms of the variables V_s, V_o , i_o , V_{AK} have to be drawn. Moreover, the followings have to be calculated:

I. The mean value of the output voltage

II. The RMS value of the output voltage

III. Both mean and RMS Value of the output current

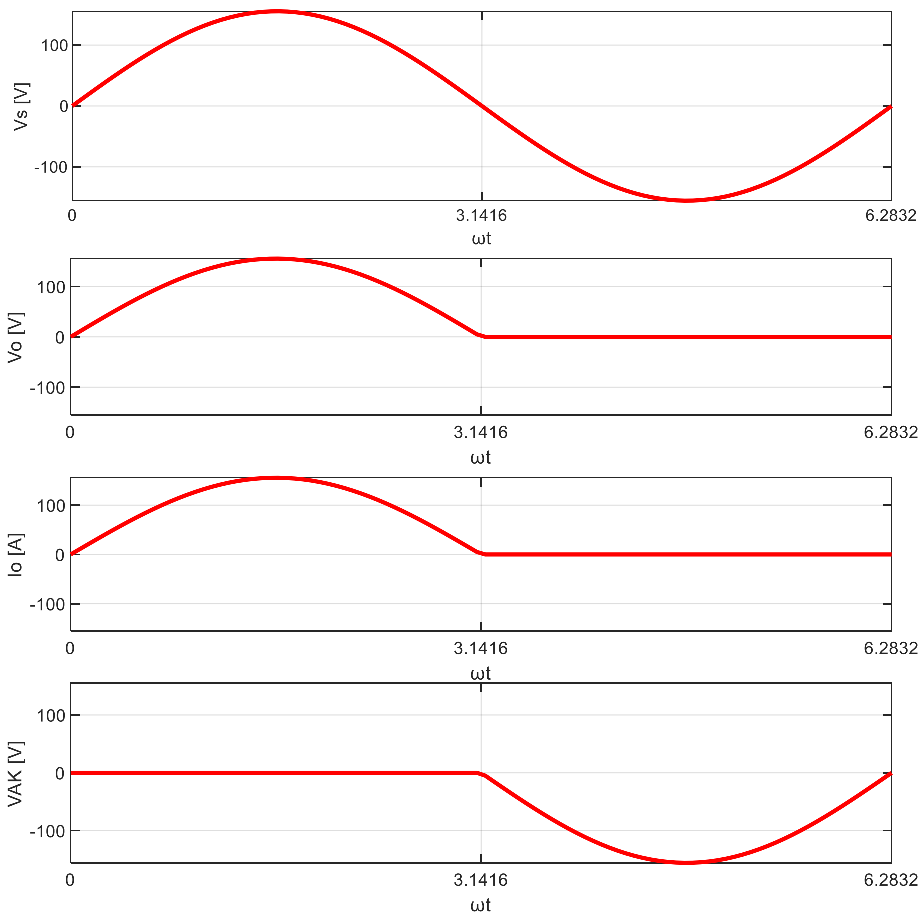
IV. Power Factor



EXERCISE 1

Solution

The waveforms of the variables of the circuit are:



EXERCISE 1

I. The mean value of the output voltage is $\bar{V}_0 = \frac{1}{2\pi}\int_0^{2\pi} V_0(t)\, d\omega t$

$$= \frac{1}{2\pi}\int_0^{\pi} 110\sqrt{2}\sin(\omega t)\, d\omega t$$

$$= \frac{110\sqrt{2}}{2\pi}(-\cos(\omega t))|_0^{\pi}$$

$$= \frac{110\sqrt{2}}{2\pi}$$

II. The RMS value of the output voltage is $\tilde{V}_o = \left[\frac{1}{2\pi}\int_0^{2\pi} V_0(t)^2\, d\omega t\right]^{1/2}$

$$= \left[\frac{1}{2\pi}\int_0^{2\pi} (110\sqrt{2}\sin(\omega t)^2\, d\omega t\right]^{1/2}$$

$$= 110\sqrt{2}\left[\frac{1}{2\pi}\int_0^{2\pi} \frac{1-\cos(2\omega t)}{2}\, d\omega t\right]^{1/2}$$

EXERCISE 1

$$= 110\sqrt{2}\left[\frac{1}{4\pi}(\omega t - \frac{\sin(2\omega t)}{2})|_0^\pi\right]^{1/2}$$

$$= 110\sqrt{2}\left[\frac{1}{4\pi}(\pi - 0 - \frac{\sin(2\pi)}{2} + \frac{\sin(0)}{2})\right]^{1/2}$$

$$= \frac{110\sqrt{2}}{2} \text{ Volts}$$

- Both mean and RMS Value of the output current are:

$$\bar{I}_o = \frac{\bar{V}_0}{R} = \frac{110\sqrt{2}}{\pi R} \text{ Amps}$$

$$\tilde{I}_o = \frac{\tilde{V}_o}{R} = \frac{110\sqrt{2}}{2R} \text{ Amps}$$

EXERCISE 1

IV. $$\text{Power Factor} = \frac{\text{Active Power}}{\text{Aparrent Power}}$$

$$\text{Input Apparent Power} = \tilde{I}_o \tilde{V}_o = \frac{110\sqrt{2}}{2R} 110 \text{ VA}$$

If we assume, that there are no losses in the circuit then we can write:

Active Input Power= Active Output Power, so:

$$= \tilde{I}_o^2 R$$

$$= \left(\frac{110\sqrt{2}}{2R} \right)^2 R$$

$$= \frac{\left(110\sqrt{2}\right)^2}{4R} \text{ Watts}$$

EXERCISE 1

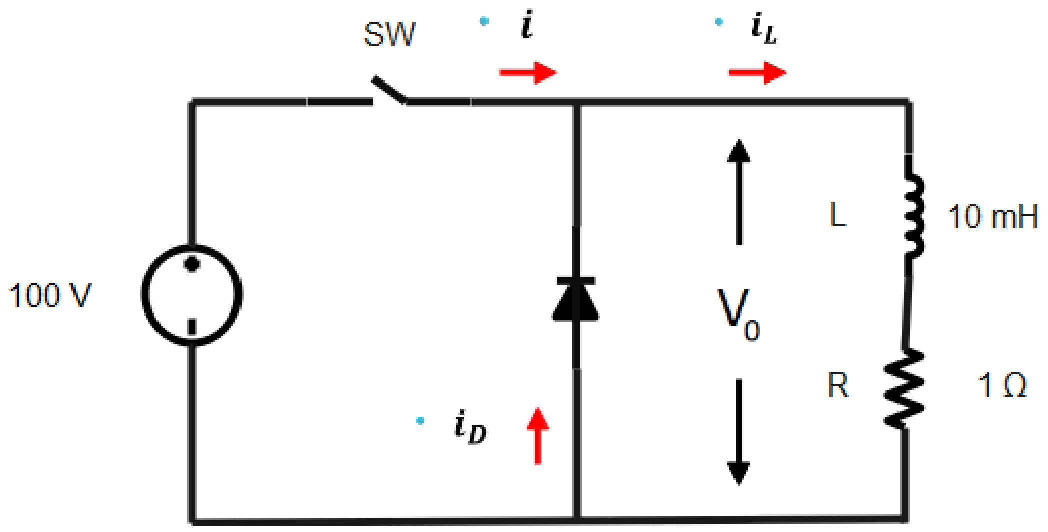
Finally:

$$\text{Power Factor} = \frac{\dfrac{\left(110\sqrt{2}\right)^2}{4R}}{\dfrac{110\sqrt{2}}{2R}110}$$

$$= \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2} = 0.707$$

EXERCISE 2

In the following circuit, the switch SW turns on at time t=0 and turns off at time t=10 msec. At time t=0 the $i_L = 0$. The waveforms of the variables i , i_L have to be found and drawn.



EXERCISE 2

Solution

When the switch SW is closed at time t = 0 the diode is inversely polarized and the current $i_L = 0$. Therefore, as soon as the switch is closed (which may be a semiconductor element), the following equation can be extracted:

$$L\frac{di}{dt}+Ri=100 \Leftrightarrow$$

$$\frac{di}{dt}+\frac{R}{L}i=\frac{100}{L} \qquad (1)$$

i= Transient component + Permanent (DC) component (2)

The permanent component of current: $i=\frac{V}{R}$ (3)

EXERCISE 2

- The transitional component of the current i is given by the solution of the homogeneous differential equation resulting from (1) and is:

$$\text{Transitive component} = Ae^{-(R/L)t} \qquad (4)$$

- Therefore, the relation (2) becomes:

$$i = Ae^{-(R/L)t} + \frac{V}{R} \qquad (5)$$

- At time t = 0 the current i = 0 and so, from relation (5) it follows that:

$$A = -\frac{V}{R}$$

EXERCISE 2

- Therefore, the equation (5) becomes:

$$i = -\frac{V}{R} e^{-(R/L)t} + \frac{V}{R}$$

$$= \frac{V}{R}(1 - e^{-(R/L)t})$$

$$= 100(1 - e^{-100t}) \qquad (6)$$

- At time t = 10 msec, the current i is equal to:

$$i = 100(1 - e^{-1}) = 63.2 \text{Amps}$$

- So, the output voltage is:

$$V_o = 100 \text{ Volts}$$

EXERCISE 2

- After turning off the switch ($t \geq 10\ msec$), the diode starts to conduct due to the reverse voltage that appears at the terminals of the coil and its current decreases. Consequently, in this case the following equations can be extracted:

$$i_D = i_L \qquad (7)$$

$$i = 0 \qquad (8)$$

$$Ri_L + L\frac{di_L}{dt} = 0 \qquad (9)$$

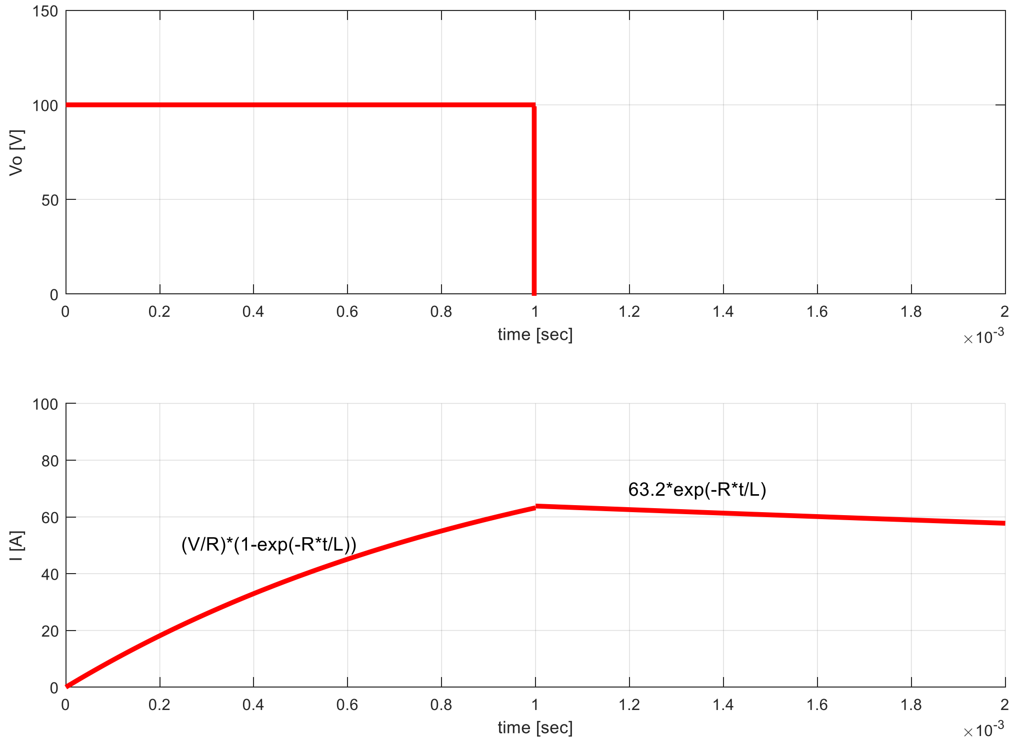
- where, $t' = t + 10msec$
- The solution of the differential equation (9) is given by the relation:

$$i_L = i_D = Ae^{-(R/L)t'} \qquad (10)$$

- At time $t' = 0$ the Current is: $i_L = i_D = 63.2Amps$

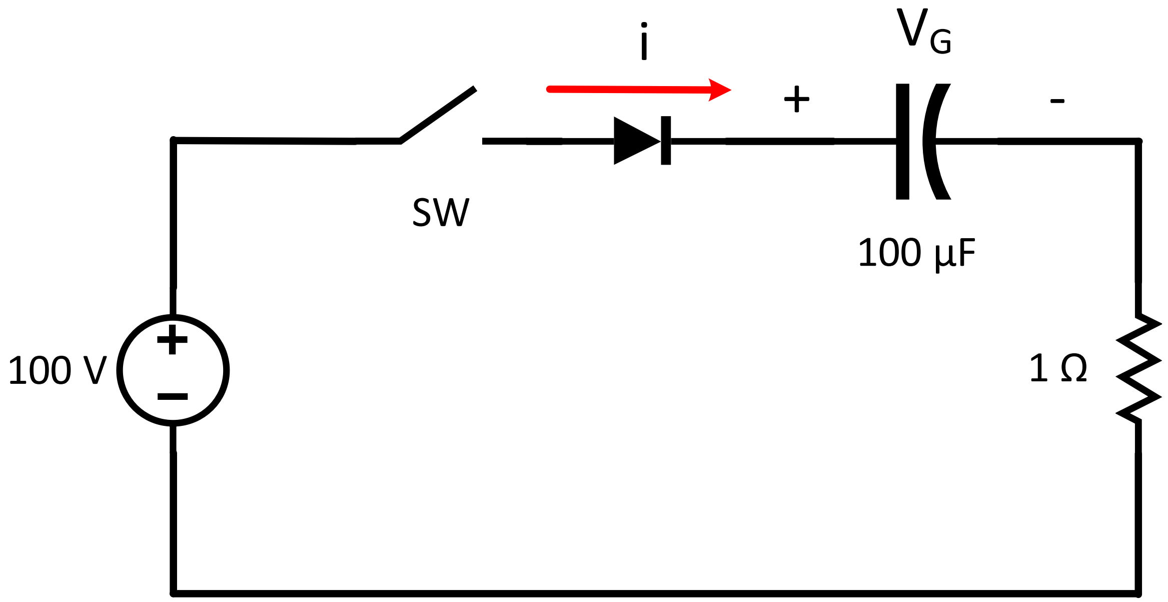
EXERCISE 2

- Therefore equation (10) becomes: $i_L = i_D = 63.2e^{-100t'}$
- When the switch is OFF, the output voltage is: $V_o = 0Volts$
- The waveforms of the circuit are the following:



EXERCISE 3

For the following circuit, the switch sw is on, when time t=0 and switches off again, when t=100 usec. When t=0, V_G=0. The functions of both the variables i and V_G have to be calculated. Moreover, draw their waveforms



EXERCISE 3

Solution

When the sw is on, then the following relationship can be extracted:

$$100 = \frac{1}{C}\int i\,dt + V_G(0) + Ri \Leftrightarrow$$

$$\frac{di}{dt} + \frac{1}{RC} i = 0$$

Solving the differential equation, the transitive current can be extracted component as follows:

$$i_N = A\,e^{-\left(\frac{i}{RC}\right)t}$$

The permanent current component i_F is equal to zero, because when t tends to be infinite and the circuit is in steady state the current of the circuit becomes zero (the capacitor is fully charged). Finally, the circuit's current is given by:

$$i = \text{Transient Component} + \text{Permanent Component}$$

$$i = i_N + i_F = A\,e^{-\left(\frac{i}{RC}\right)t} + 0 = A\,e^{-\left(\frac{i}{RC}\right)t} \qquad (1)$$

EXERCISE 3

A is a constant, which it can be calculated via the initial conditions of the circuit. In the beginning the capacitor is uncharged, so at the moment that the sw switches on $V_C(0)=0$. So the following equations can be extracted in time domain:

$$100 = V_R = Ri \qquad (2)$$

$$i(0) = \frac{V}{R} \qquad (3)$$

Taking the relationships (1), (2), (3) into consideration the constant A is

$$A = \frac{V}{R} \qquad (4)$$

(4)→(1), it can be extracted that:

$$i = \frac{V}{R} e^{-\left(\frac{i}{RC}\right)t}$$

$$= 100\, e^{-10^4 t} \qquad (5)$$

EXERCISE 3

For t=100 usec, the current is $i = 100\,e^{-1} = 36{,}8$ Amps

$$V_C = \frac{1}{C}\int_0^t i\,dt + V_C(0)$$
$$= \frac{V}{RC}\int_0^t e^{-\left(\frac{i}{RC}\right)t}\,dt + V_C(0)$$

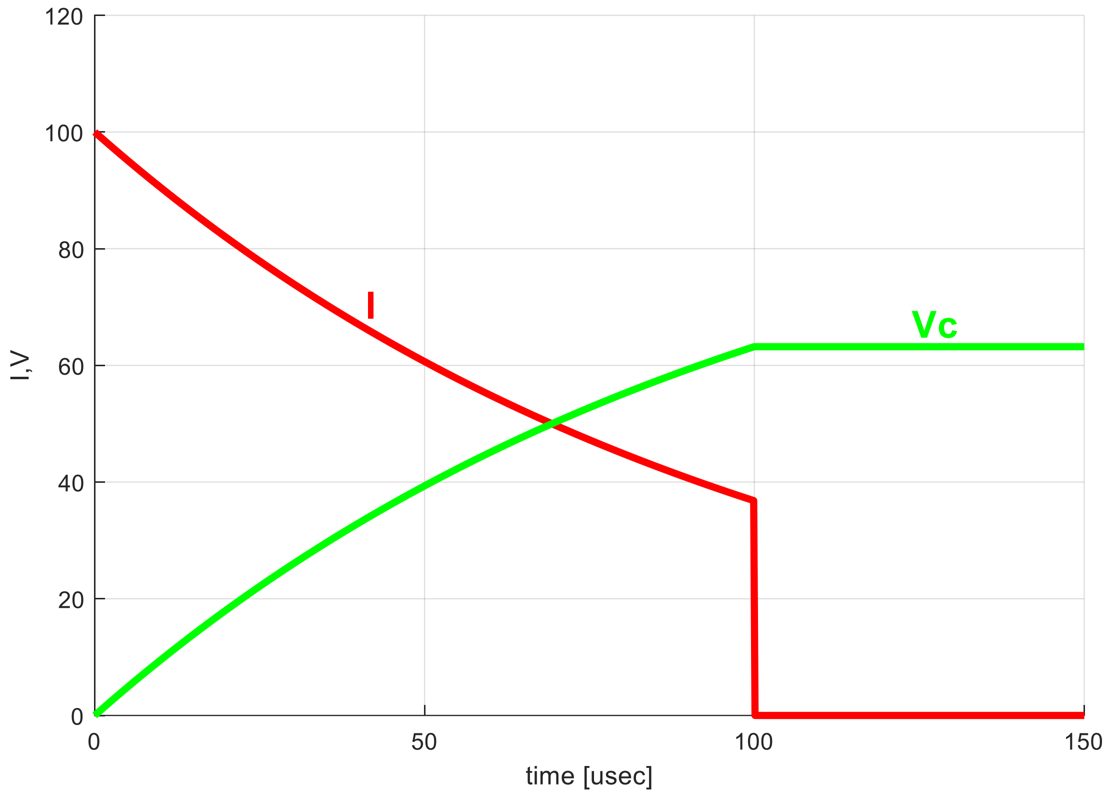
But $V_C(0)=0$, so:

$$V_C = 100\left(1 - e^{-\left(\frac{i}{RC}\right)t}\right)$$
$$= 100\left(1 - e^{-10^4 t}\right)$$

For t=100 usec, the voltage is $V_C=100(1-e^{-1})=63{,}2$ Volts

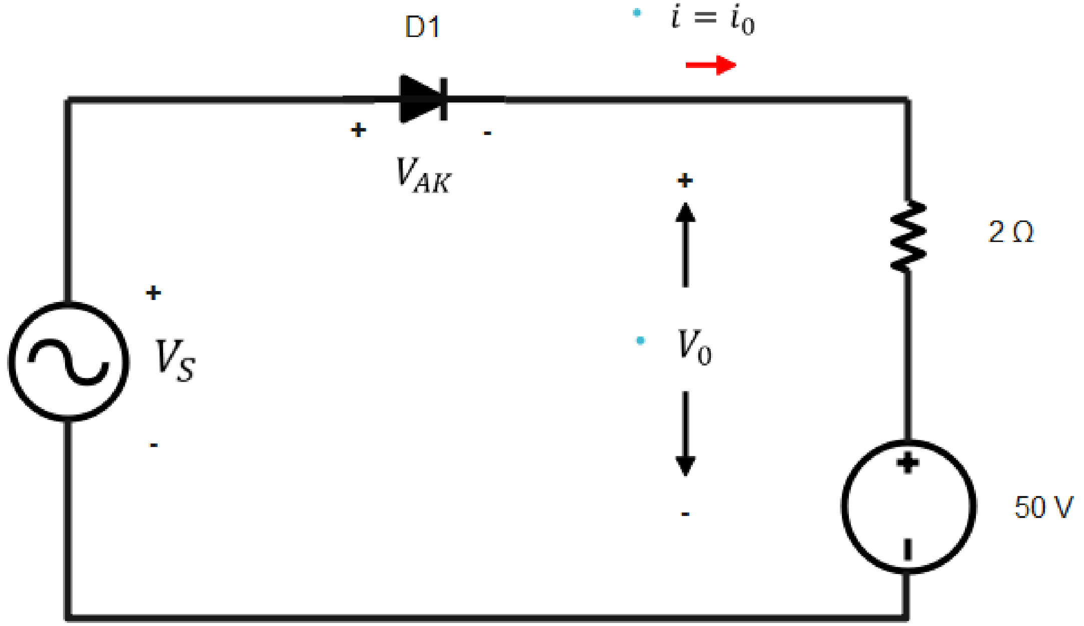
EXERCISE 3

Taking the previous equations into consideration, the waveforms of the current and Voltage are:



EXERCISE 4

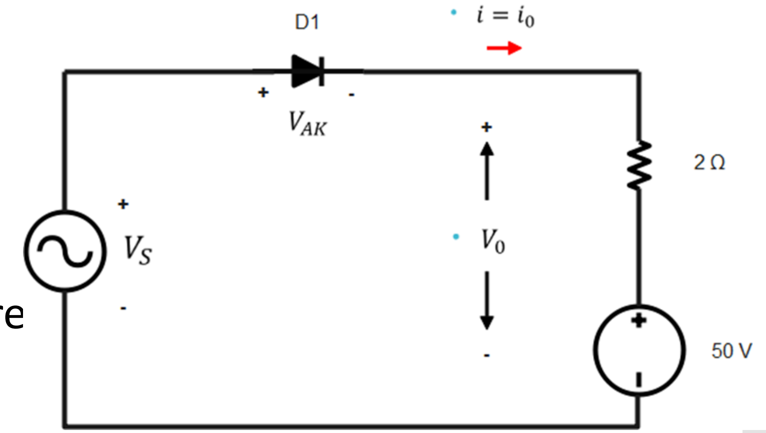
For the following circuit are given $V_s = 110\sqrt{2}\sin 120\pi t$, R=2Ω and V=50 Volts. The waveforms of the variables V_s, i , i_o , i_D , V_o have to be drawn. Moreover, the mean value of the output voltage and output current have to be calculated.

