Financial Assets

Time Value of Money, Annuities, Stocks, Bonds and the Efficient Market Hypothesis

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Assume that

- You will be paid **€10,000** for a job.
- **When** do you prefer to get the money?
  - Now
  - 1 year from now
- The value of money is **different** over time
- Why?
- **Growth** potential of money
- **Inflation** considerations

$$i = \pi + r$$

Present Value → Future Value

- Deposit an amount **PV = present value**
- 2 cases:

- **No compounding**
- Interest is not re-invested

$$FV = PV(1 + i * t)$$

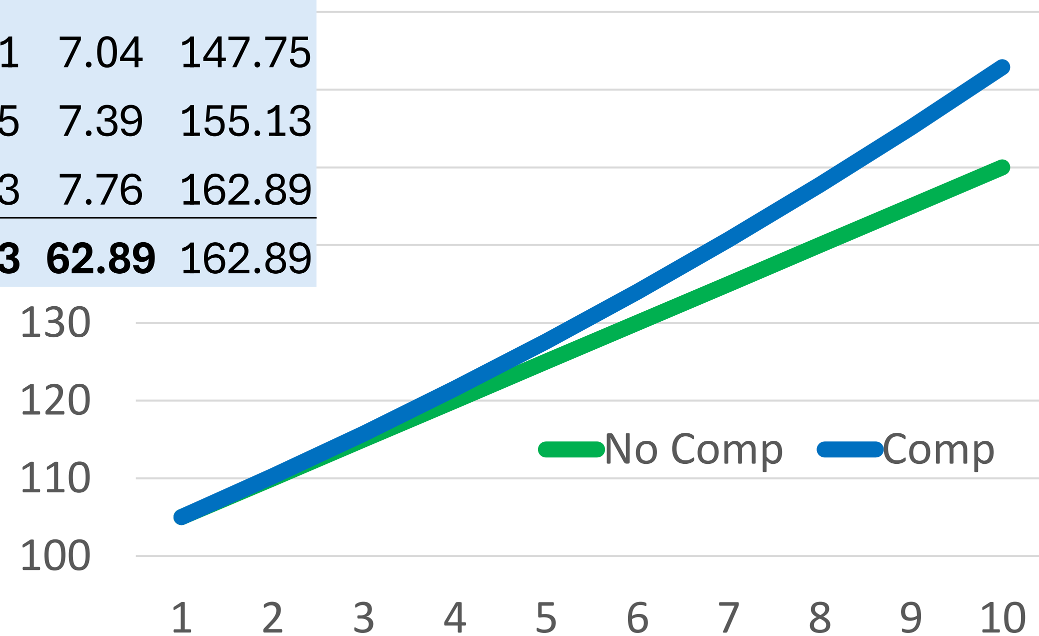
- **With compounding**
- Interest is re-invested at interest rate i