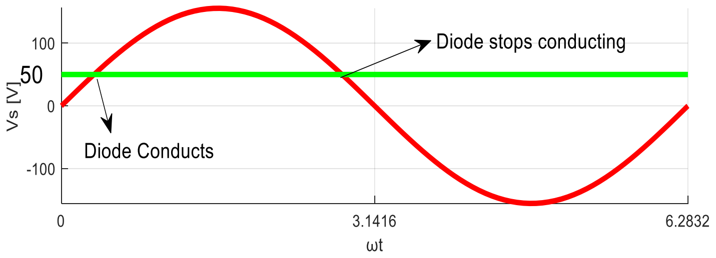


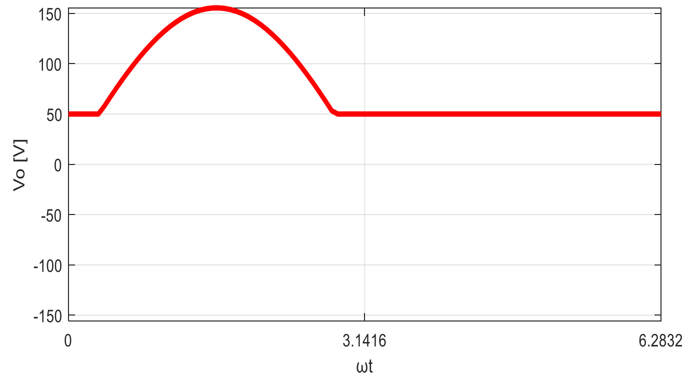
EXERCISE 4

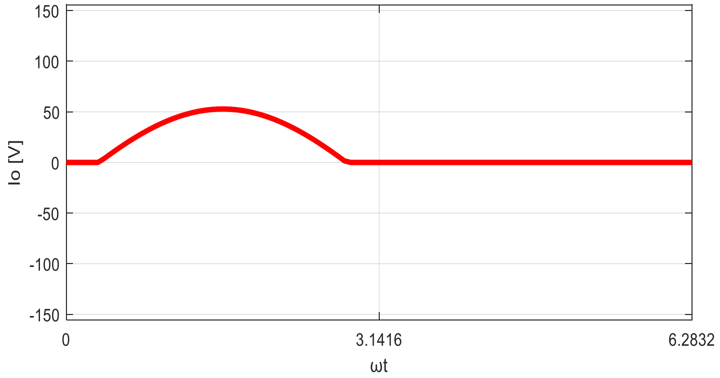
Solution

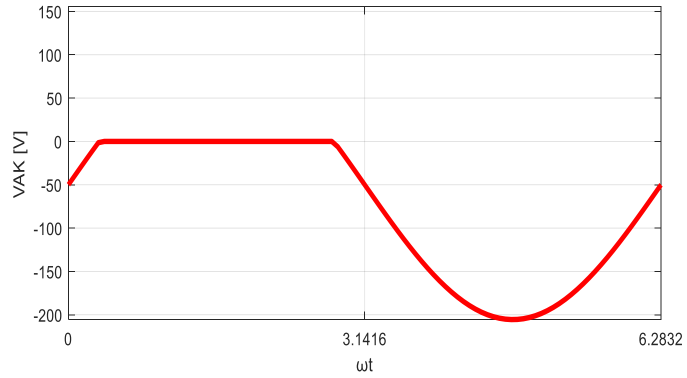
The waveforms of the variables of the circuit are











EXERCISE 4

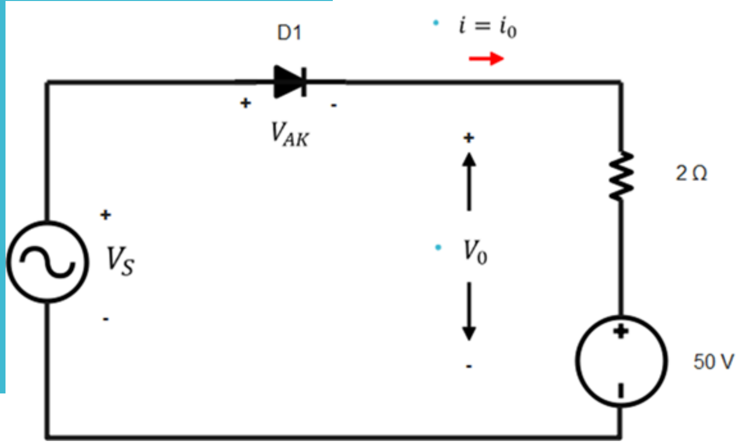
The angle α where the diode conducts is given by the equation:

$$110\sqrt{2}\sin\alpha = 50 \Leftrightarrow$$

$$\alpha = \sin^{-1}\left[\frac{50}{110\sqrt{2}}\right] = 18.75^\circ \qquad (1)$$

Moreover, the extinction angle of the diode is given by the equation:

$$\beta = \pi - \alpha$$
$$= 180^\circ - 18.75^\circ$$
$$= 161.25^\circ \qquad (2)$$



EXERCISE 4

- The mean value of the output voltage is $\bar{V}_0 = \frac{1}{2\pi}\int_0^{2\pi} V_0 d\omega t$

$$= \frac{1}{2\pi}\left[\int_0^{\alpha} 50 d\omega t + \int_{\alpha}^{\beta} 110\sqrt{2}\sin\omega t d\omega t + \int_{\beta}^{2\pi} 50 d\omega t\right]$$

$$= \frac{1}{2\pi}\left[50\alpha - 110\sqrt{2}\cos\beta + 110\sqrt{2}\cos\alpha + 50(2\pi) - 50\beta\right]$$

$$= \frac{1}{2\pi}\left[50(0.1\pi) + 294.62 + 50(2\pi) - 50(\pi - 0.1\pi)\right]$$

$$= \frac{483.62}{2\pi} = 76.9 \text{Volts}$$

EXERCISE 4

- From the circuit it is concluded that :

$$V_0 - 2i_0 - 50 = 0 \Leftrightarrow$$
$$i_0 = \frac{V_0 - 50}{2}$$

Finally:

The mean value of the output current is:

$$\bar{I}_0 = \frac{\bar{V}_0 - 50}{2} = \frac{76.9 - 50}{2}$$
$$= 13.45 Amps$$

EXERCISE 6

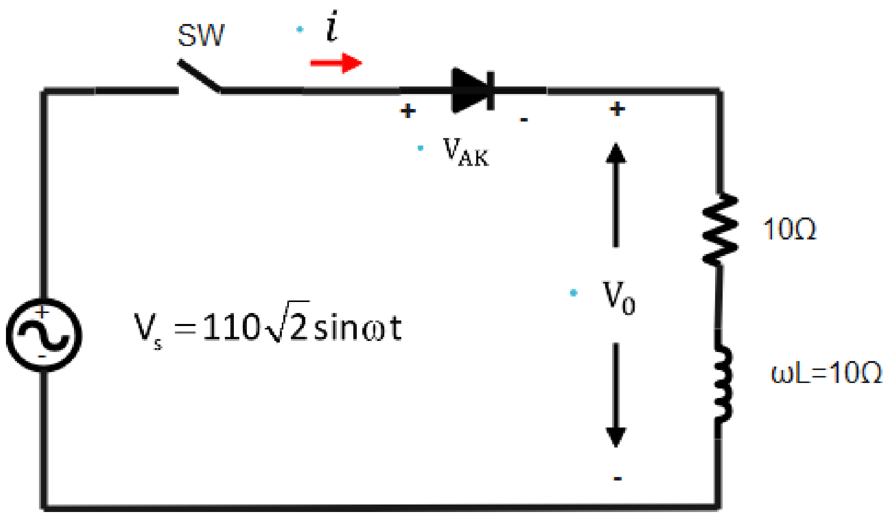
For the following circuit are given $V_s = 110\sqrt{2}\sin\omega t$, ω=377 rad/sec,

R=10Ω and ωL=10Ω.

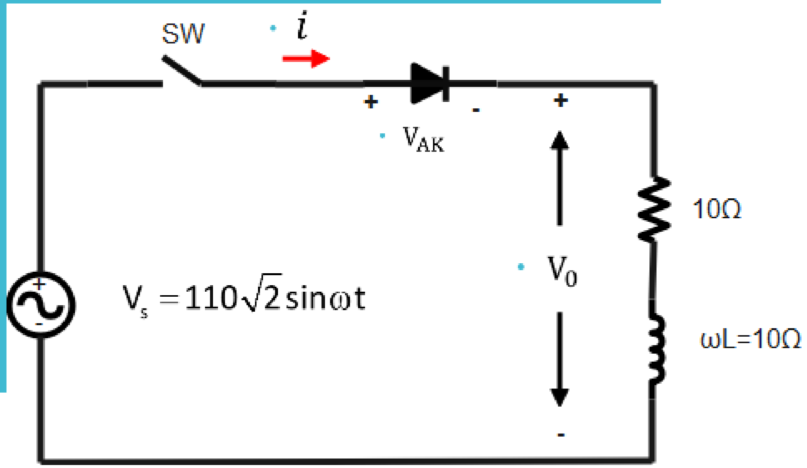
The switch SW turns on at time t= 0. The waveforms of the variables V, i ,V_o , V_{AK} have to be drawn. Moreover, the followings have to be calculated:

I. Both mean and RMS value of the output voltage

II. Both mean and RMS value of the output current i



EXERCISE 6



Solution

When the switch SW is on at time t = 0, the diode will conduct and therefore the following equation can be extracted:

$$L\frac{di}{dt} + Ri = V_s \Leftrightarrow$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{110\sqrt{2}}{L}\sin\omega t \qquad (1)$$

i= Transitive component + Permanent component

$$i = i_N + i_F \qquad (2)$$

$$i_F = \text{Permanent component}$$

$$= \frac{100\sqrt{2}\sin(\omega t - \varphi)}{|Z|} \qquad (3)$$

EXERCISE 6

- Where, $\varphi = \tan^{-1}(\frac{\omega L}{R}) = \tan^{-1}(\frac{10}{10}) = 45^\circ$

$$|Z| = \sqrt{R^2 + (\omega L)^2}$$

$$= \sqrt{10^2 + 10^2}$$

$$= 14.14\Omega$$

The transitional component of the i current is given by the solution of the homogeneous equation resulting from (1) and is:

i_N= Transient component

$$= Ae^{-(R/L)t} \qquad (4)$$

EXERCISE 6

Therefore, substituting relations (3), (4) in (2) it follows that:

$$i = \frac{110\sqrt{2}}{14.14}\sin(\omega t - 45^{\circ}) + Ae^{-(R/L)t}$$

$$= 11\sin(\omega t - 45)^{\circ} + Ae^{-(R/L)t} \qquad (5)$$

Using the initial condition, where at time t = 0 the current i = 0, the constant A can be calculated from the relation (5)

$$i = 0 \rightarrow$$

$$11\sin(-45^{\circ}) + Ae^{0} = 0$$

$$A = 11\sin(45^{\circ}) = 7.78$$

EXERCISE 6

- Therefore, the relation (5) becomes:

$$i = 11\sin(\omega t - 45^\circ) + 7.78e^{-(R/\omega L)\omega t}$$

$$= 11\sin(\omega t - 45^\circ) + 7.78e^{-\omega t} \qquad (6)$$

When the switch is on at time t = 0 or at angle ωt = 0°, then it can be said that the firing angle of the diode α = 0°. But it isn't known the extinction angle β, where at this angle the current becomes equal to zero. Using equation (6) and setting the condition i = 0 for ωt = β, the extinction angle β of the diode can be calculated as follows:

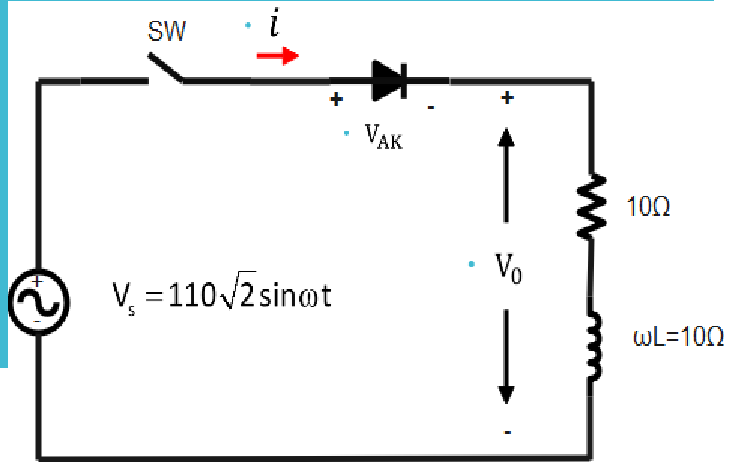
$$i = 0 \Leftrightarrow$$

$$11\sin(\beta - 45^\circ) + 7.78e^{-\beta} = 0 \Leftrightarrow$$

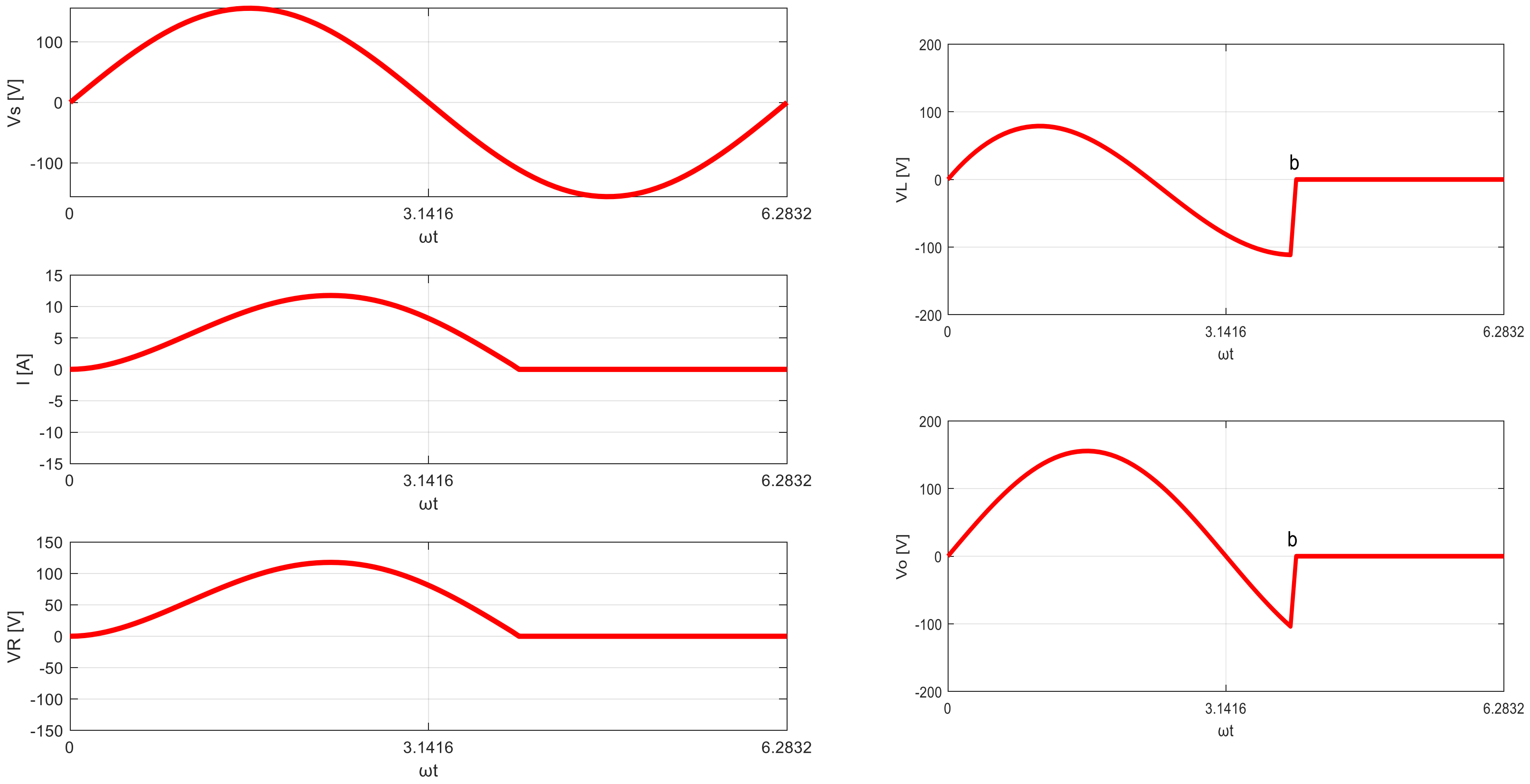
$$\sin(\beta - 45^\circ) = -0.707e^{-\beta} \Leftrightarrow$$

$$\beta = 225^\circ$$

EXERCISE 6



- consequently, using the above equations the waveforms of the circuit are the following:



EXERCISE 6

- From the above waveforms eventuate the following :

I. Mean value of the output voltage $\bar{V}_0 = \frac{1}{2\pi}\int_0^{2\pi} V_0 \, d\omega t$

$$= \frac{110\sqrt{2}}{2\pi}(-\cos\omega t)\Big|_0^{225}$$

$$= \frac{110\sqrt{2}}{2\pi}(-\cos(225^\circ) + \cos(0^\circ))$$

$$= 42.26\text{Volts}$$

RMS value of the output voltage $\tilde{V}_0 = \left[\frac{1}{2\pi}\int_0^{2\pi} V_0^2 d\omega t\right]^{1/2}$

$$= \left[\frac{1}{2\pi}\int_0^{225} (110\sqrt{2}\sin\omega t)^2 d\omega t\right]^{1/2}$$

EXERCISE 6

$$= \left[\frac{110}{\pi} \int_0^{225} \frac{1 - \cos 2\omega t}{2} d\omega t \right]^{1/2}$$

$$= \left[\frac{110^2}{\pi} \left[\omega t - \frac{\sin 2\omega t}{2} \right] \Big|_0^{225} \right]^{1/2}$$

$$= \left[\frac{110^2}{2\pi} \left[1.25\pi - 0 - \frac{\sin 2.5\pi}{2} + \frac{\sin 0}{2} \right] \right]^{1/2}$$

$$= 81.24 \text{Volts}$$

I. Mean value of the output current i

$$\bar{I}_0 = \frac{\bar{V}_0}{R} = \frac{42.26}{10} = 4.226 \text{Amps}$$

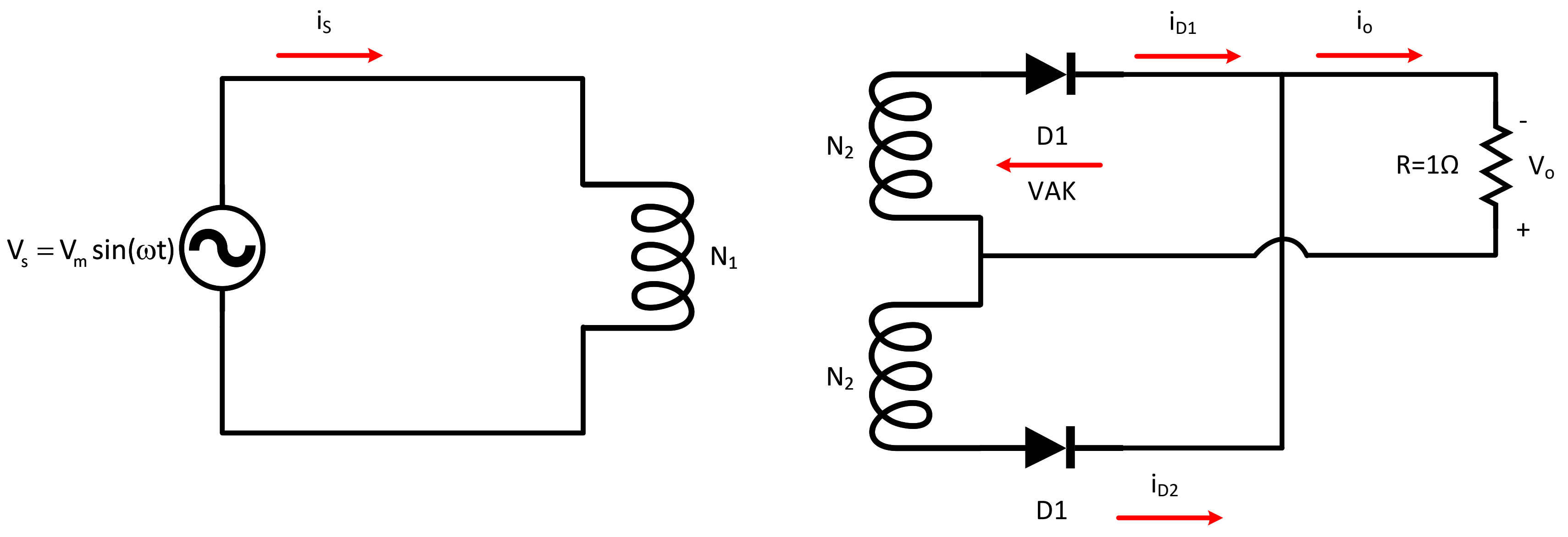
RMS value of the output current i

$$\tilde{I}_0 = \frac{\tilde{V}_0}{R} = \frac{81.24}{10} = 8.124 \text{Amps}$$

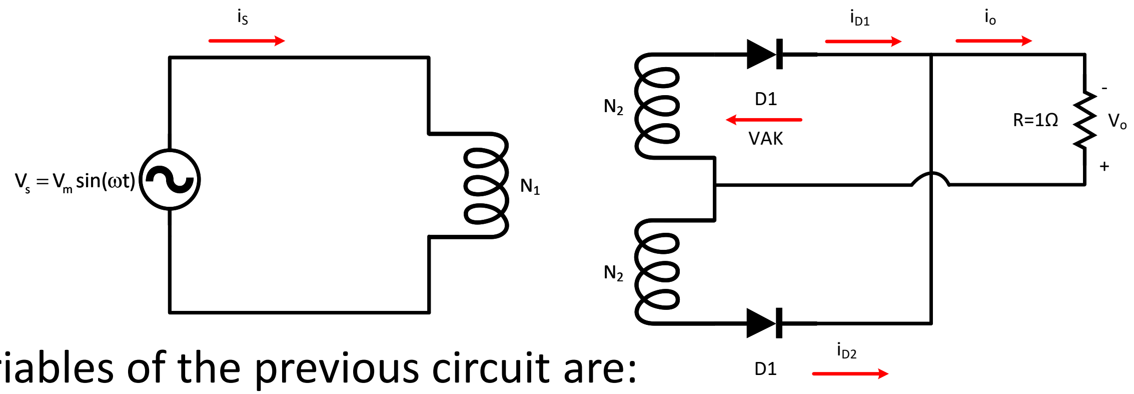
EXERCISE 7

For the following circuit, $V_s = V_m \sin(\omega t)$

The following variables V_s, i_o, i_S, i_{D1}, V_{AK}, V_o , i_{D2} have to be drawn. Moreover, the mean value of both output voltage and output current have to be calculated.

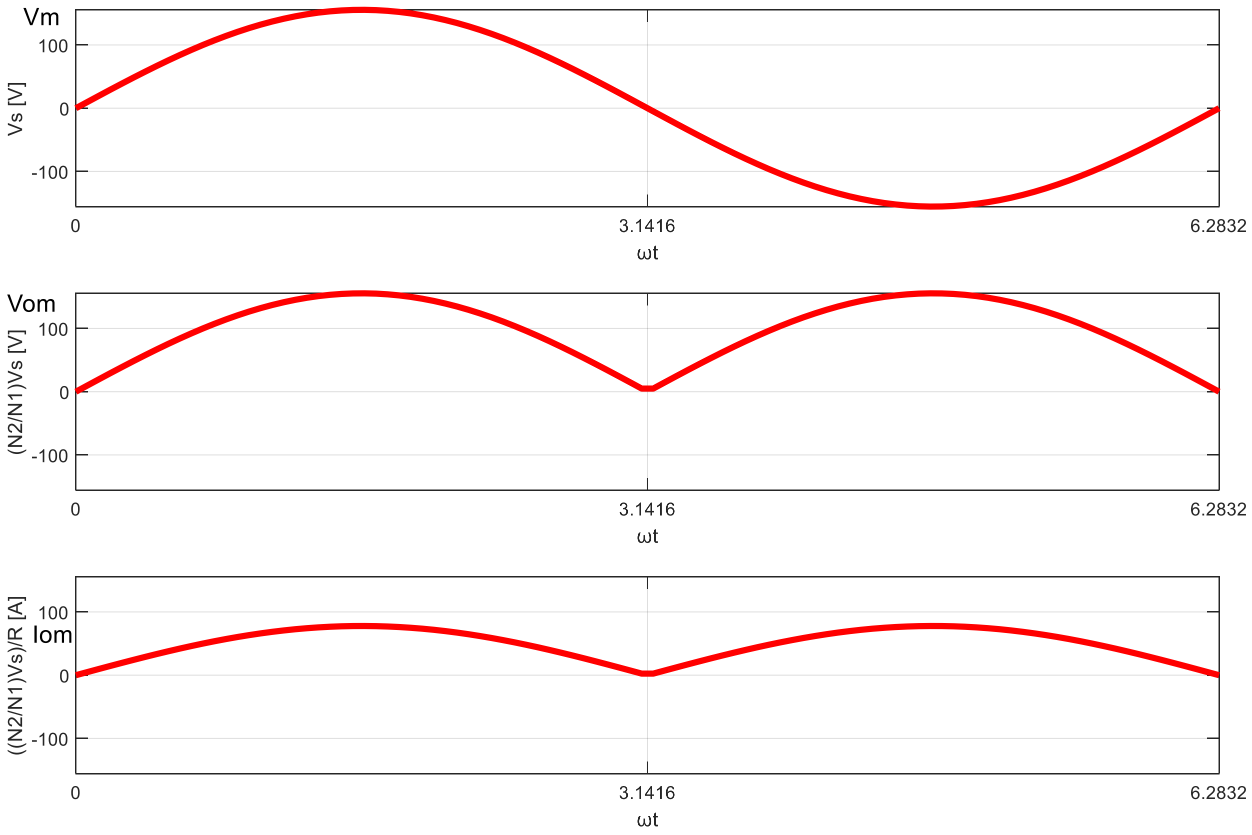


EXERCISE 7



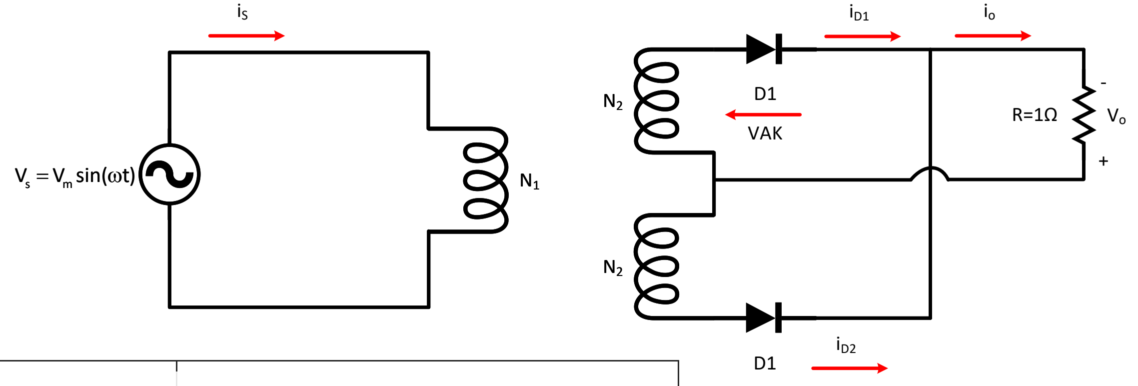
Solution

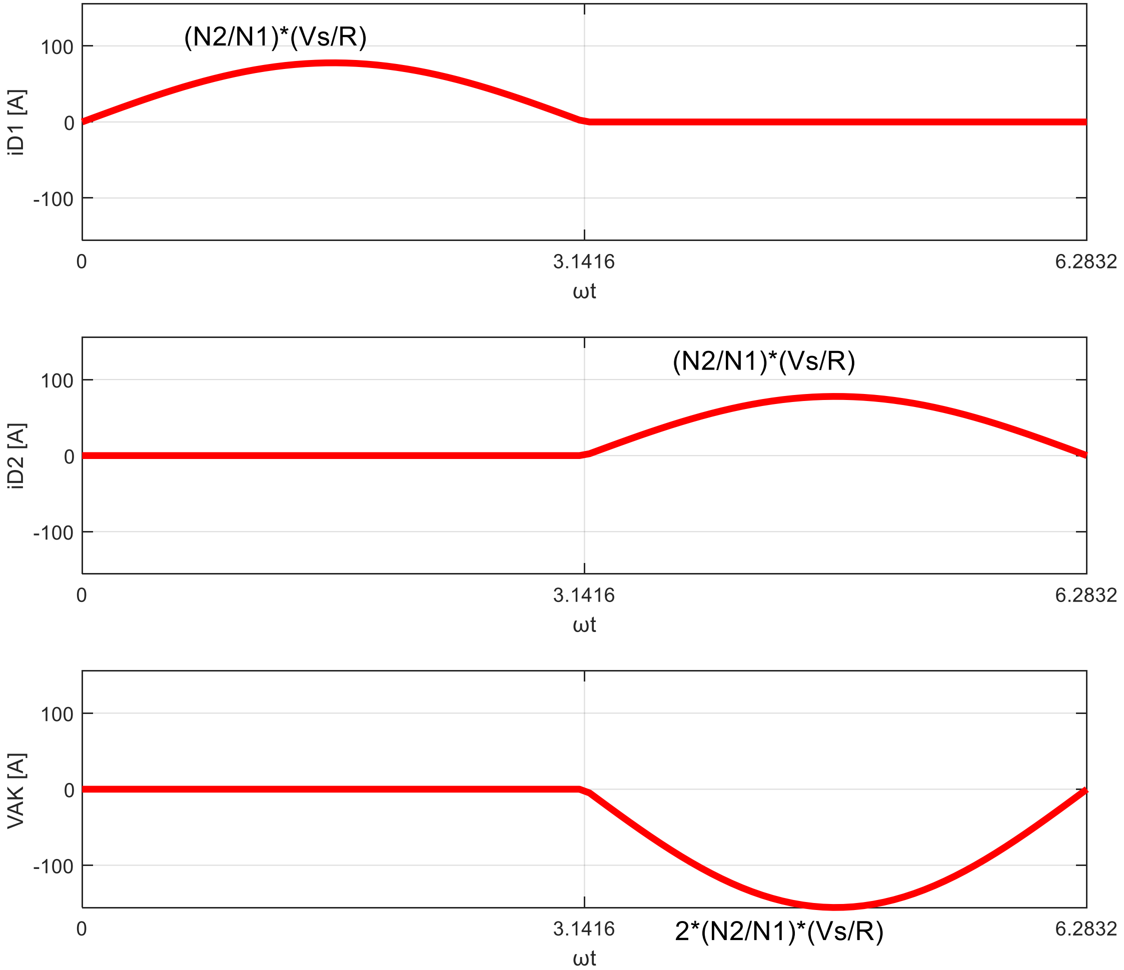
The waveforms of the variables of the previous circuit are:



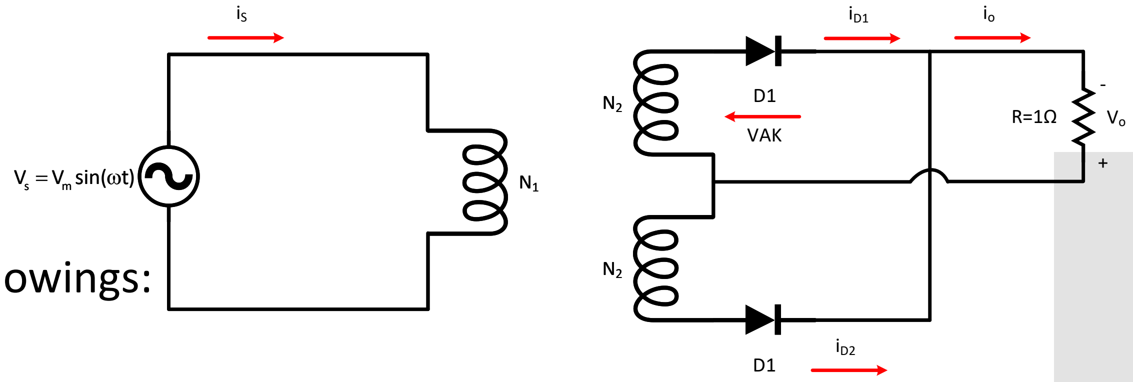
EXERCISE 7

Solution





EXERCISE 7



Solution

So they can be extracted the followings:

Mean value of output voltage:

$$\overline{V}_o = \frac{1}{2\pi}\int_0^{2\pi} V_o \, d\omega t$$

$$= \frac{1}{\pi}\int_0^{\pi} V_m \left(\frac{N_2}{N_1}\right) \sin(\omega t)\, d\omega t$$

$$= \left(\frac{V_m}{\pi}\right)\left(\frac{N_2}{N_1}\right)\left(-\cos(\omega t)\right)\Big|_0^{\pi}$$

$$= \left(\frac{2V_m}{\pi}\right)\left(\frac{N_2}{N_1}\right) \text{Volts}$$

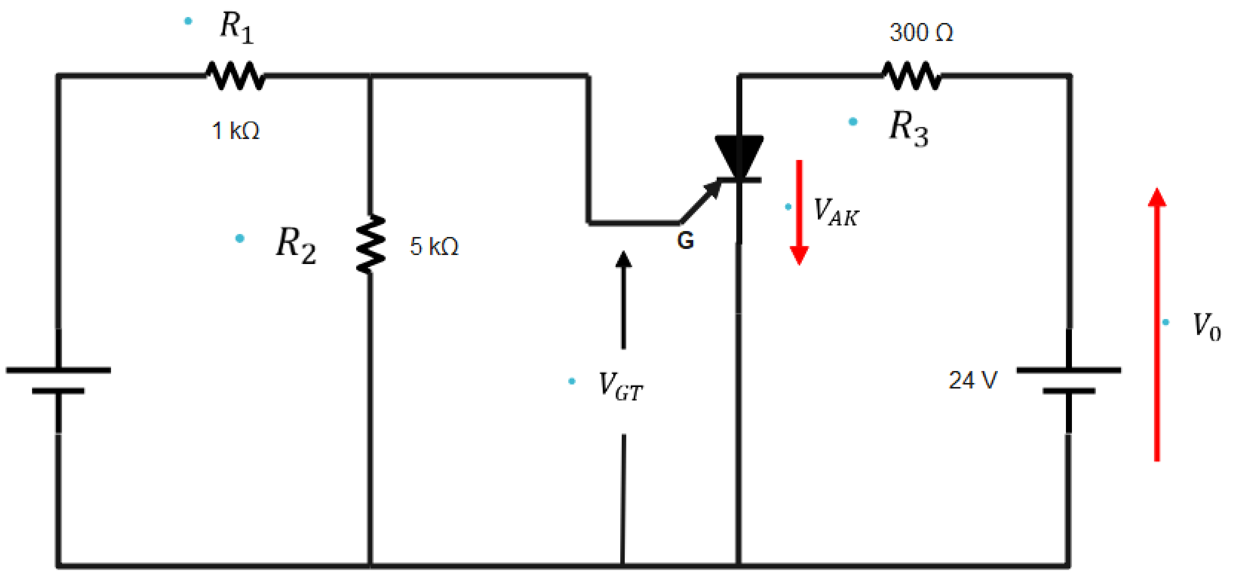
Mean value of output current:

$$\overline{I}_o = \frac{\overline{V}_o}{R} = \left(\frac{2V_m}{\pi R}\right)\left(\frac{N_2}{N_1}\right) \text{Amps}$$

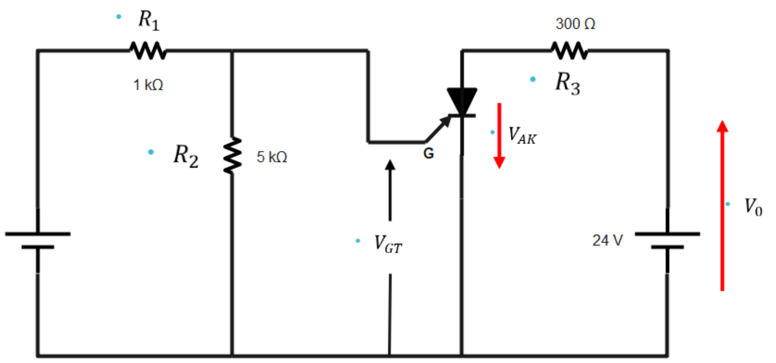
EXERCISE 8

For the following circuit are given: $R_1 = 1\ k\Omega,\ R_2 = 5\ k\Omega, R_3 = 300\ \Omega$.

The thyristor requires forward voltage $V_{GT} = 1.2\ Volts$ between the gate and the cathode. The value of voltage V_i that is required has to be calculated, that the firing of the thyristor has been ensured. Moreover, both the power that is consumed by the thyristor and the power that is consumed by the resistor R_3 have to be calculated, if the voltage drop at the terminals of thyristor when it conducts is $V_{AK} = 1.5\ Volts$. It is given $V_0 = 24\ Volts$.



EXERCISE 8



Solution

From the above circuit it turns down that:

$$V_{GT} = \frac{R_2}{R_1 + R_2} V_i \Leftrightarrow$$

$$V_i = \frac{R_2}{R_1 + R_2} V_{GT} = \frac{(1+5)k\Omega}{5\,k\Omega} 1.2\,Volts = 1.4\,Volts$$

The current that circulates in the external circuit of thyristor is:

$$i = \frac{V_0 - V_{AK}}{R_3} = \frac{(24-1.5)\,Volt}{300\Omega} = 75\,mA$$

Power that is consumed by the thyristor =

$$= V_{AK} \cdot i = 1.5 \cdot 75 \cdot 10^{-3} = 0.1125\text{ Watts}$$

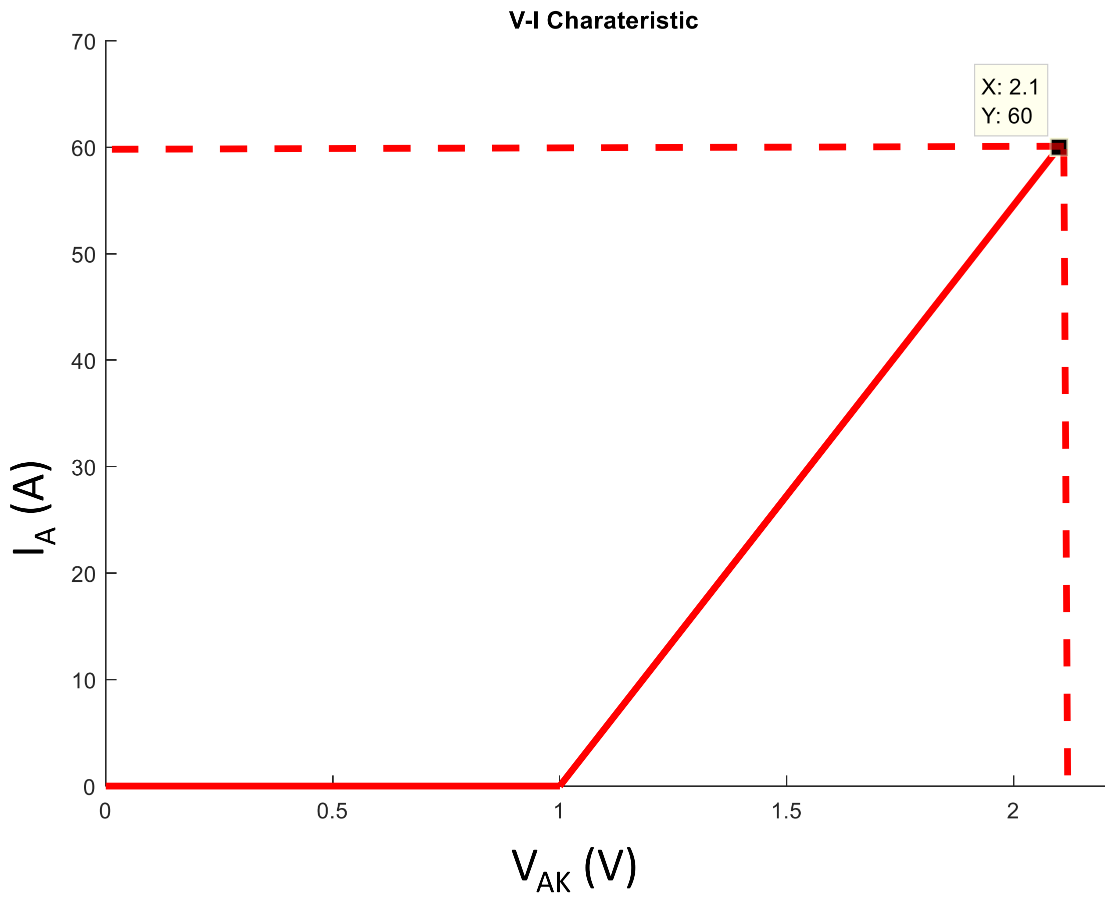
EXERCISE 8

Power that is consumed by the resistor R_3 =

$$= i^2 \cdot R_3 = (75 \cdot 10^{-3})^2 \cdot 300 = 1.6875 \text{ Watts}$$

EXERCISE 9

The V-I characteristic of a thyristor is given by the following waveform:



The mean value of power losses of thyristor for the following cases:

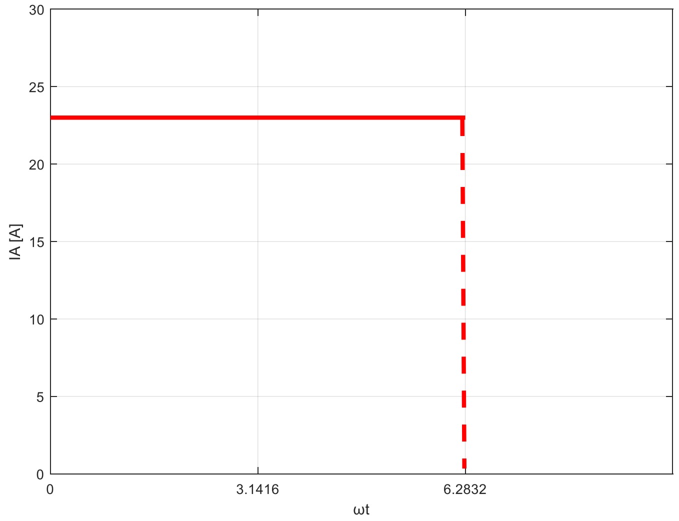
EXERCISE 9

Solution

According to the above characteristic, for any current, which flows through the thyristor, the voltage in the terminals of thyristor is given by the following equation:

$$V_{AK} = 1 + \frac{1.1}{60} i$$

I.



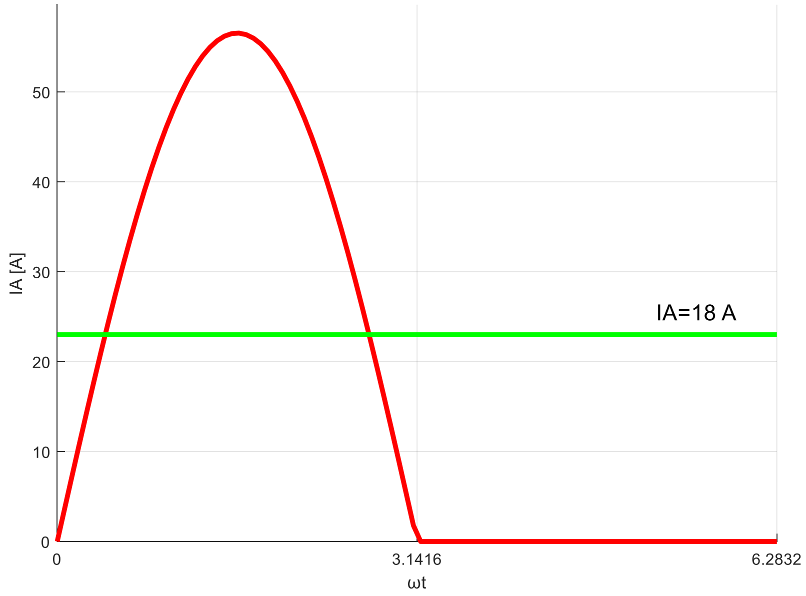
For current i=23 A, the voltage V_{AK} is:

$$V_{AK} = 1 + \frac{1.1}{60} 23 = 1.422 \text{ Volts}$$

So, the mean value of power losses of thyristor = 1.422 * 23=32.71 Watts

EXERCISE 9

II.