$$FV = PV(1 + i)^t$$

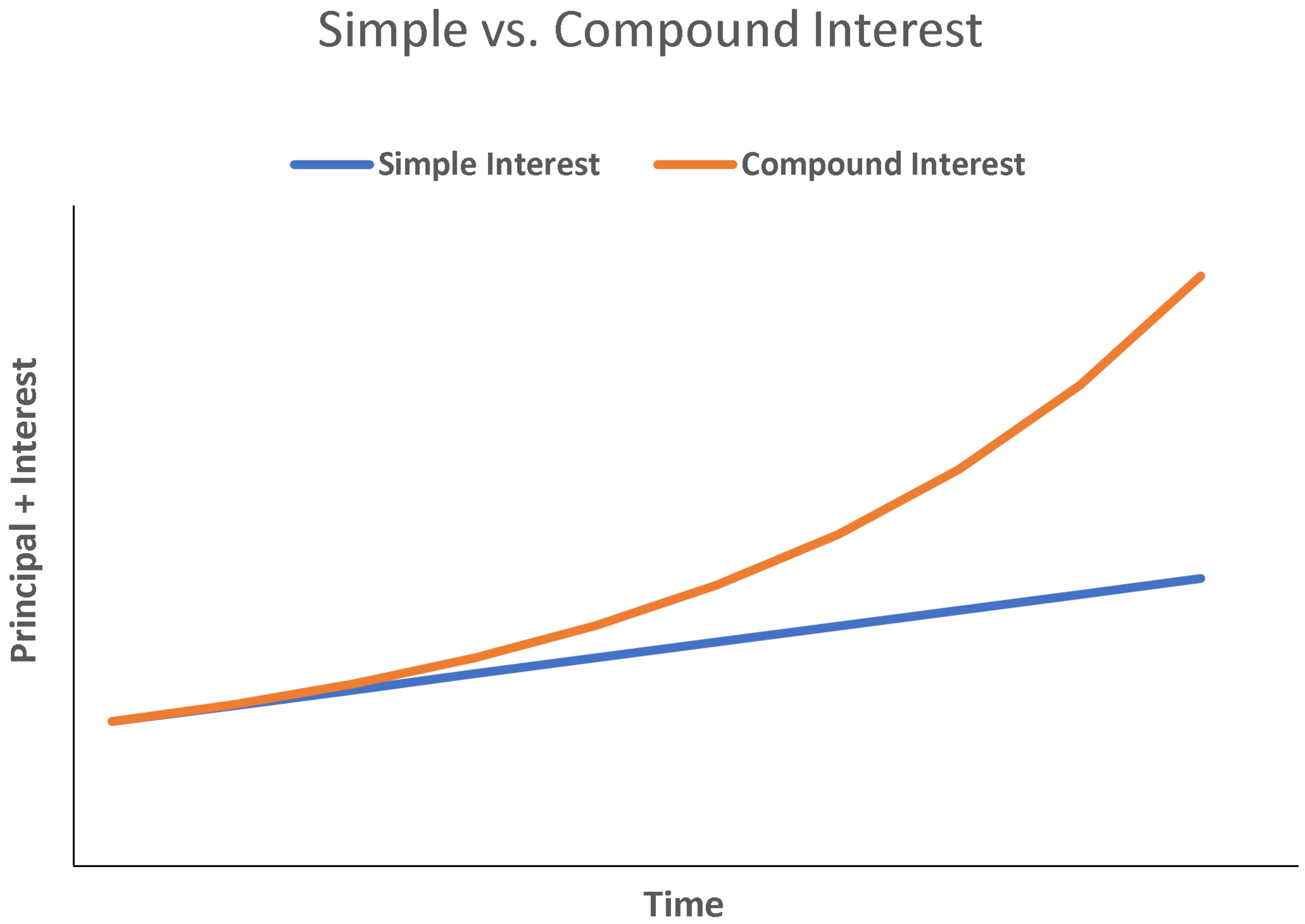
Future Value Comparison

No compounding				Compounding			
Year	Deposit	Interest	No Comp	Year	Deposit	Interest	Comp
1	100	5	105	1	100	5	105
2	100	5	110	2	105	5.25	110.25
3	100	5	115	3	110.25	5.51	115.76
4	100	5	120	4	115.76	5.79	121.55
5	100	5	125	5	121.55	6.08	127.63
6	100	5	130	6	127.63	6.38	134.01
7	100	5	135	7	134.01	6.70	140.71
8	100	5	140	8	140.71	7.04	147.75
9	100	5	145	9	147.75	7.39	155.13
10	100	5	150	10	155.13	7.76	162.89
Final	**100**	**50**	150	**Final**	**155.13**	**62.89**	162.89

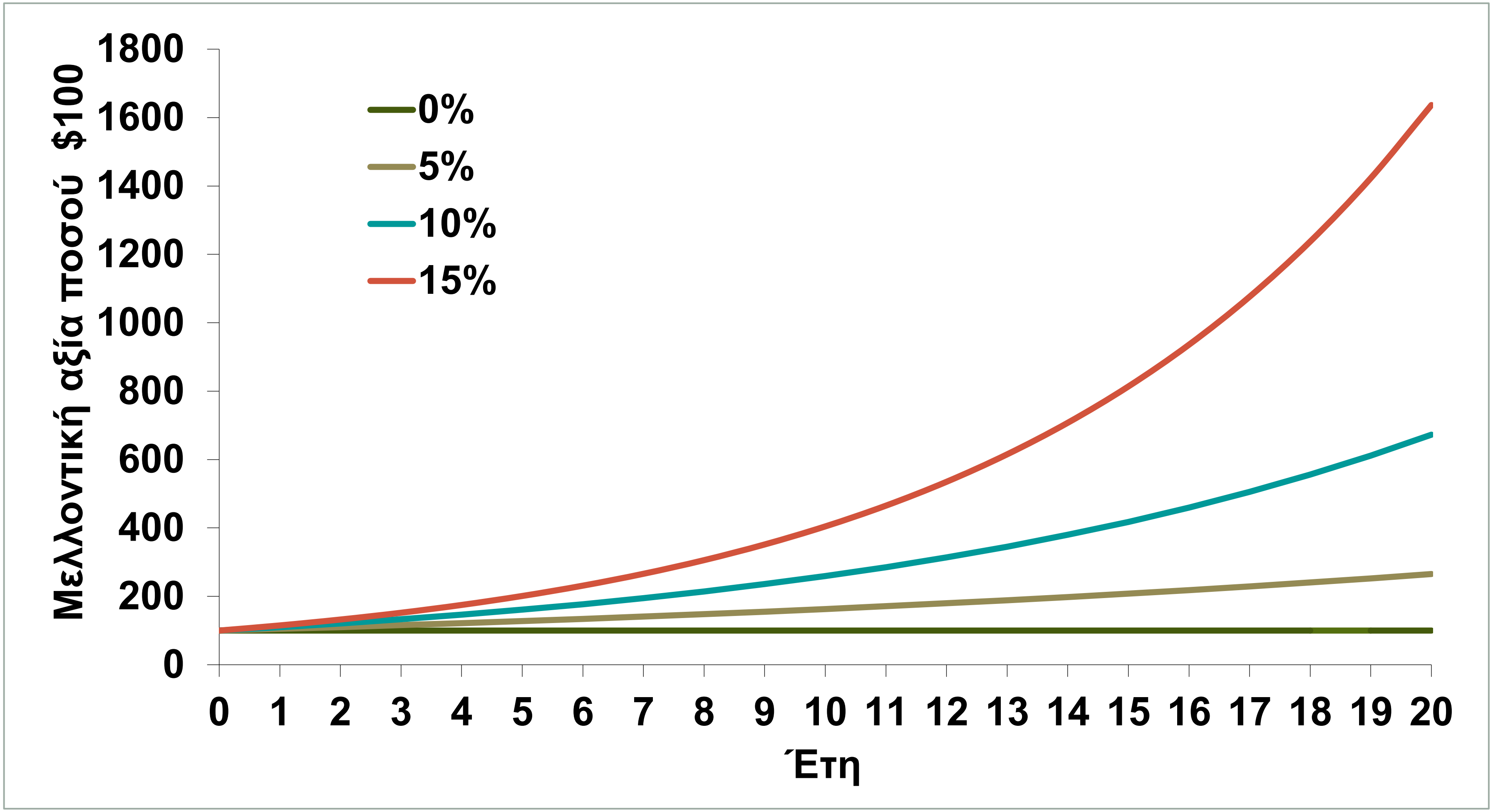
Deposit **€100**
Years **10**
Interest rate **5%**