The current that flows in the circuit is given by the following equation:

$$i = 18\pi \sin(\omega t) \qquad \text{for} \quad 0 \leq \omega t \leq \pi$$

EXERCISE 9

So, the mean value of power losses of thyristor $= \frac{1}{2\pi}\int_0^{2\pi} V_{AK}\, i d\omega t$

$$= \frac{1}{2\pi}\int_0^{2\pi} V_{AK}\, i d\omega t$$

$$= \frac{1}{2\pi}\int_0^{2\pi} (1+\frac{1.1}{60}(18\pi\sin(\omega t)))\,(18\pi\sin(\omega t))d\omega t$$

$$= \frac{1}{2\pi}\left(\int_0^{\pi}(18\pi\sin(\omega t))\,d\omega t+\int_0^{\pi}\left(\frac{1.1*(18\pi)^2}{60}\sin^2(\omega t)\right)d\omega t\right)$$

$$= \frac{1}{2\pi}\left(-18\pi\cos(\omega t)\,|_0^{\pi}+\frac{1.1*(18\pi)^2}{60}\left(\frac{\omega t}{2}-\frac{\sin(\omega t)}{4}\right)|_0^{\pi}\right)$$

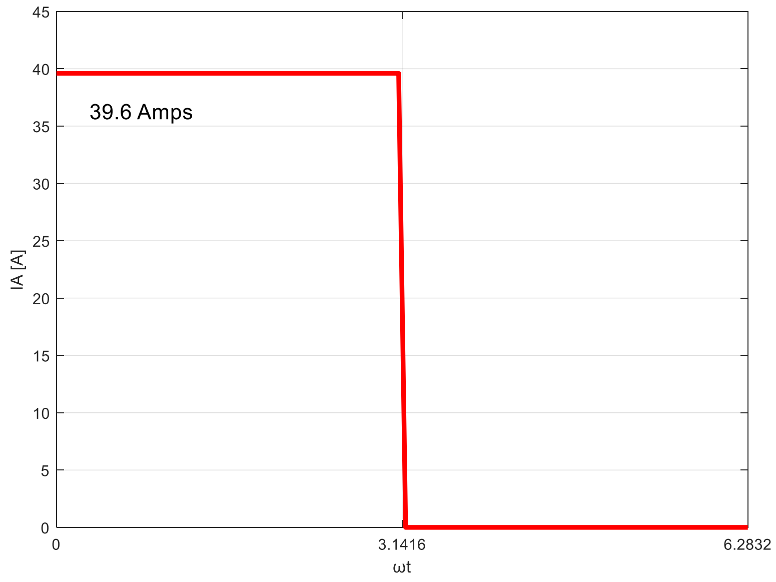
$$= \frac{1}{2\pi}\left(-18\pi\cos(\pi)+18\pi\cos(0)+\frac{1.1*(18\pi)^2}{60}\left(\frac{\pi}{2}\right)\right)$$

$$= \frac{1}{2\pi}\left(18\pi+18\pi+\frac{1.1*(18\pi)^2}{60}\left(\frac{\pi}{2}\right)\right)$$

$$= 32.66 \text{ Watts}$$

EXERCISE 9

III.



So, the mean value of power losses of thyristor $= \frac{1}{2\pi}\int_0^{2\pi} V_{AK}\, i d\omega t$

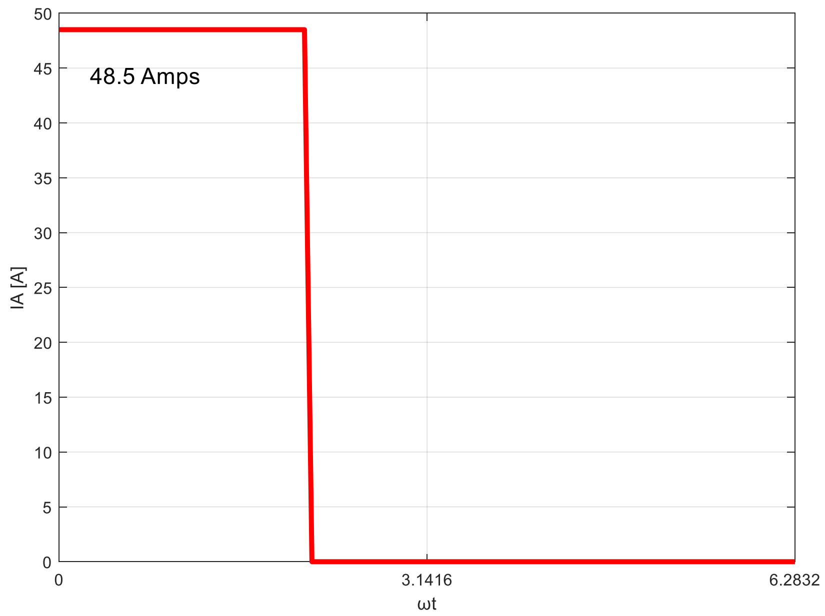
$$= \frac{1}{2\pi}\int_0^{\pi} (1 + \frac{1.1}{60} 39.6)\,(39.6) d\omega t$$

$$= \frac{1}{2\pi} (1 + \frac{1.1}{60} 39.6)\,(39.6)\pi$$

$$= 34.18 \text{ Watts}$$

EXERCISE 9

IV.



So, the mean value of power losses of thyristor $= \frac{1}{2\pi}\int_0^{2\pi} V_{AK}\, i d\omega t$

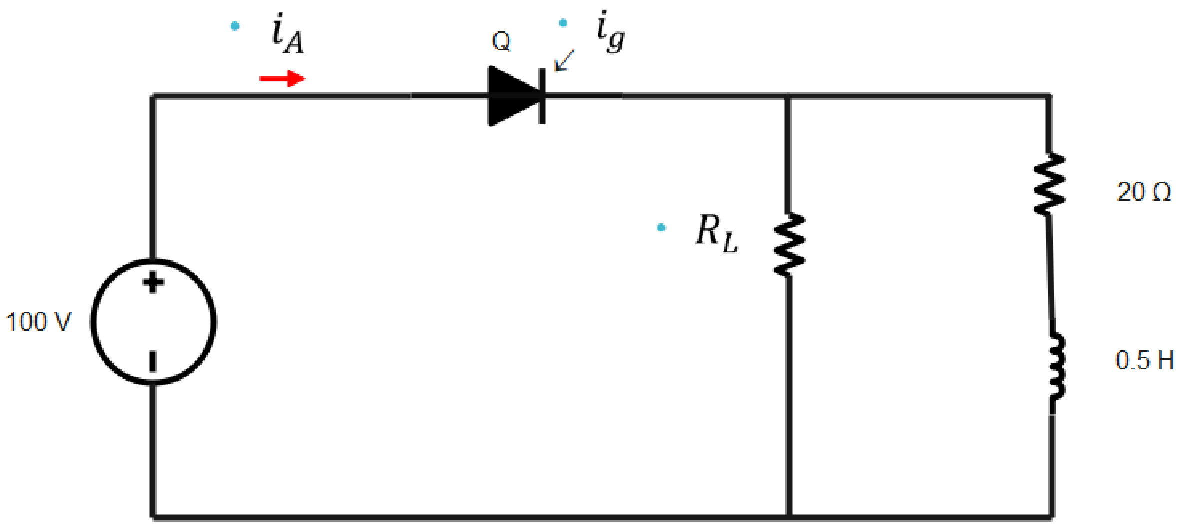
$$= \frac{1}{2\pi}\int_0^{2\pi/3} (1+\frac{1.1}{60}48.5)\,(48.5) d\omega t$$

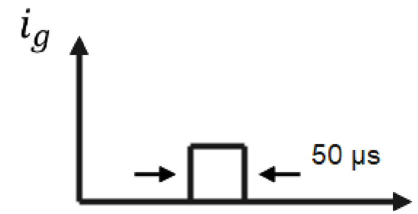
$$= \frac{1}{2\pi}(1+\frac{1.1}{60}48.5)\,(48.5) 2\pi/3$$

$$= 20.4 \text{ Watts}$$

EXERCISE 10

The thyristor of the following circuit has a holding current value of 50 mA. They are required firing pulses of 50 usec width. It has to be proven that without resistor R_L it would be impossible for thyristor to switch to the conduction state after the end of the firing pulse. The value of resistor R_L has to be calculated to ensure the conduction of the thyristor. The voltage drop of the thyristor (it is the voltage at thyristor terminals when it conducts) to be considered insignificant.

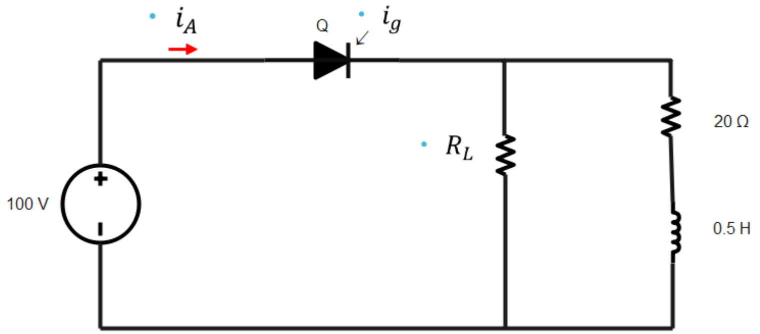




EXERCISE 10

Solution

Without the resistor R_L , the current of thyristor increases exponentially and is given by the following equation:

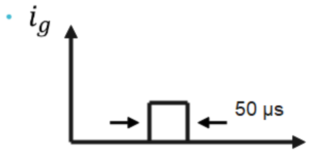


$$i = I(1 - e^{-t/\tau})$$

Where,

$$I = \frac{100}{20} = 5\ A$$

and



$$\tau = \frac{0.5H}{20\Omega} = 0.025\ sec$$

Therefore, after 50 usec from the beginning of thyristor's operation, which is the end of firing pulse , the current that flows through thyristor is:

$$i = 5(1 - e^{50\cdot10^{-6}/0.025}) = 10\ mA$$

EXERCISE 10

Therefore, after the end of firing pulse, thyristor’s current would not be able to reach the holding value of 50 mA, thus it would not be able to transit in to conduction state. By adding the resistor R_L this problem can be avoided, because resistor R_L absorbs a current 100/R_L [A] from the input source. In this case:

$$i = \frac{V}{R} = (50 - 10) \geq 40\ \text{mA}$$

$$R = \frac{100}{40 \cdot 10^{-3}} \leq 2.5\ \text{k}\Omega$$

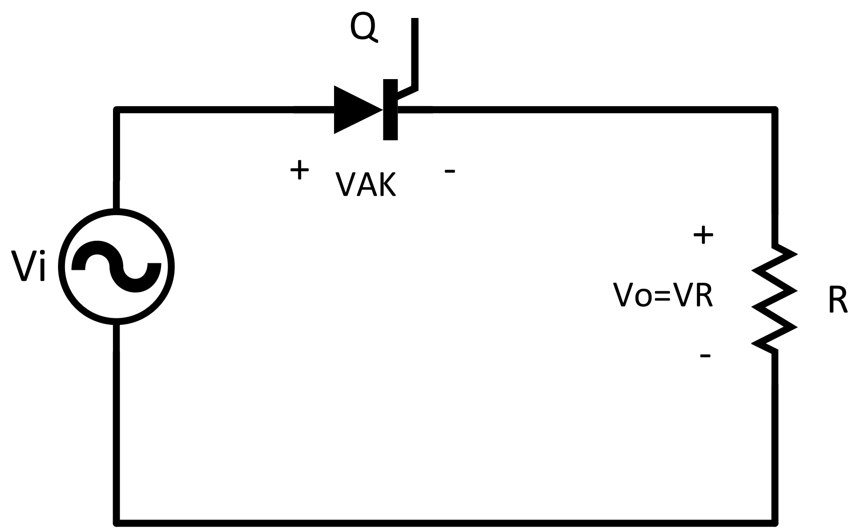
EXERCISE 11

For the following circuit, are given:

- $V_i = 220\sqrt{2}\sin(\omega t)$
- R=1 Ω
- Firing angle a=π/10

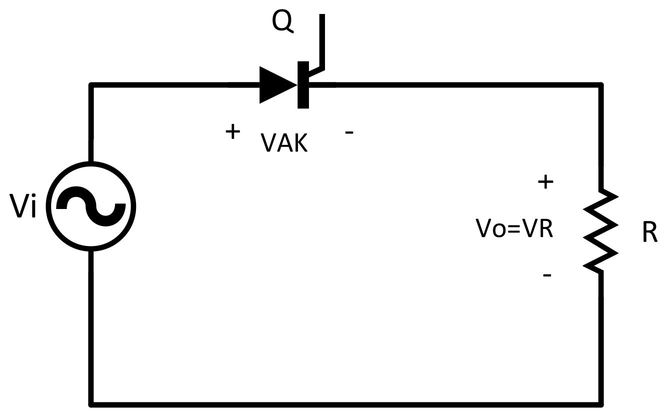
The followings have to calculated:

I. Mean value of the output Voltage
II. RMS value of the output voltage
III. Both mean and RMS value of the current
IV. Power Factor

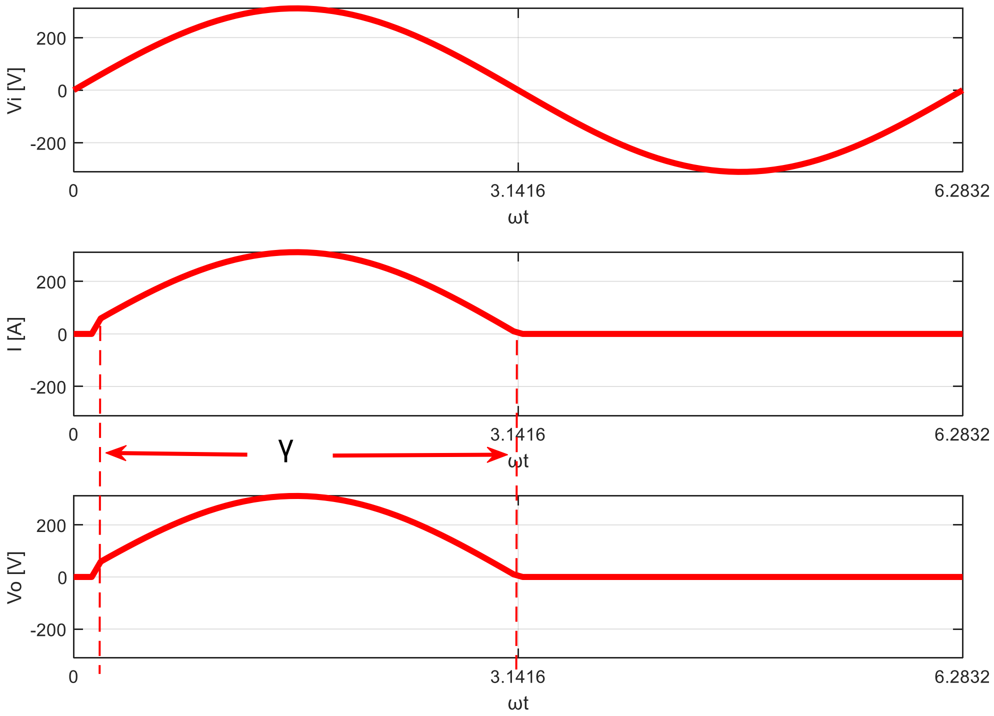


EXERCISE 11

Solution



The waveforms of the above circuit are:

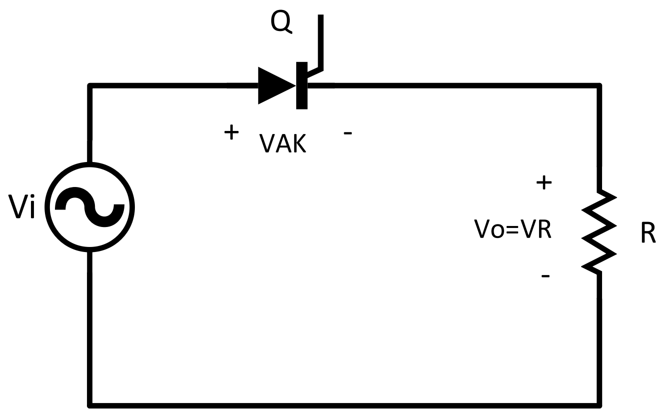


EXERCISE 11

From the above waveforms, the followings can be extracted:

I. The mean value of the output voltage is $\bar{V}_0 = \frac{1}{2\pi}\int_0^{2\pi} V_0(t)\, d\omega t$

$$= \frac{1}{2\pi}\int_{\pi/10}^{\pi} 220\sqrt{2}\sin(\omega t)\, d\omega t$$
$$= \frac{220\sqrt{2}}{2\pi}(-\cos(\omega t))|_{\pi/10}^{\pi}$$
$$= \frac{220\sqrt{2}}{2\pi}(-\cos(\pi)+\cos(\pi/10))$$
$$= 98.3 \text{ Volts}$$



II. RMS value of the output voltage $\tilde{V}_o = \left[\frac{1}{2\pi}\int_0^{2\pi} V_0^2\, d\omega t\right]^{1/2}$

EXERCISE 11

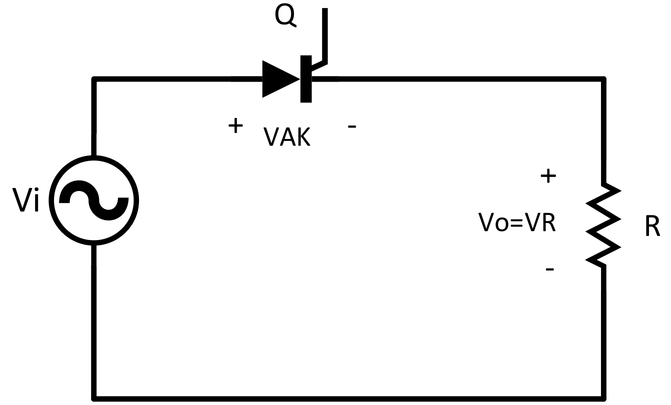
$$=\left[\frac{1}{2\pi}\int_{\pi/10}^{\pi}(220\sqrt{2}\sin(\omega t)^2\, d\omega t\right]^{1/2}$$

$$=110\sqrt{2}\left[\frac{1}{2\pi}\int_{\pi/10}^{\pi}\frac{1-\cos(2\omega t)}{2}\, d\omega t\right]^{1/2}$$

$$=220\sqrt{2}\left[\frac{1}{4\pi}(\omega t-\frac{\sin(2\omega t)}{2})|_{\pi/10}^{\pi}\right]^{1/2}$$

$$=220\sqrt{2}\left[\frac{1}{4\pi}(\pi-\frac{\pi}{18}-\frac{\sin(2\pi)}{2}+\frac{\sin(\pi/9)}{2})\right]^{1/2}$$

$$=155.5 \text{ Volts}$$



III. Mean value of the output Current

$$\overline{I}_o=\frac{\overline{V}_o}{1}$$

$$=\frac{98.3}{1}=98.3\text{Amps}$$

EXERCISE 11

RMS value of the output Current $\tilde{I}_0 = \frac{\tilde{V}_0}{R} = \frac{155.5}{1} = 155.5$ Amps

IV. Mean value of the output Power

$P_o(t) = V_o(t)\, i_o(t) \rightarrow$ instantaneous output Power

$$P_o = \frac{1}{2\pi}\int_0^{2\pi} V_o(t) i_o(t) d\omega t$$

$$= \frac{1}{2\pi}\int_{\pi/10}^{\pi} (220\sqrt{2}\sin(\omega t)^2\ d\omega t$$

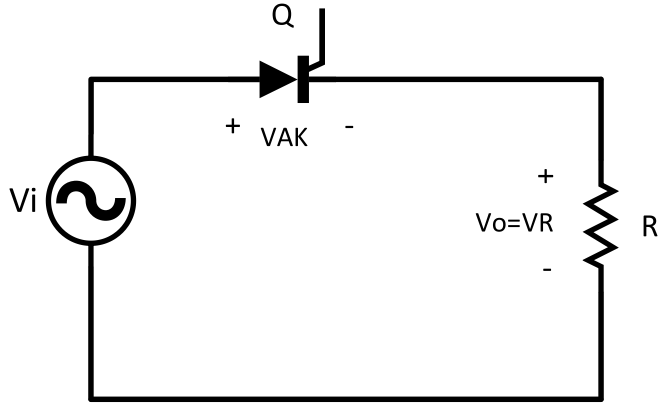
$$= \frac{220^2}{\pi}\int_{\pi/10}^{\pi} \sin^2(\omega t)\ d\omega t$$

$$= \frac{220^2}{\pi}\int_{\pi/10}^{\pi} \frac{1-\cos(2\omega t)}{2}\ d\omega t$$

$$= \frac{220^2}{2\pi}(\omega t - \frac{\sin(2\omega t)}{2})|_{\pi/10}^{\pi}$$

$$= \frac{220^2}{2\pi}\left[(\pi - \frac{\pi}{18} - \frac{\sin(2\pi)}{2} + \frac{\sin(\pi/9)}{2})\right]$$

$$= 24172.8 \text{ Watts}$$



EXERCISE 11

Both the RMS value of the power and the mean value of the power are consumed on the resistance of the load R. Moreover, the RMS value of the power can be calculated by the following equation:

$$P_o = R\,\tilde{I}_o^2$$
$$= 1155.5^2$$
$$= 24172.8 \text{ Watts}$$

V. Input power factor $= \dfrac{\text{active power}}{\text{apparent power}}$

$$\text{apparent power} = \tilde{V}_i\,\tilde{I}_i$$

If it can be assumed that there are no losses, then:

$$P_o = P_i \rightarrow \text{RMS value}$$

So:

$$PF = \frac{24172.8}{34210} = 0.71$$

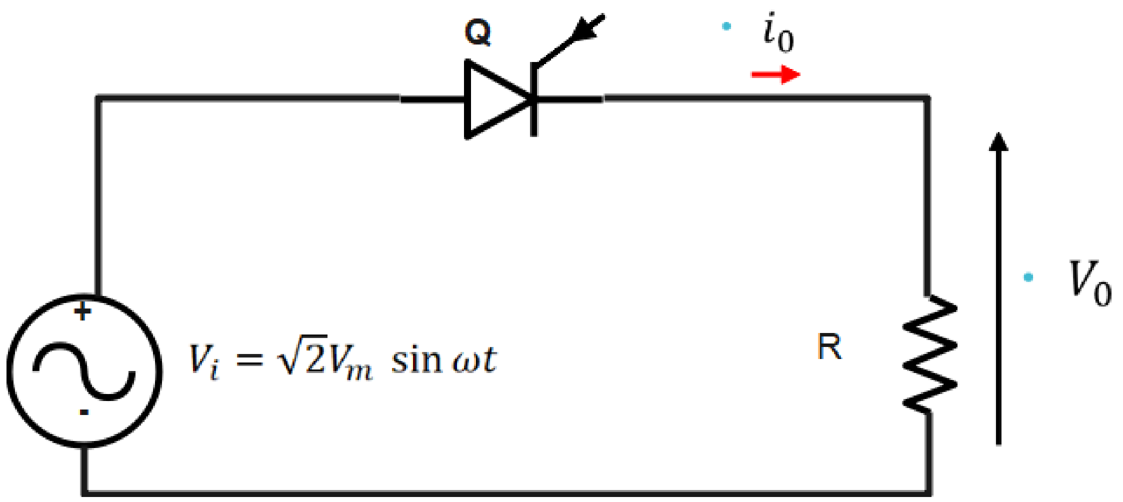
EXERCISE 12

For the following circuit, firing pulse is given to thyristor at angle α. The waveforms of the variables V_i, V_o, i_o have to be drawn. Moreover, the followings have to be calculated:

I. The RMS value of the output current

II. The mean value of the power or active power

III. The input power factor



EXERCISE 12

Solution

The equations of load current and output voltage are the followings:

$$i_0 = \frac{V_i}{R}\Big|_\alpha^\pi = \frac{\sqrt{2}V_m \sin\omega t}{R}\Big|_\alpha^\pi$$

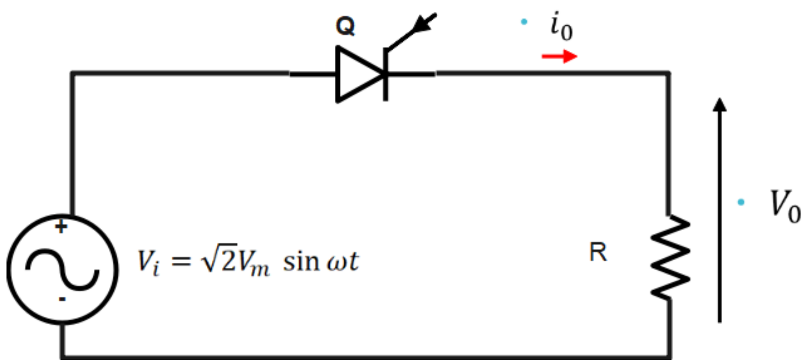
$$V_0 = V_i\Big|_\alpha^\pi = \sqrt{2}V_m \sin\omega t\Big|_\alpha^\pi$$

I. The RMS value of the output current is: $\tilde{I}_0 = \left[\frac{1}{2\pi}\int_0^{2\pi} i_0^2\, d\omega t\right]^{1/2}$

$$= \left[\frac{1}{2\pi}\int_\alpha^\pi \left[\frac{\sqrt{2}V_m \sin\omega t}{R}\right]^2 d\omega t\right]^{1/2}$$

$$= \left[\frac{1}{2\pi}\int_\alpha^\pi \frac{2V_m^2}{R^2}\left(\frac{1-\cos\omega t}{2}\right) d\omega t\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi R^2}\left(\omega t - \frac{\sin 2\omega t}{2}\right)\Big|_\alpha^\pi\right]^{1/2}$$



EXERCISE 12

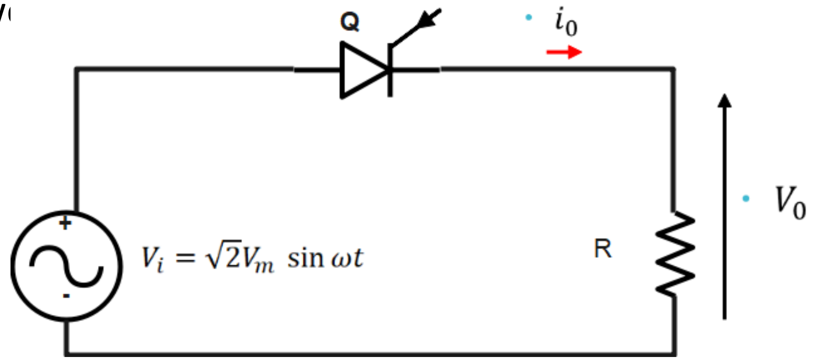
$$= \left[\frac{V_m^2}{2\pi R^2} \left(\pi - \alpha - \frac{\sin 2\pi}{2} + \frac{\sin 2\alpha}{2} \right) \right]^{1/2}$$

$$= \left[\frac{V_m^2}{4\pi R^2} \left(2(\pi - \alpha) + \frac{\sin 2\alpha}{2} \right) \right]^{1/2}$$

$$= \frac{V_m}{2R} \left[\frac{2(\pi - \alpha) + \sin 2\alpha}{\pi} \right]^{1/2}$$

I. The mean value of the power or active power is: $P_0 = \frac{1}{T} \int_0^T P_0(t)\, dt$

Where, $P_0(t) = \text{instantaneous output pow}$



EXERCISE 12

$$P_0 = \frac{1}{2\pi}\int_0^{2\pi} V_0(\omega t)\, i_0(\omega t)\, d\omega t$$

$$= \frac{1}{2\pi}\int_\alpha^{\pi}\left(\sqrt{2}V_m \sin\omega t\right)\left(\frac{\sqrt{2}V_m \sin\omega t}{R}\right) d\omega t$$

$$= \frac{1}{2\pi R}\int_\alpha^{\pi} 2V_m^2 \sin^2(\omega t)\, d\omega t$$

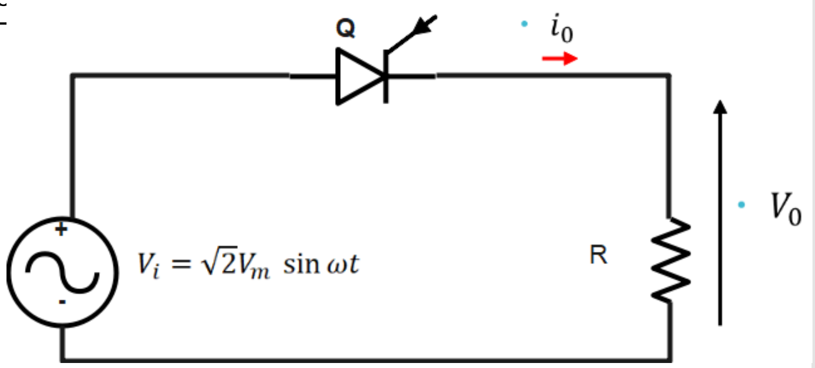
$$= \frac{1}{2\pi R}\int_\alpha^{\pi} 2V_m^2 \left[\frac{1-\cos 2\omega t}{2}\right] d\omega t$$

$$= \frac{V_m^2}{2\pi R}\left(\omega t - \frac{\sin 2\omega t}{2}\right)\Big|_\alpha^\pi$$

$$= \frac{V_m^2}{2\pi R}\left(\pi - \alpha - \frac{\sin 2\pi}{2} + \frac{\sin 2\alpha}{2}\right)$$

$$= \frac{V_m^2}{2\pi R}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)$$

$$= \frac{V_m^2}{4\pi R}\left[2(\pi - \alpha) + \sin 2\alpha\right]$$



EXERCISE 12

The active power value can also be calculated from the following equation:

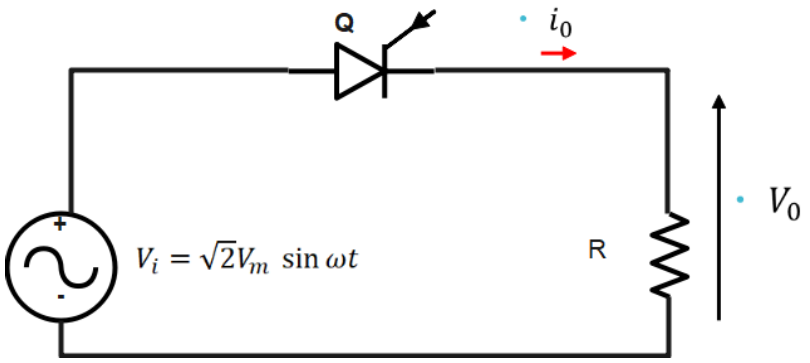
$$P_0 = \tilde{I}_0^2 \cdot R$$

$$= \frac{V_m^2}{4\pi R}\left[2(\pi-\alpha)+\sin 2\alpha\right]$$

I. Finally, $\text{Power Factor} = \dfrac{\text{input active power}}{\text{apparent power}}$

$$= \frac{P_0}{\tilde{V}_i \cdot \tilde{I}_0}$$

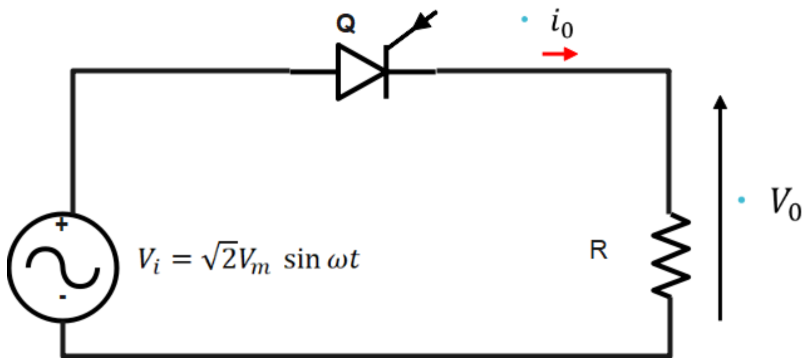
$$= \frac{\dfrac{V_m^2}{4\pi R}\left[2(\pi-\alpha)+\sin 2\alpha\right]}{\left(\sqrt{2}V_m\right)\cdot\left[\dfrac{V_m}{2R}\left(\dfrac{2(\pi-\alpha)+\sin 2\alpha}{\pi}\right)^{1/2}\right]}$$



EXERCISE 12

$$= \frac{V_m^2 2R\sqrt{\pi}\left[2(\pi-\alpha)+\sin 2\alpha\right]}{\sqrt{2}V_m^2 4\pi R\left[2(\pi-\alpha)+\sin 2\alpha\right]^{1/2}}$$

$$= \frac{1}{\sqrt{2\pi}}\left[2(\pi-\alpha)+\sin 2\alpha\right]^{1/2}$$



EXERCISE 13

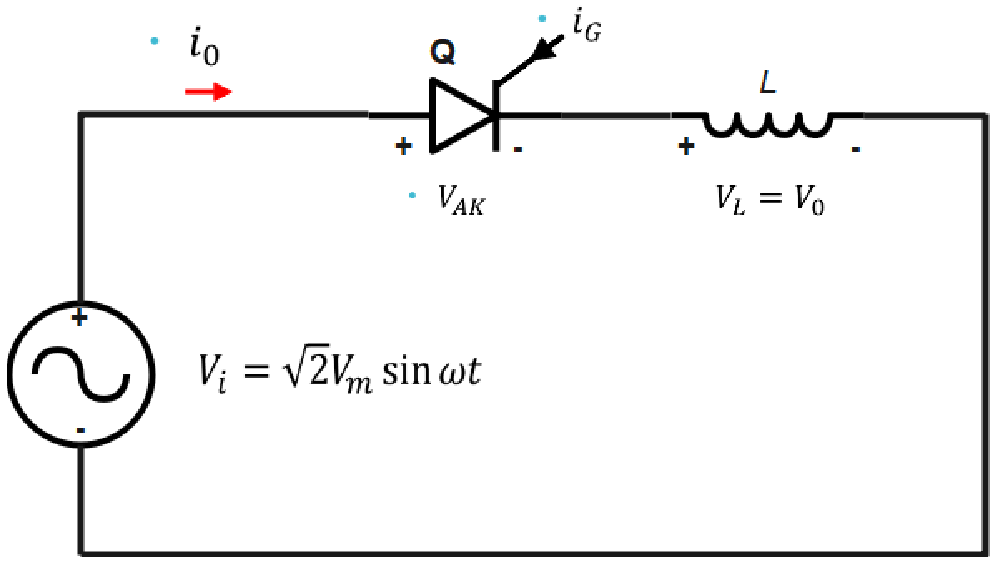
For the following circuit, firing pulse is given at angle α=60°. Moreover, are given: $V_i = \sqrt{2}V_m \sin\omega t$ $V_m = 220$ Volts, $\omega L = 1\,\Omega$

The waveforms of the variables V_i, V_o , i_o have to be drawn. Moreover, the followings have to be calculated:

I. Both mean and RMS value of the output voltage

II. Both mean and RMS value of the output current

III. The input power factor



EXERCISE 13

Solution

When the thyristor conducts then the following relationship is valid:

$$V_i = V_0 \Leftrightarrow$$

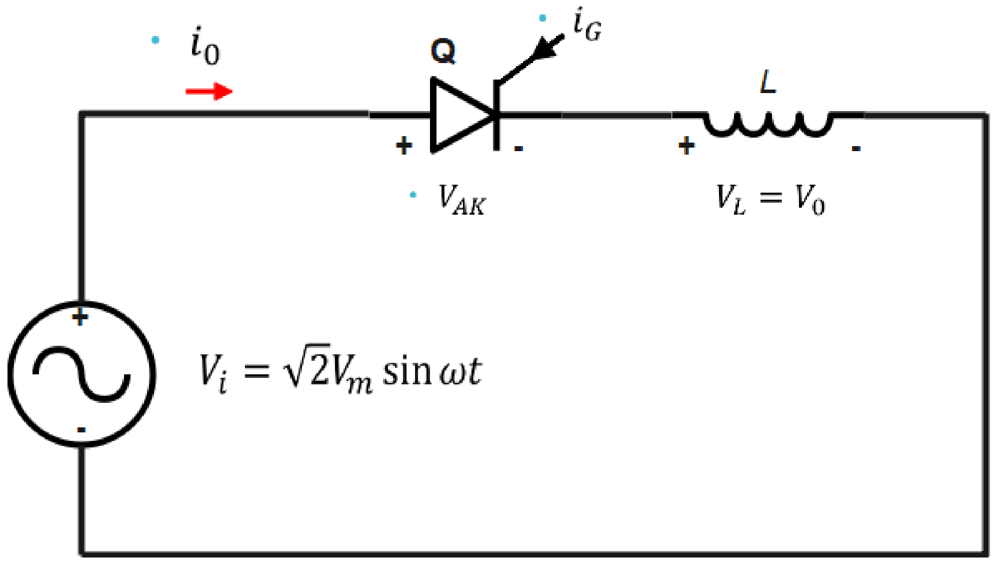
$$\sqrt{2}V_m \sin\omega t = L\frac{di_0}{dt}$$

Therefore,

$$i_0(\omega t) = \frac{1}{L}\int_{\alpha}^{\omega t} \sqrt{2}\, V_m \sin\omega t\, d\omega t$$

$$= \frac{\sqrt{2}V_m}{\omega L}\left[-\cos\omega t\, |_{\alpha}^{\omega t}\right]$$

$$= \frac{\sqrt{2}V_m}{\omega L}\left[\cos\alpha - \cos\omega t\right]$$



EXERCISE 13

From the above relationship it is found that for ωt = α then i = 0. Moreover, for ωt = π the output current is maximum. Therefore, the conduction angle of thyristor is:

$$\beta = 2\pi - \alpha = 360^\circ - 60^\circ = 300^\circ$$

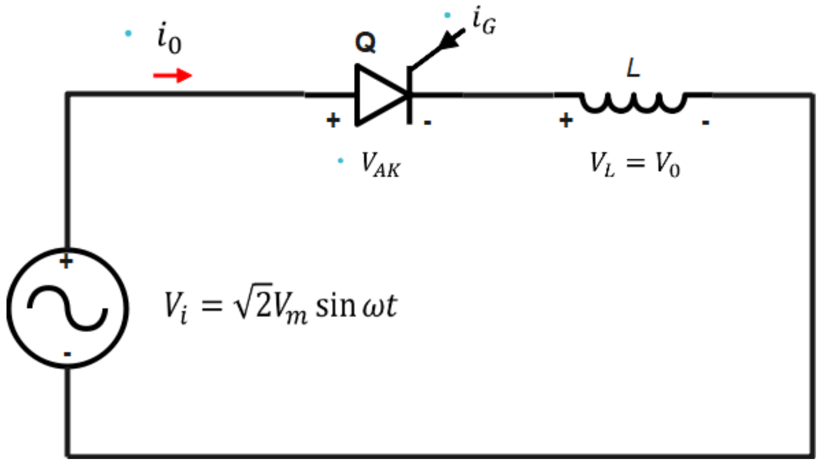
$$V_0 = \sqrt{2}V_m \sin\omega t$$

$$\text{for } \alpha \le \omega t \le \beta$$

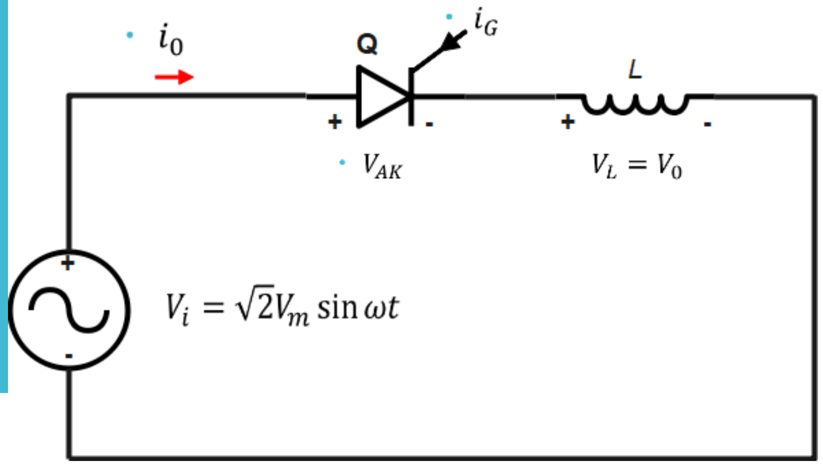
The output voltage is given by the equation:

$$V_0 = \sqrt{2}V_m \sin\omega t$$

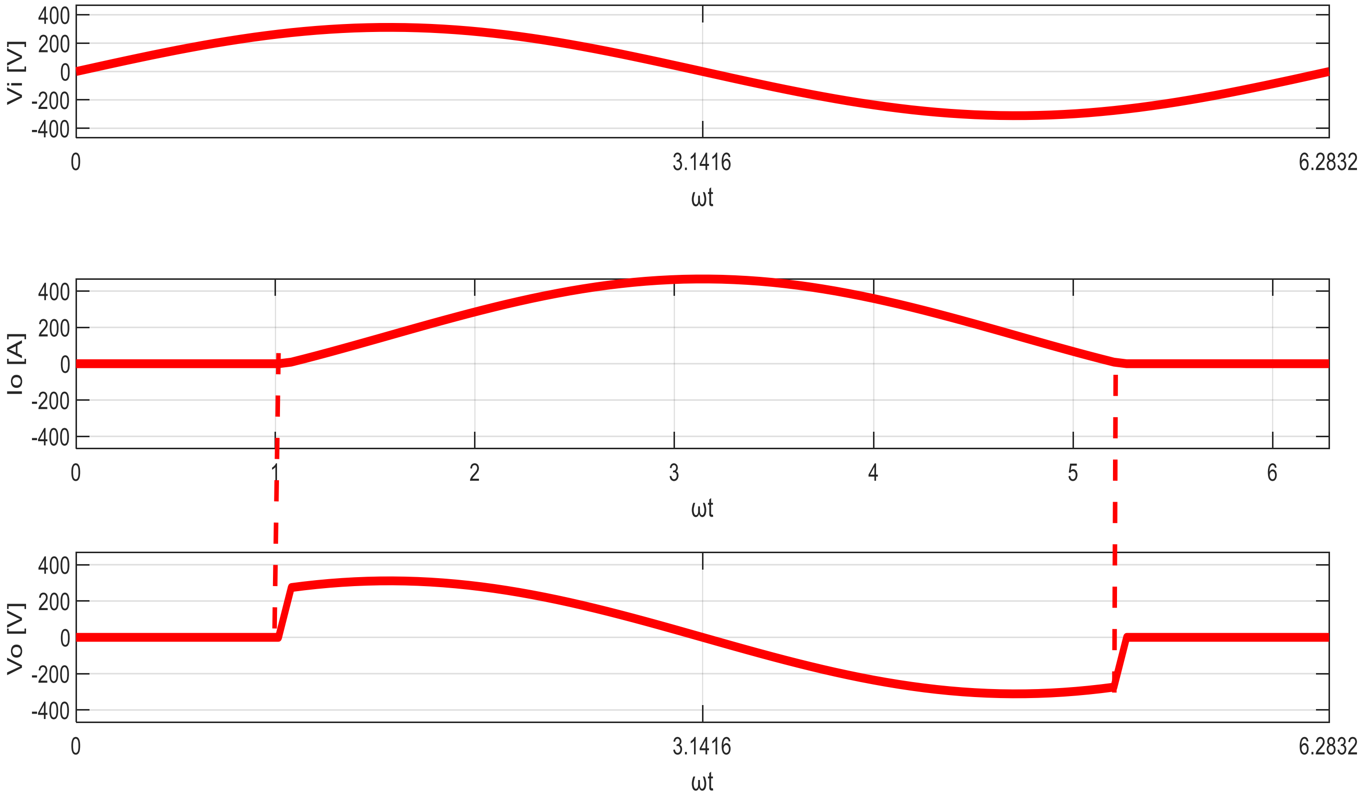
$$\text{for } \alpha \le \omega t \le \beta$$



EXERCISE 13



The waveforms of the variables of the circuit are:



EXERCISE 13

From the above waveforms, the followings can be extracted:

Mean value of the output voltage:

$$\bar{V}_0 = \frac{1}{2\pi}\int_0^{2\pi} V_0 \, d\omega t$$

$$= \frac{1}{2\pi}\int_\alpha^{2\pi-\alpha} 220\sqrt{2}\sin\omega t \, d\omega t = 0$$

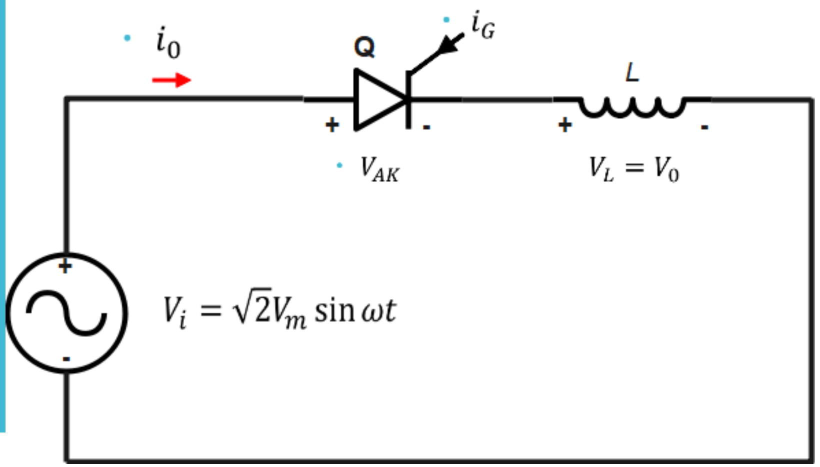
RMS value of the output voltage:

$$\tilde{V}_0 = \left[\frac{1}{2\pi}\int_0^{2\pi} V_0^2 \, d\omega t\right]^{1/2}$$

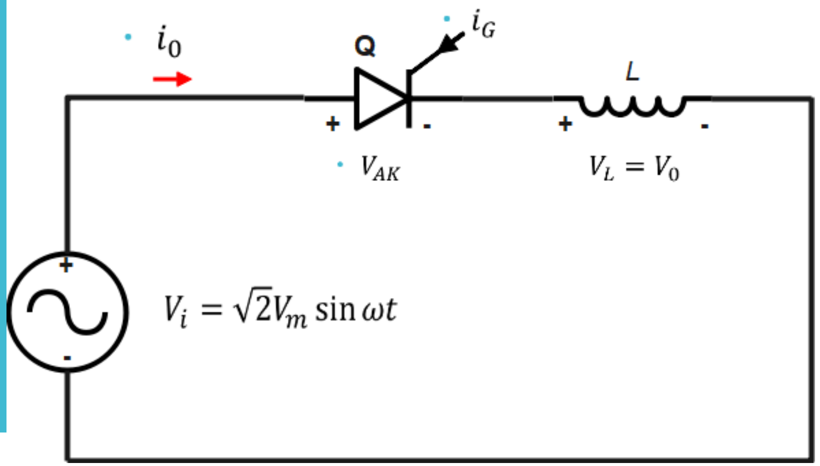
$$= \left[\frac{1}{2\pi}\int_\alpha^{\beta}\left[\sqrt{2}V_m \sin\omega t\right]^2 d\omega t\right]^{1/2}$$

$$= \left[\frac{1}{2\pi}\int_\alpha^{\beta} 2V_m^2\left(\frac{1-\cos 2\omega t}{2}\right) d\omega t\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi}\left(\omega t - \frac{\sin 2\omega t}{2}\right)\Big|_\alpha^\beta\right]^{1/2}$$



EXERCISE 13



$$= \left[\frac{V_m^2}{2\pi} \left(\beta - \alpha - \frac{\sin 2\beta}{2} + \frac{\sin 2\alpha}{\alpha} \right) \right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi} \left(2\pi - \alpha - \alpha \frac{\sin 2(2\pi - \alpha)}{2} + \frac{\sin 2\alpha}{\alpha} \right) \right]^{1/2}$$

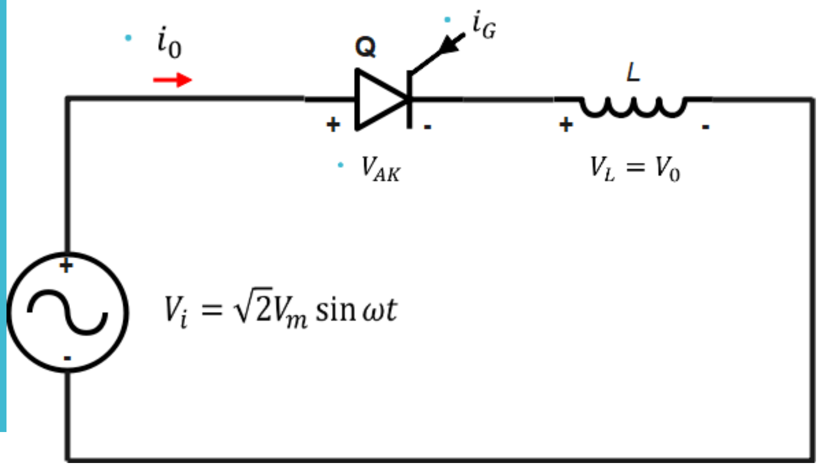
$$= \left[\frac{V_m^2}{2\pi} (2\pi - 2\alpha + \sin 2\alpha) \right]^{1/2}$$

$$= \left[\frac{V_m^2}{\pi} \left(\pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{1/2}$$

$$= \left[\frac{220^2}{\pi} \left(\pi - \frac{\pi}{3} + \frac{\sin\left(2\pi/3 \right)}{2} \right) \right]^{1/2}$$

$$= 197.25 \text{ Volts}$$

EXERCISE 13



Mean value of the output current:

$$\overline{I_0} = \frac{1}{2\pi}\int_0^{2\pi} i_0 \, d\omega t$$

$$= \frac{1}{2\pi}\int_\alpha^\beta \frac{\sqrt{2}V_m}{\omega L}\left[\cos\alpha - \cos\omega t\right] d\omega t$$

$$= \frac{\sqrt{2}V_m}{2\pi\omega L}\left[\int_\alpha^\beta \cos\alpha \, d\omega t - \int_\alpha^\beta \cos\omega t \, d\omega t\right]$$

$$= \frac{\sqrt{2}V_m}{2\pi\omega L}\left[(\cos\alpha)(\beta-\alpha) - \sin\omega t \, |_\beta^\alpha\right]$$

$$= \frac{\sqrt{2}V_m}{2\pi\omega L}\left[(\cos\alpha)(\beta-\alpha) - \sin\beta - \sin\alpha\right]$$

$$= \frac{\sqrt{2}V_m}{2\pi\omega L}\left[(\cos\alpha)(2\pi-\alpha-\alpha) - \sin(2\pi-\alpha) + \sin\alpha\right]$$

$$= \frac{220\sqrt{2}}{2\pi}\left[\cos\left(\pi/3\right)\left(2\pi - 2\pi/3\right) + 2\sin\left(\pi/3\right)\right]$$

$$= 189.5 \text{ Amps}$$

EXERCISE 13

RMS value of the output current:

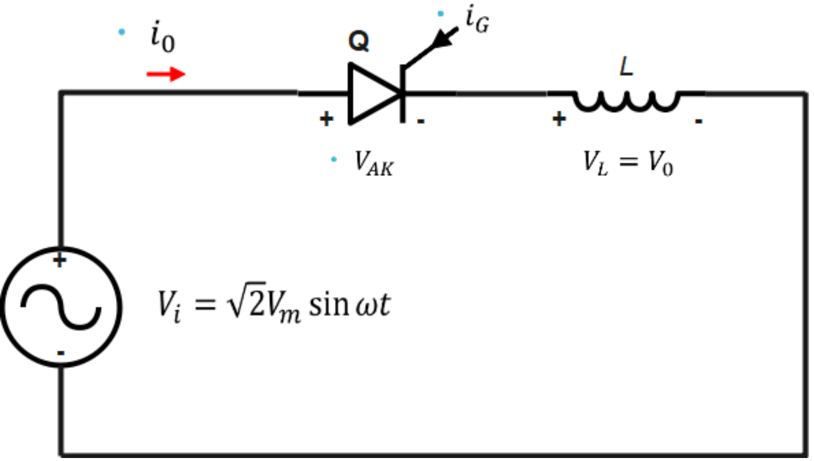
$$\tilde{I}_0 = \left[\frac{1}{2\pi} \int_0^{2\pi} i_0^2 \, d\omega t \right]^{1/2}$$

$$= \left[\frac{1}{2\pi} \int_\alpha^\beta \left[\frac{\sqrt{2}V_m}{\omega L} (\cos\alpha - \cos\omega t) \right]^2 d\omega t \right]^{1/2}$$

$$= \left[\frac{V_m^2}{\pi(\omega L)^2} \int_\alpha^\beta \left[(\cos\alpha)^2 - 2(\cos\alpha)(\cos\omega t) + (\cos\omega t)^2 \right] d\omega t \right]^{1/2}$$

$$= \left[\frac{V_m^2}{\pi(\omega L)^2} \int_\alpha^\beta \left[(\cos\alpha)^2 (\omega t)|_\alpha^\beta - 2(\cos\alpha)(\sin\omega t)|_\alpha^\beta + \frac{1}{2}\left(\omega t + \frac{\sin 2\omega t}{2} \right)|_\alpha^\beta \right] \right]^{1/2}$$

$$= \left[\frac{V_m^2}{\pi(\omega L)^2} \int_\alpha^\beta \left[(\cos\alpha)^2 (2\pi - \alpha - \alpha) - 2(\cos\alpha)\left[\sin(2\pi - \alpha) - \sin\alpha \right] + \frac{1}{2}\left(2\pi - \alpha - \alpha + \frac{\sin 2(2\pi - \alpha) - \sin 2\alpha}{2} \right) \right] \right]^{1/2}$$



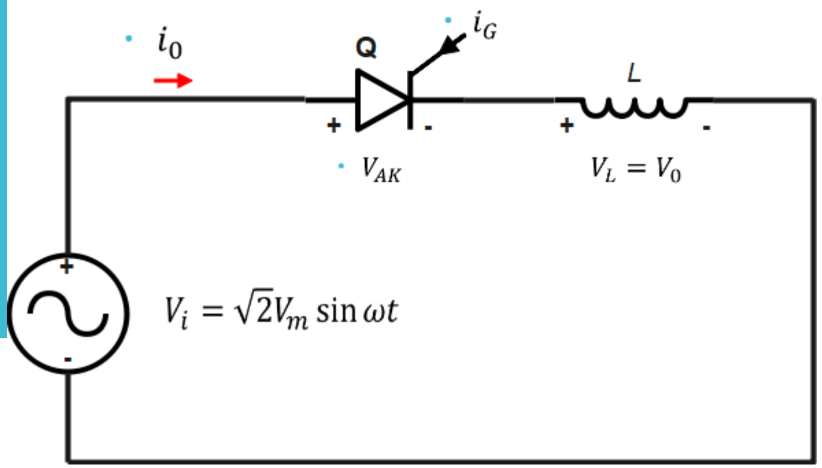
EXERCISE 13

$$= \left[\frac{V_m^2}{\pi(\omega L)^2} \left[(\cos\alpha)^2 (2\pi - 2\alpha) + 4(\cos\alpha)(\sin\alpha) + \frac{1}{2}(2\pi - 2\alpha - \sin 2\alpha) \right] \right]^{1/2}$$

$$= \frac{V_m}{\omega L} \left[\frac{4(\cos\alpha)^2 (\pi - \alpha) + 4\sin 2\alpha + 2(\pi - \alpha) - \sin 2\alpha}{2\pi} \right]^{1/2}$$

$$= \frac{V_m}{\omega L} \left[\frac{2(\pi - \alpha) + \left[2(\cos\alpha)^2 + 1 \right] + 3\sin 2\alpha}{2\pi} \right]^{1/2}$$

The power factor of the circuit is equal to zero because the input active power is equal to zero. This is a purely inductive load.



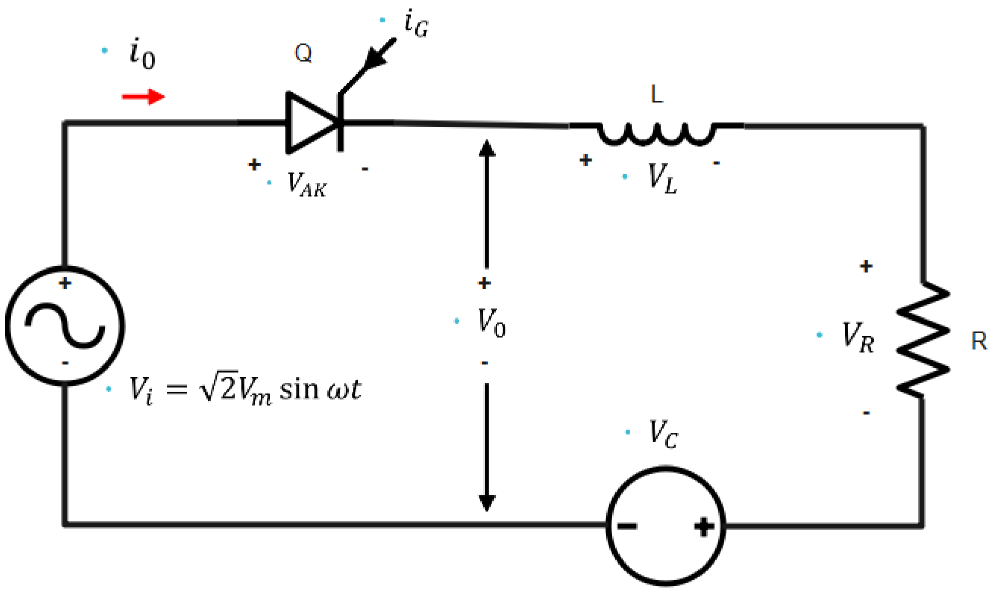
EXERCISE 14

For the following circuit are given: $V_i = 240\sqrt{2}\sin\omega t \quad \omega = 100\pi \text{ rad/sec}$

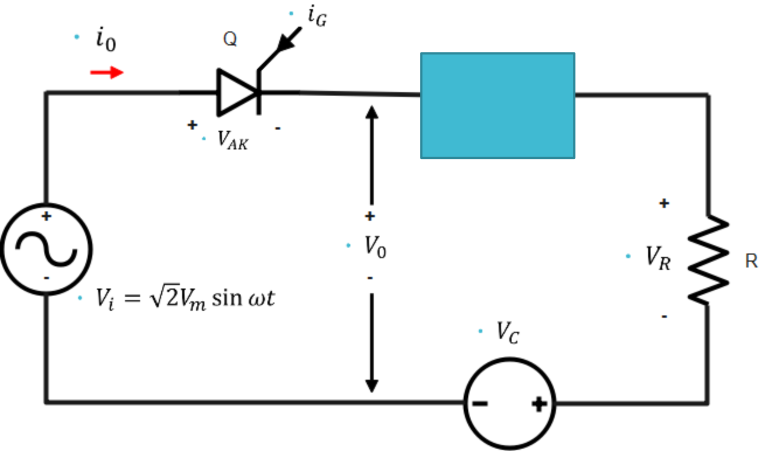
firing angle $\alpha = 60^\circ$ and $V_C = 100$ Volts

The waveforms of the variables V_i, V_o, i_o have to be drawn for the following cases:

a) Ohmic load R=10 Ω
b) Purely inductive load
c) Ohmic-Inductive load R=10 Ω ωL=10 Ω



EXERCISE 14



Solution

a) For the above circuit, attention must be paid, if the firing angle α of thyristor is before or after the angle γ at which the output voltage has the same value as the voltage V_C , therefore:

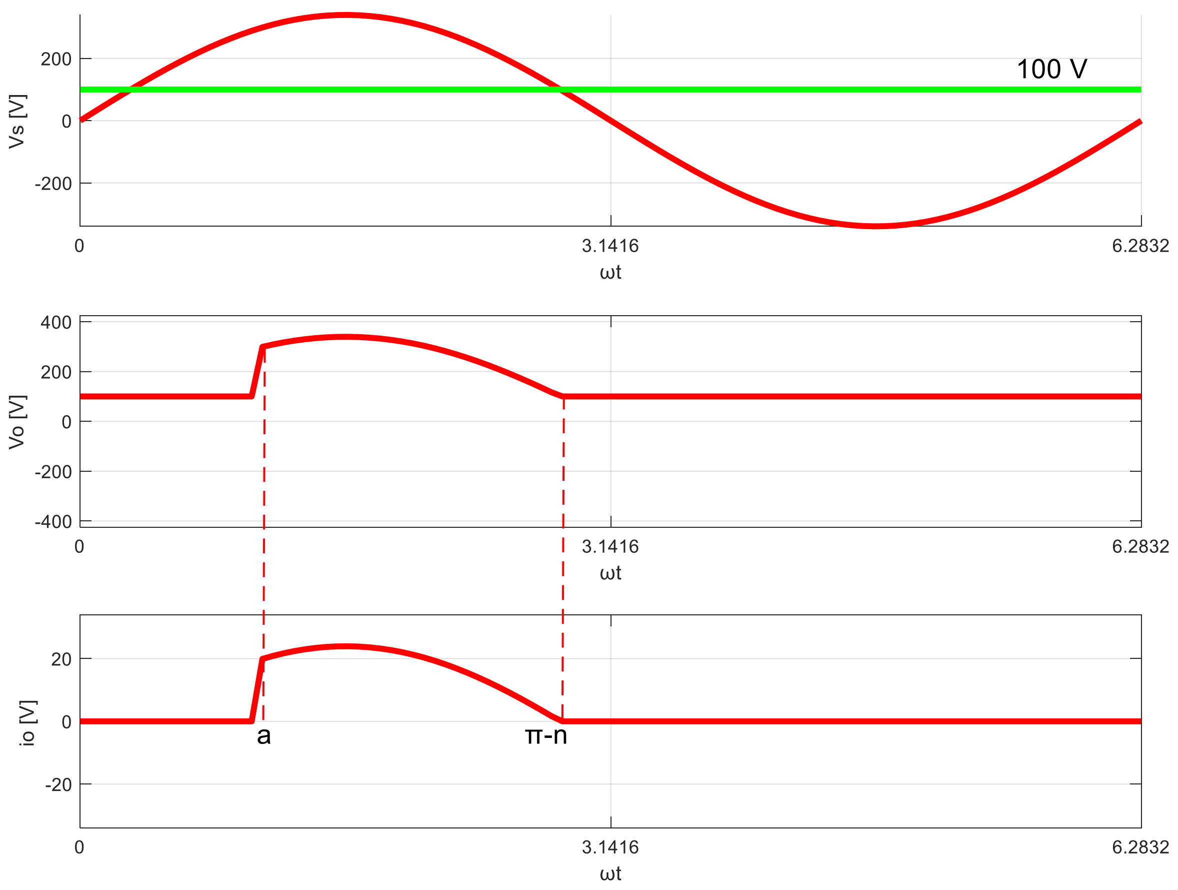
$$V_i = V_C \Leftrightarrow$$
$$240\sqrt{2}\sin(\gamma) = 100 \Leftrightarrow$$
$$\gamma = \sin^{-1}\left[\frac{100}{240\sqrt{2}}\right] \Leftrightarrow$$
$$\gamma = 17^\circ$$

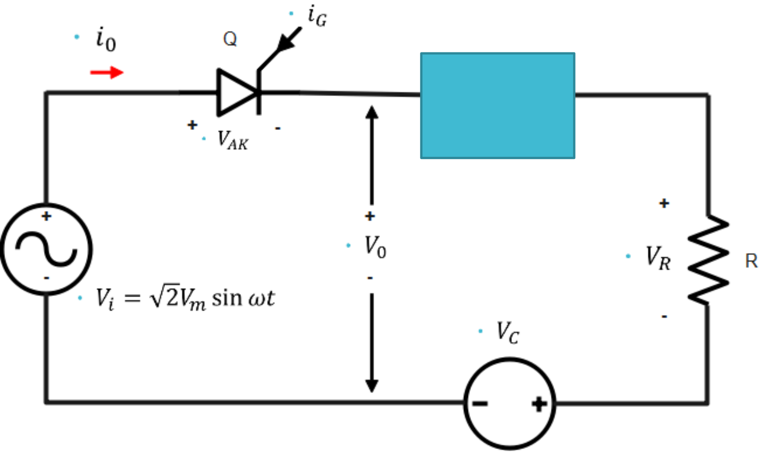
Because the angle α is greater than the angle γ, thyristor conducts to angle α = 60° and turns off at angle β (when the input voltage becomes equal to V_C). Therefore, during extinguishing:

$$V_i = V_C \Leftrightarrow$$
$$240\sqrt{2}\sin\beta = 100 \Leftrightarrow$$
$$\beta = \sin^{-1}\left[\frac{100}{240\sqrt{2}}\right] \Leftrightarrow$$
$$\beta = 17^\circ \text{ or } \beta = 163^\circ$$

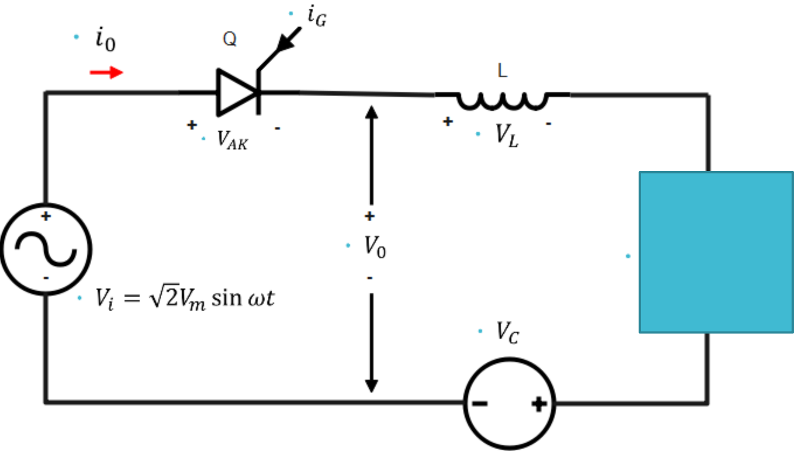
EXERCISE 14

From the two values obviously β = 163° is chosen. The required waveforms of the variables of the circuit for ohmic load are:





EXERCISE 14



b) When thyristor conducts then the following equation can be extracted:

$$V_i = V_0 \Leftrightarrow$$

$$\sqrt{2}V_m \sin\omega t = V_C + L\frac{di_0}{dt}$$

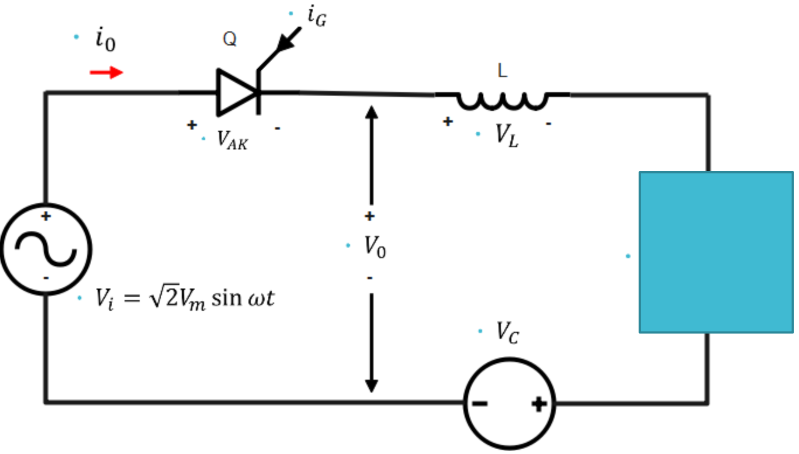
Therefore,

$$i_0(\omega t) = \frac{1}{\omega L}\int_{\alpha}^{\omega t}\sqrt{2}V_m \sin\omega t \, d\omega t - \frac{1}{\omega L}\int_{\alpha}^{\omega t} V_C \, d\omega t$$

$$= \frac{\sqrt{2}V_m}{\omega L}\left[-\cos\omega t|_{\alpha}^{\omega t}\right] - \frac{V_C}{\omega L}\left[\omega t|_{\alpha}^{\omega t}\right]$$

$$= \frac{\sqrt{2}V_m}{\omega L}\left[\cos\alpha - \cos\omega t\right] - \frac{V_C}{\omega L}\left[\omega t - \alpha\right]$$

EXERCISE 14



From the above relationship it is found that for ωt = α then i = 0. The maximum current value shows up at the angle ωt, where the following equation can be extracted:

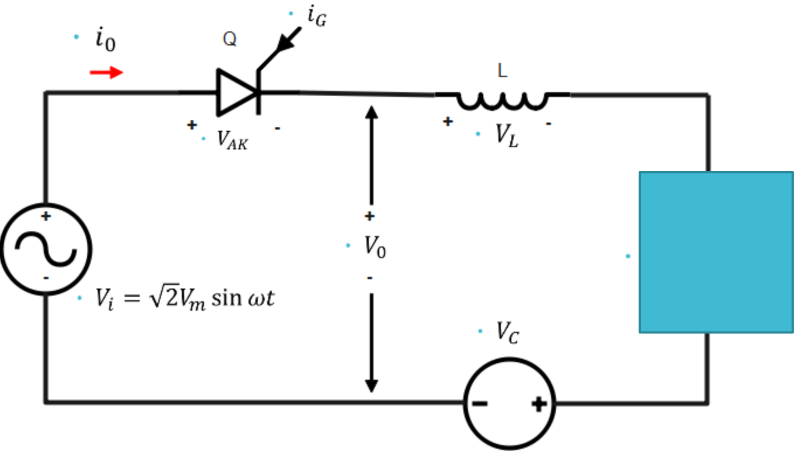
$$\frac{di_0(\omega t)}{d(\omega t)} = 0 \Leftrightarrow$$

$$\frac{\sqrt{2}V_m}{L}\sin\omega t = \frac{V_C}{L} \Leftrightarrow$$

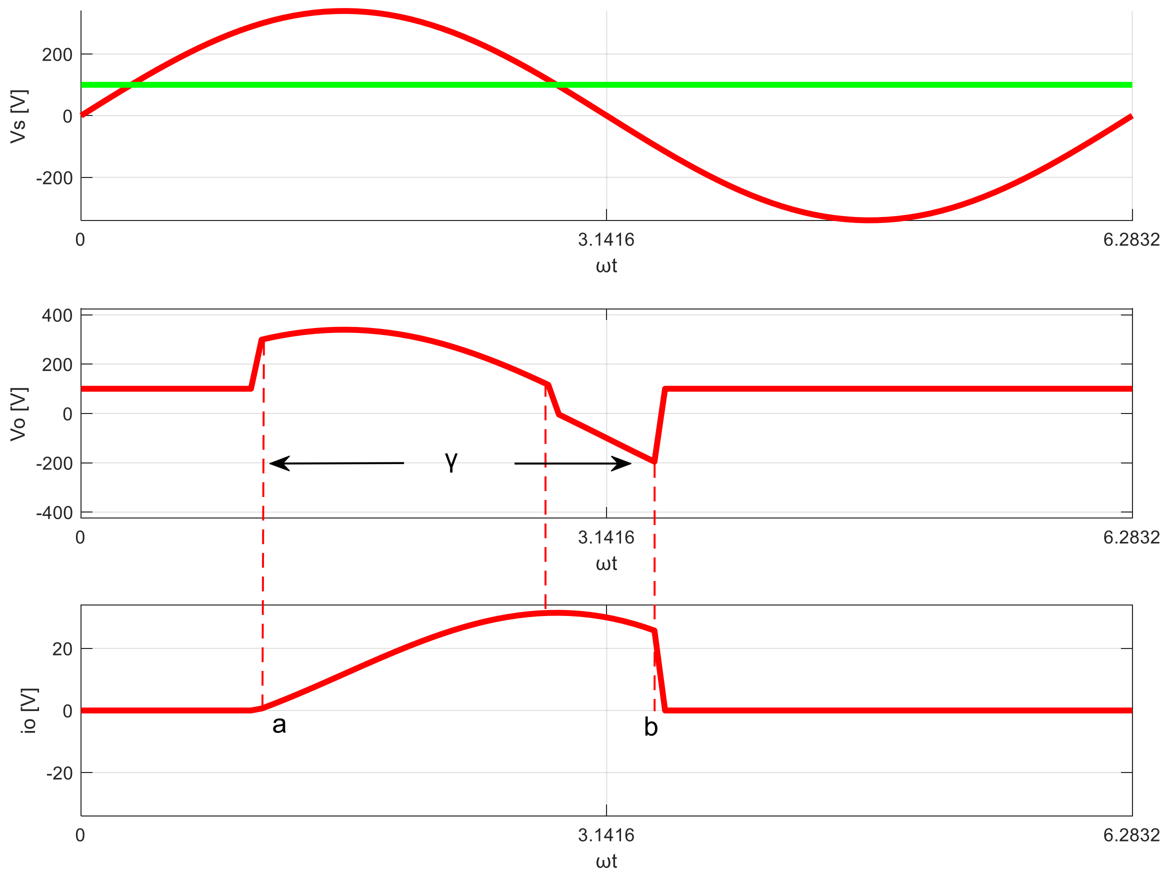
$$\sin(\omega t) = \frac{V_C}{\sqrt{2}V_m} \Leftrightarrow$$

$$\omega t = \sin^{-1}\left[\frac{100}{240\sqrt{2}}\right] \text{ or } \omega t = 163^\circ$$

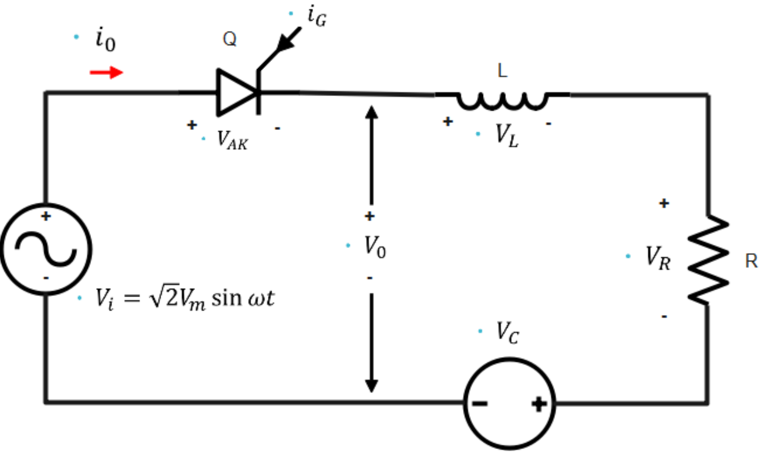
EXERCISE 14



The waveforms of the variables of the circuit are:



EXERCISE 14



c) The current of the above circuit for ohmic-inductive load is given by the equation:

$$i_0 = \frac{V_m\sqrt{2}}{|Z|}\left[\sin(\omega t - \varphi) - \frac{m}{\cos\varphi} + Be^{(\alpha-\omega t)/\tan\phi}\right]$$

for $\alpha \le \omega t \le \alpha + \gamma$

Where,

$\varphi =$ angle of current shifting

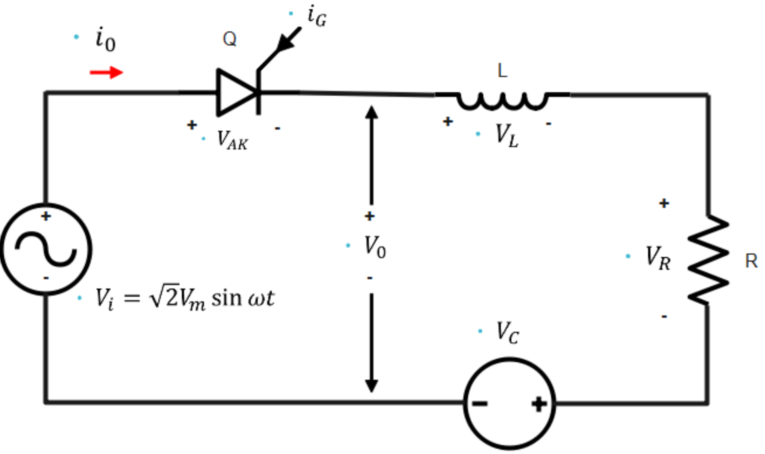
$$= \tan^{-1}\left[\frac{\omega L}{R}\right]$$

$$= \tan^{-1}\left[\frac{10}{10}\right]$$

$$= 45^\circ$$

$\alpha =$ firing angle of thyristor $= 60^\circ$

EXERCISE 14

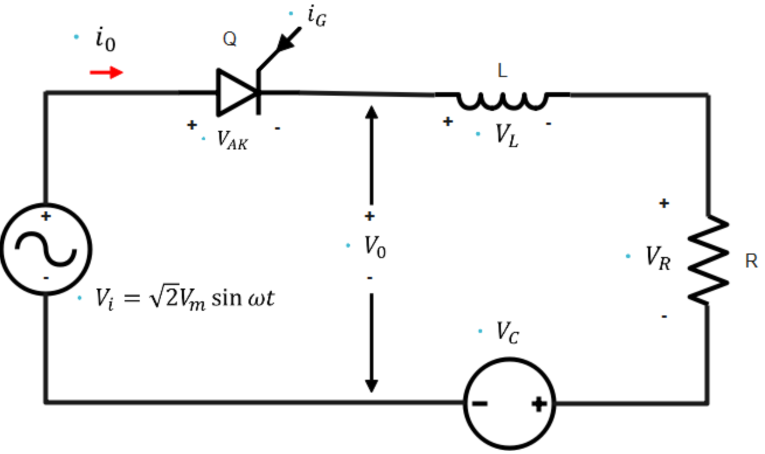


$$\rightarrow |Z| = \sqrt{R^2 + (\omega L)^2}$$
$$= \sqrt{10^2 + 10^2}$$
$$= 14.14\ \Omega$$

$$\rightarrow m = \frac{V_c}{240\sqrt{2}} = \frac{100}{240\sqrt{2}} = 0.3$$

$$\rightarrow B = \frac{m}{\cos\phi} - \sin(\alpha - \varphi)$$
$$= \frac{0.3}{\cos 45^\circ} - \sin^{-1}(60^\circ - 45^\circ) = 0.165$$

EXERCISE 14



Therefore,

$$i_0 = \frac{240\sqrt{2}}{14.14}\left[\sin\left(\omega t - 45^\circ\right) - \frac{0.3}{\cos 45^\circ} + 0.165e^{\left(\pi/3 - \omega t\right)/\tan 45^\circ}\right]$$

$$= 24\left[\sin\left(\omega t - 45^\circ\right) - 0.42 + 0.165e^{\left(\pi/3 - \omega t\right)/\tan 45^\circ}\right]$$

$$= 24\left[\sin\left(\omega t - 45^\circ\right) - 0.42 + 0.47e^{-\omega t}\right]$$

for $\alpha \le \omega t \le \alpha + \gamma$

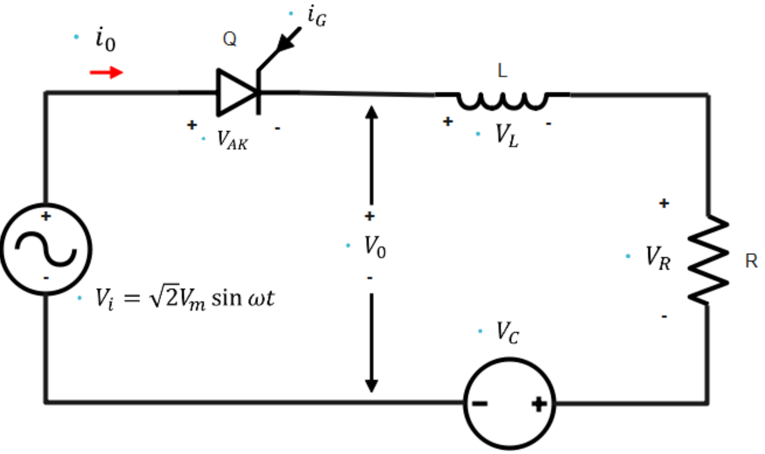
From the above equation for i=0 , the extinction angle β of thyristor is:

$$\beta = 202^\circ$$

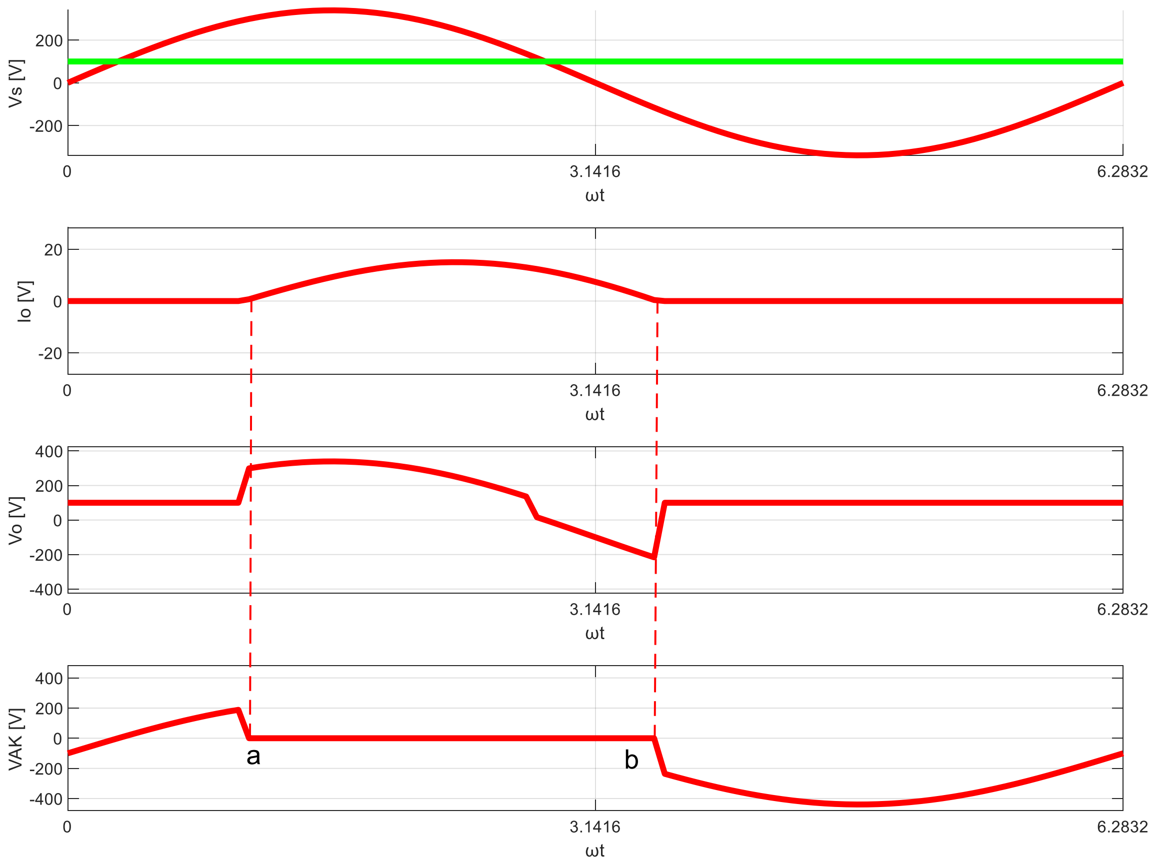
Therefore, the conduction angle of thyristor is:

$$\gamma = \beta - \alpha = 202 - 60 = 142^\circ$$

EXERCISE 14



The waveforms of the variables of the circuit for this load are:

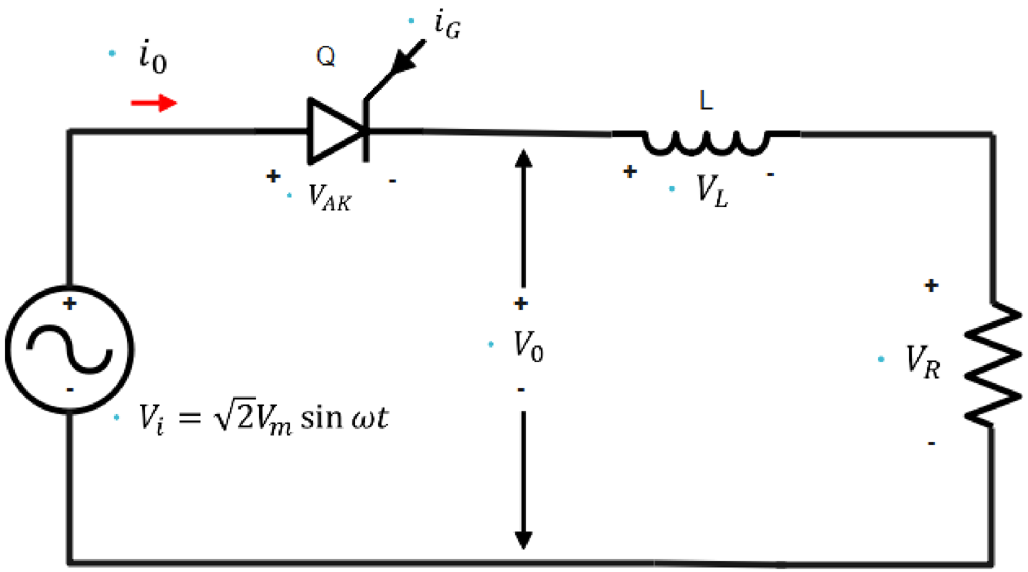


EXERCISE 15

For the following circuit the RMS value of the output voltage is 240 Volts and grid frequency is 50 Hz. If the voltage drop at the thyristor terminals is considered negligible and the firing angle of thyristor is $\alpha = 70°$, the followings have to be calculated:

a) For purely ohmic load R=10 Ω, the waveforms of the variables V_i, V_o, i_o, V_{AK} have to be drawn and both mean value of output voltage and output current have to be found.

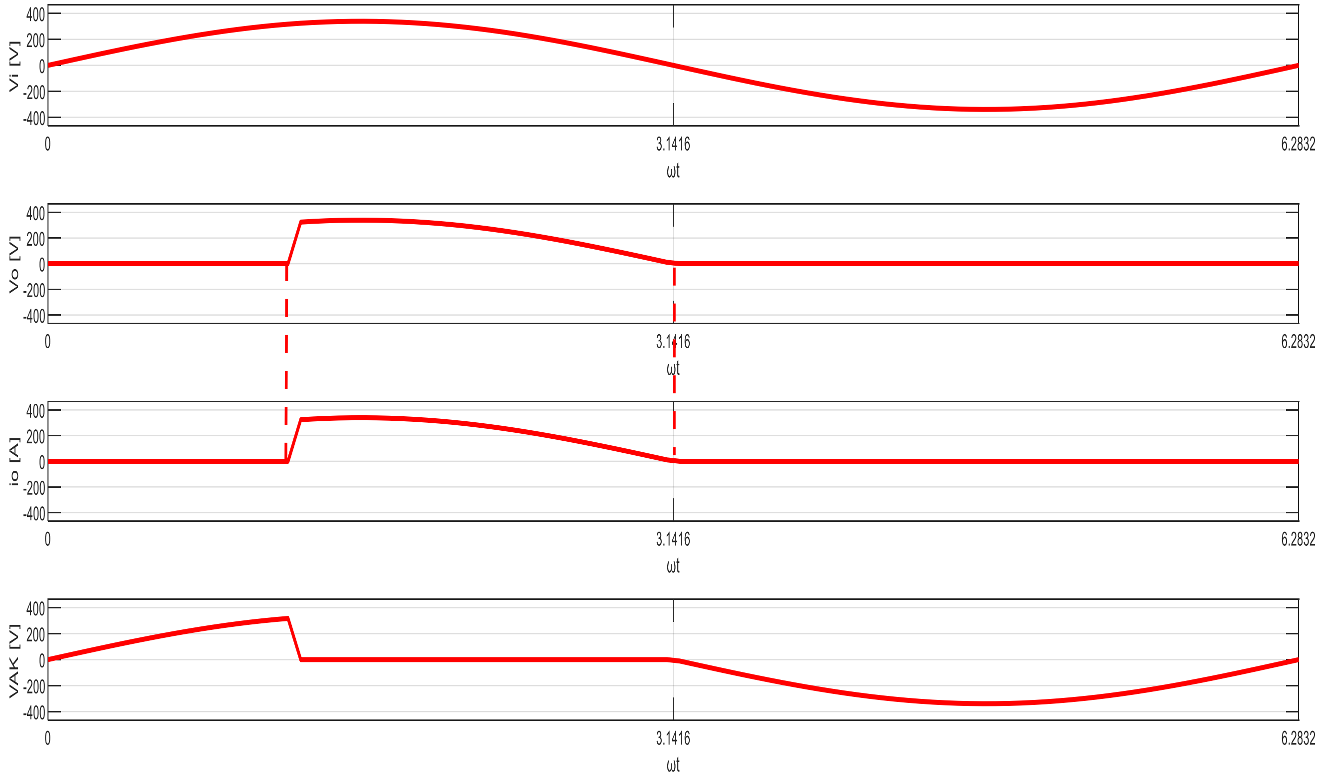
b) For purely inductive load L=0.1 H, the waveforms of the variables V_i, V_o, i_o, V_{AK} have to be drawn and both mean value of output voltage and output current have to be found.



EXERCISE 15

Solution

a) For purely ohmic load the waveforms of the variables of the circuit are:



EXERCISE 15

Therefore, from the above waveforms it follows that:

Mean value of the output voltage $\overline{V}_0 = \frac{1}{2\pi}\int_0^{2\pi} V_0 \, d\omega t$

$$= \frac{1}{2\pi}\int_\alpha^{\pi} 240\sqrt{2}\sin\omega t \, d\omega t$$

$$= \frac{240\sqrt{2}}{2\pi}\left(-\cos\omega t \,|_\alpha^\pi\right)$$

$$= \frac{240\sqrt{2}}{2\pi}\left(-\cos\pi + \cos 70^\circ\right)$$

$$= \frac{240\sqrt{2}}{2\pi}\left(1 + 0.34\right)$$

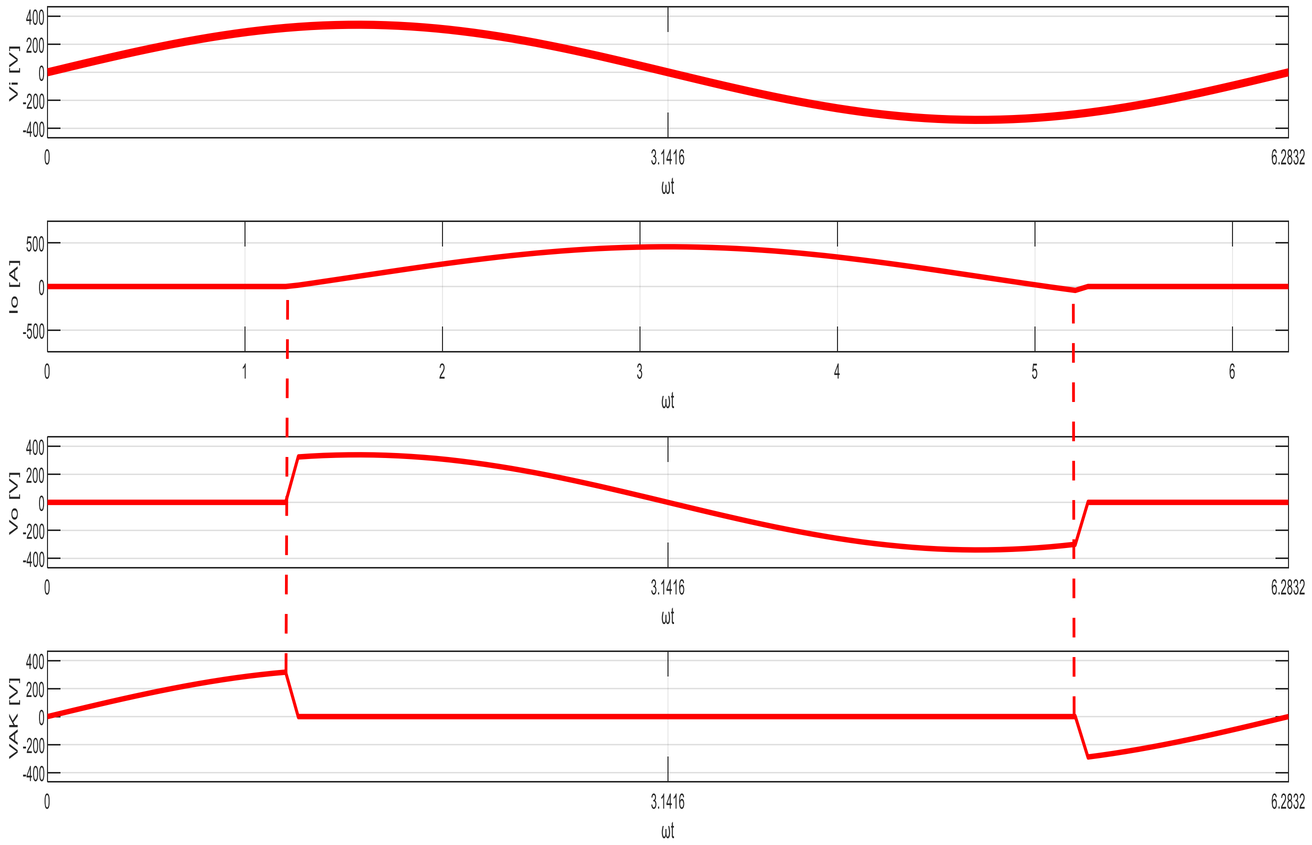
$$= 72.5 \text{ Volts}$$

Mean value of the output current $\overline{I}_0 = \frac{\overline{V}_0}{R}$

$$= \frac{72.5}{10} = 7.25 \text{ Amps}$$

EXERCISE 15

b) For purely inductive load the waveforms of the variables of the circuit are:



EXERCISE 15

Therefore, from the above waveforms it follows that:

$$\text{Mean value of the output voltage } \overline{V}_0 = \frac{1}{2\pi}\int_0^{2\pi} V_0 \, d\omega t$$

$$= \frac{1}{2\pi}\int_\alpha^{2\pi-\alpha} 240\sqrt{2}\sin\omega t \, d\omega t$$

$$= 0 \text{ Volts}$$

When thyristor conducts then the following equation can be extracted :

$$V_i = V_0 \Leftrightarrow$$

$$240\sqrt{2}\sin\omega t = L\frac{di_0}{dt}$$

$$i_0(\omega t) = \frac{1}{\omega t}\int_\alpha^{\omega t} 240\sqrt{2}\sin\omega t \, d\omega t$$

$$= \frac{240\sqrt{2}}{\omega L}\left(-\cos\omega t \, |_\alpha^{\omega t}\right)$$

$$= \frac{240\sqrt{2}}{\omega L}\left(-\cos\omega t + \cos 70^\circ\right)$$

$$= \frac{240\sqrt{2}}{\omega L}\left(0.34 - \cos\omega t\right)$$

EXERCISE 15

At the angle ωt=2π-α the current $i_0(a) = 0$.

Also at angle ωt=2π-α the current $i_0(2\pi - a) = 0$.

Therefore, the extinction angle of thyristor is:

$$\beta = 2\pi - \alpha$$

Mean value of the output voltage $\bar{I}_0 = \frac{1}{2\pi}\int_0^{2\pi} i_0 \, d\omega t$

$$= \frac{1}{2\pi}\int_\alpha^{2\pi-\alpha} \frac{240\sqrt{2}}{\omega L}\left(0.34 - \cos\omega t\right)$$

$$= \frac{240\sqrt{2}}{2\pi\omega L}\left[2(\pi-\alpha)\cos\alpha - \sin(2\pi-\alpha) + \sin\alpha\right]$$

$$= \frac{240\sqrt{2}}{2\pi\omega L}\left[(\pi-\alpha)\cos\alpha + \sin\alpha\right]$$

$$= \frac{240\sqrt{2}}{\pi(2\pi 50)(0.1)}\left[\left(\pi - \frac{7\pi}{18}\right)\cos\left(\frac{7\pi}{18}\right) + \sin\left(\frac{7\pi}{18}\right)\right]$$

$$= 5.49 \text{ Amps}$$

EXERCISE 16

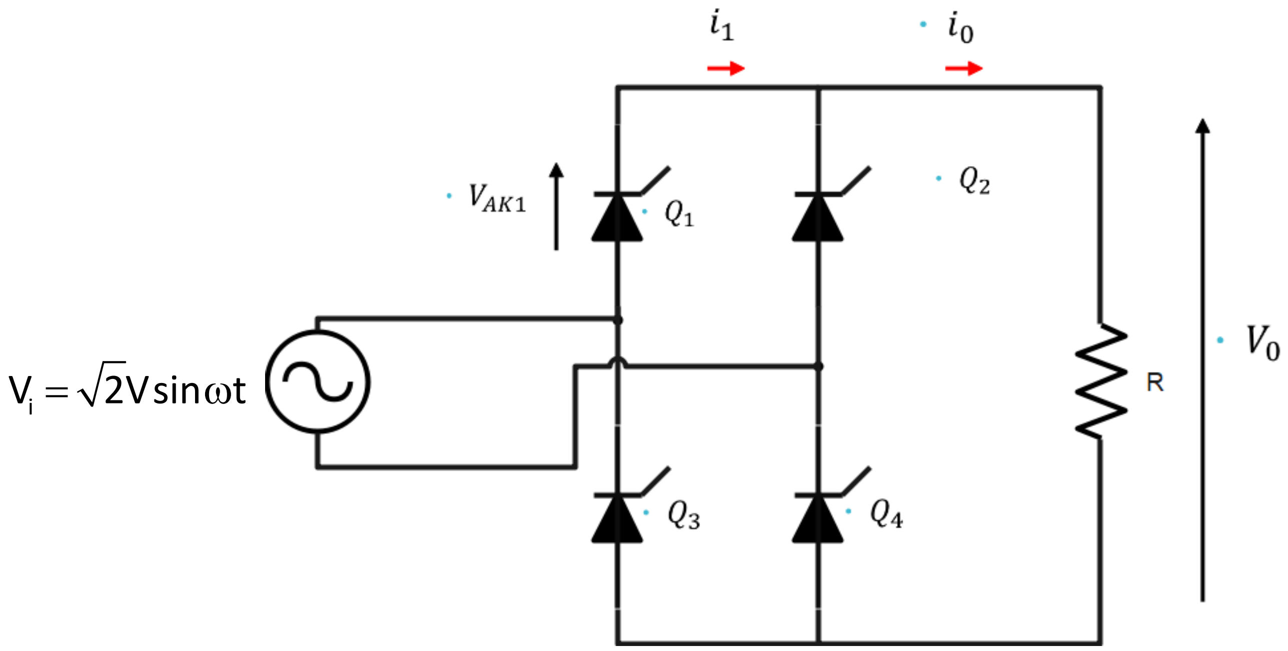
For the single-phase controlled rectification of the following circuit prove that:

a)The mean value of the output voltage is given by the relationship

$$\bar{V}_0 = \frac{\sqrt{2}V}{\pi}(1+\cos\alpha) \text{ Volts}$$,where a = firing angle of thyristor,

b)The RMS value of the output current is given by the relationship

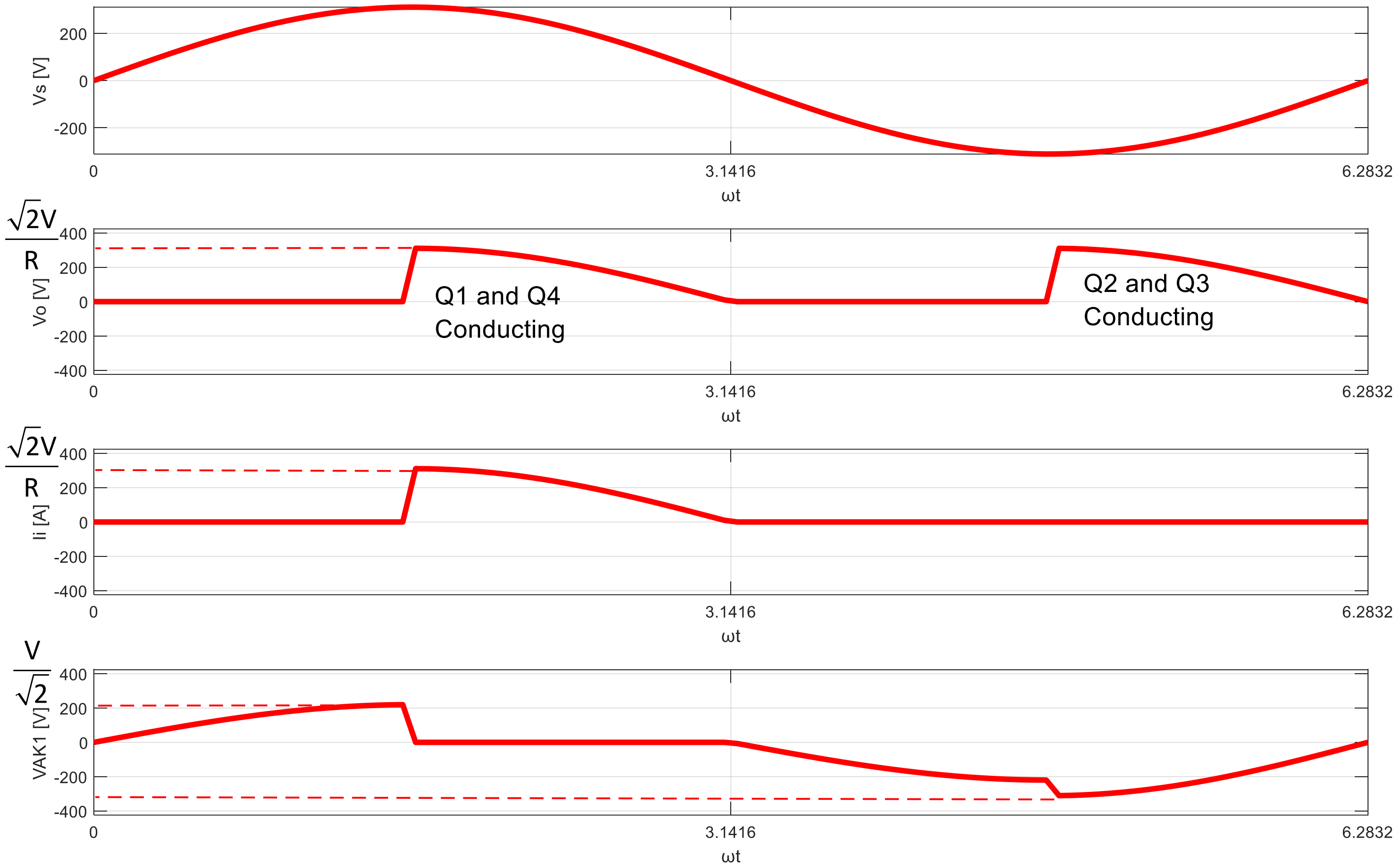
$$\tilde{I}_0 = \frac{V}{R}\left[\frac{1}{\pi}\left[(\pi-\alpha)+\frac{1}{2}\sin 2\alpha\right]\right]^{1/2} \text{ Amps}$$



EXERCISE 16

Solution

For firing angle α and ohmic load, the waveforms of the single-phase controlled rectification are:



EXERCISE 16

Using the above waveforms the followings are extended:

a) $$\bar{V}_0 = \frac{1}{\pi}\int_{\alpha}^{\pi} \sqrt{2}V \sin\omega t \, d\omega t = \frac{\sqrt{2}V}{\pi}(1+\cos\alpha)$$

b) $$\tilde{I}_0 = \left[\frac{1}{\pi}\int_{\alpha}^{\pi}\left(\frac{\sqrt{2}V}{R}\sin\omega t\right)^2 d\omega t\right]^{1/2}$$

$$= \frac{V}{R}\left[\frac{1}{\pi}\left[(\pi-\alpha)+\frac{1}{2}\sin 2\alpha\right]\right]^{1/2} \text{ Amps}$$

EXERCISE 17

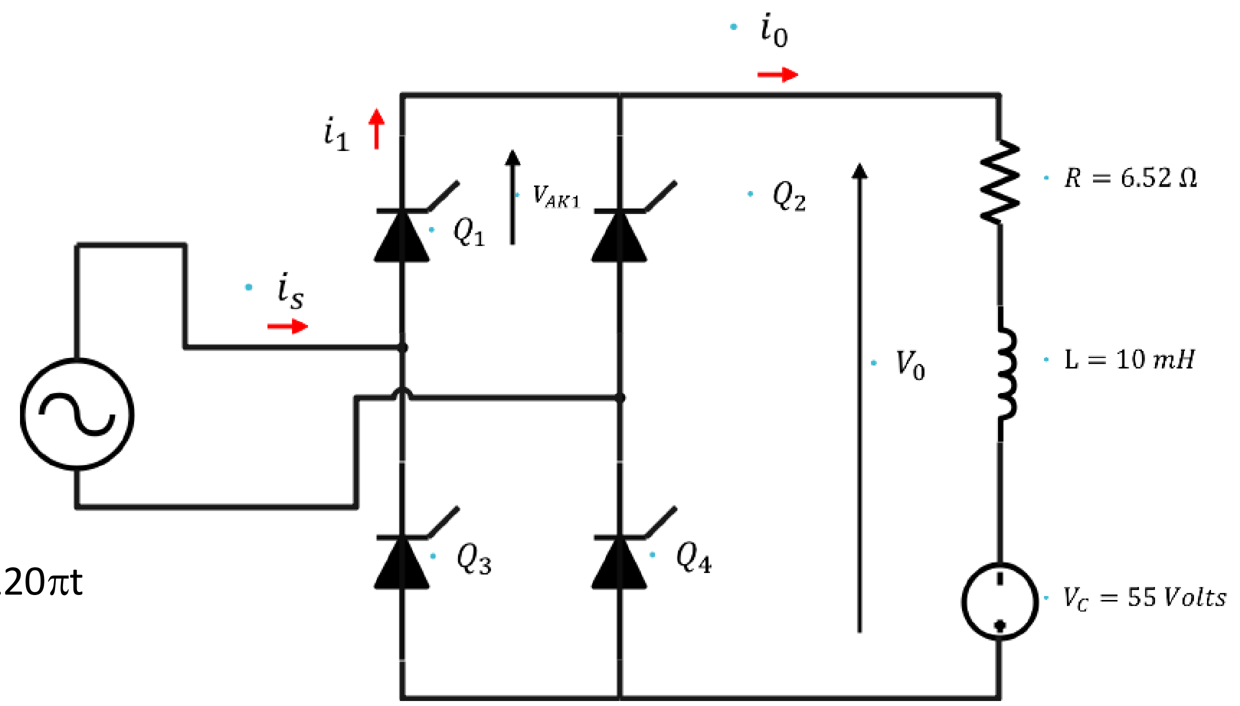
For the single-phase controlled rectification of the following circuit the firing angle of thyristor α=45° and the load is such that continuous conduction mode is considered. The followings have to be calculated:

a)The mean value of the output current

b)The RMS value of the output current

c)The RMS value of thyristor current

d)The input power factor (pf)

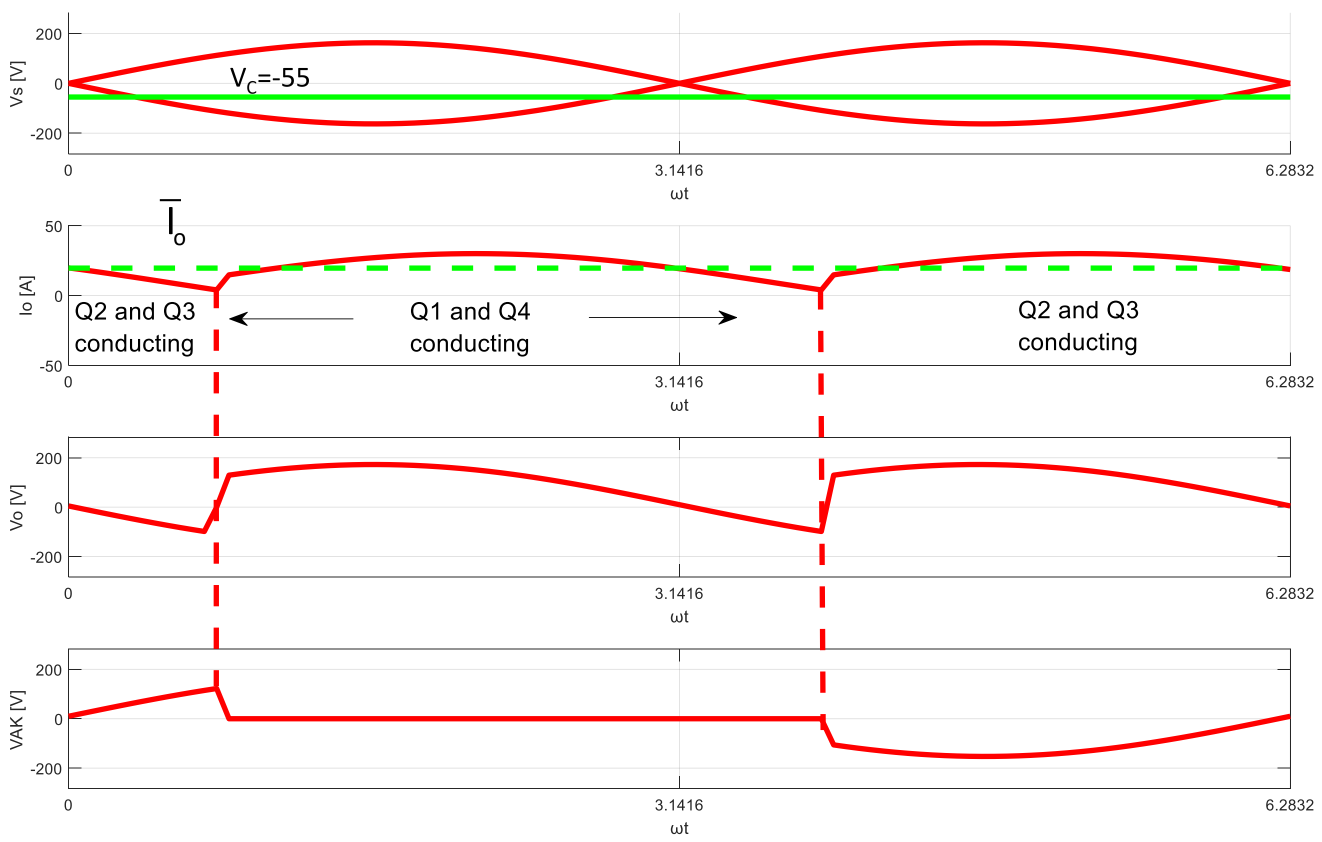


$V_i = 115\sqrt{2}\sin 120\pi t$

EXERCISE 17

Solution

For direct output current the waveforms of the variables of the circuit are:



EXERCISE 17

Using the above waveforms the followings are extracted:

a)

$$\bar{V}_0 = \frac{1}{\pi}\int_{45^\circ}^{225^\circ} 115\sqrt{2}\sin\omega t \, d\omega t$$

$$= \frac{2\sqrt{2}\cdot 115}{\pi}\cos 45^\circ$$

$$= 73.21 \text{ Volts}$$

Therefore,

$$\bar{I}_0 = \frac{\bar{V}_0 - V_C}{R}$$

$$= \frac{73.21 + 55}{6.52}$$

$$= 19.66 \text{ A}$$

EXERCISE 17

b) $$\tilde{I}_0 = \sqrt{\overline{I}_0^2 + \tilde{I}_{0,2}^2 + \tilde{I}_{0,4}^2 + \ldots\ldots} \approx \sqrt{\overline{I}_0^2 + \tilde{I}_{0,2}^2}$$

Where,

$\tilde{I}_{0,2}$ = RMS value of the second harmonic component of output current

$\tilde{I}_{0,4}$ = RMS value of the fourth harmonic component of output current

$$\tilde{I}_{0,2} \gg \tilde{I}_{0,4}$$

$$\tilde{I}_{0,2} = \frac{\tilde{V}_{0,2}}{|Z_{0,2}|}$$

Where,

$\tilde{V}_{0,2}$ = RMS value of the second harmonic component of output voltage

$|Z_{0,2}|$ = Measure of compound resistance at frequency of the second harmonic component

EXERCISE 17

$$|Z| = \sqrt{R^2 + (2\omega L)^2}$$
$$= \sqrt{6.52^2 + (2 \cdot 3.77)^2}$$
$$= 7.54\ \Omega$$

The amplitude of the n_{th} harmonic component of the output voltage is given by:

$$\hat{V}_{0,n} = \sqrt{a_n^2 + b_n^2}$$

Where,

$$a_n = \frac{2\sqrt{2}V}{\pi}\left[\frac{\sin(n+1)\alpha}{n+1} - \frac{\sin(1-n)\alpha}{1-n}\right]$$
$$b_n = \frac{2\sqrt{2}V}{\pi}\left[\frac{\cos(n+1)\alpha}{n+1} - \frac{\cos(n-1)\alpha}{n-1}\right]$$

EXERCISE 17

Therefore, for the second harmonic component :

$$\hat{V}_{0,2} = \sqrt{a_2^2 + b_2^2}$$

Where,

$$a_2 = \frac{2\sqrt{2}\cdot 115}{\pi}\left[\frac{\sin(3\cdot 45^\circ)}{3} - \sin 45^\circ\right]$$

$$= 103.5(0.235 - 0.707)$$

$$= 46.86 \text{ Volts}$$

$$b_2 = 103.5\left[\frac{\cos(3\cdot 45^\circ)}{3} - \cos 45^\circ\right]$$

$$= 103.5(-0.235 - 0.707)$$

$$= -97.5 \text{ Volts}$$

EXERCISE 17

And

$$\hat{V}_{o,2} = \sqrt{(-48.86)^2 + (-97.5)^2}$$
$$= 109.05 \text{ Volts}$$

$$\tilde{I}_{o,2} = \frac{109.05 / \sqrt{2}}{7.54} = 10.226 \text{ A}$$

Therefore,

$$\tilde{I}_0 = \sqrt{(19.66)^2 + (10.226)^2} = 22.16 \text{ Amps}$$

c) $\tilde{I}_Q = \text{RMS value of current of thyristor} = \frac{\tilde{I}_0}{\sqrt{2}} = 15.67 \text{ Amps}$

EXERCISE 17

As it is known: $$\text{pf} = \frac{\text{actual or active input power}}{\text{apparent input power}}$$

If there are no losses in the rectification, then:

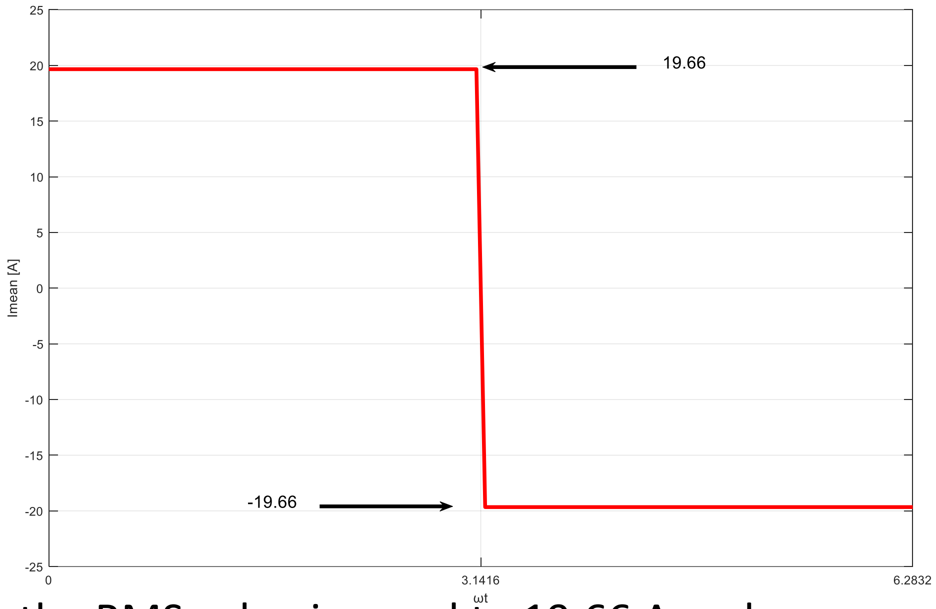
$$\text{actual input power} = \text{actual output power} =$$
$$\text{power consumed in the resistor} - \text{power supplied by the battery to the input source}$$

$$= \tilde{I}_0^2 R - V_C \bar{I}_0$$
$$= (22.16)^2 \times 6.52 - 55 \times 19.66$$
$$= 3202 - 1081$$
$$= 2120.7 \text{ Watts}$$

$$\text{apparent input power} = \tilde{V}_i \times \tilde{I}_s$$

EXERCISE 17

It should be noted that the input current has the following waveform:



Therefore, the RMS value is equal to 19.66 A and so:

$$\text{apparent power} = 115 \times 19.66 = 2260.9 \text{ VA}$$

Finally,

$$\text{pf} = \frac{2120.7}{2260.9} = 0.938$$