The power of compounding
Exponential growth

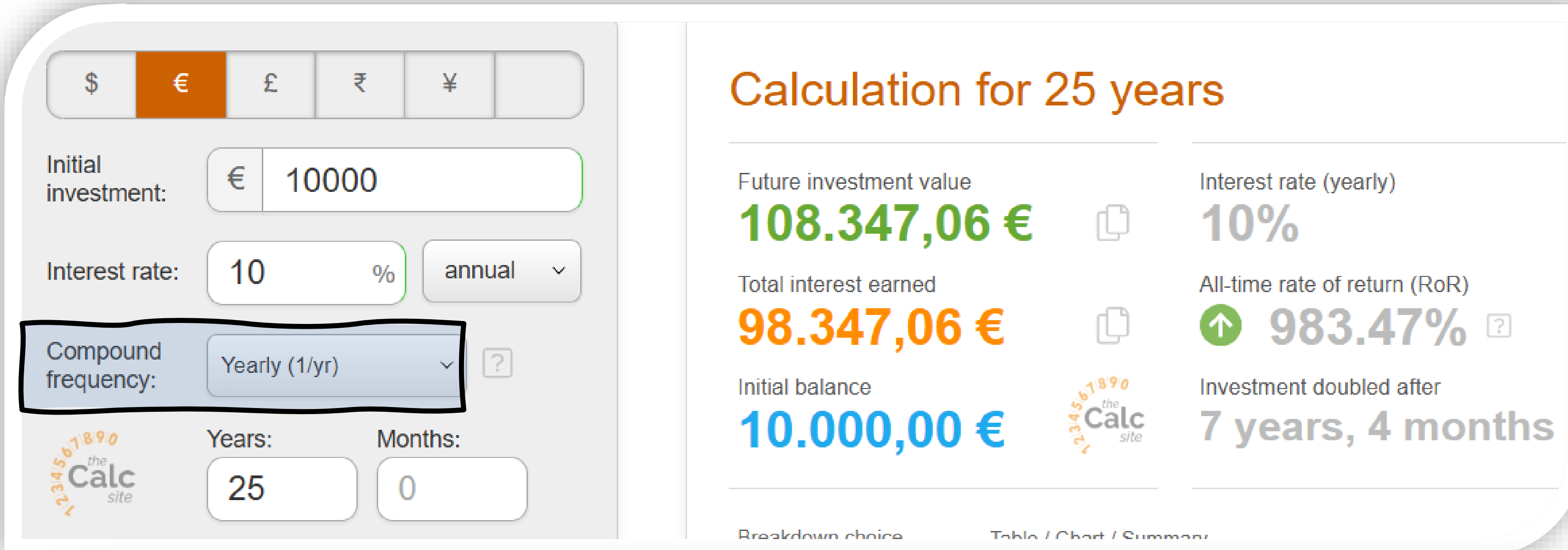


The power of the interest rate

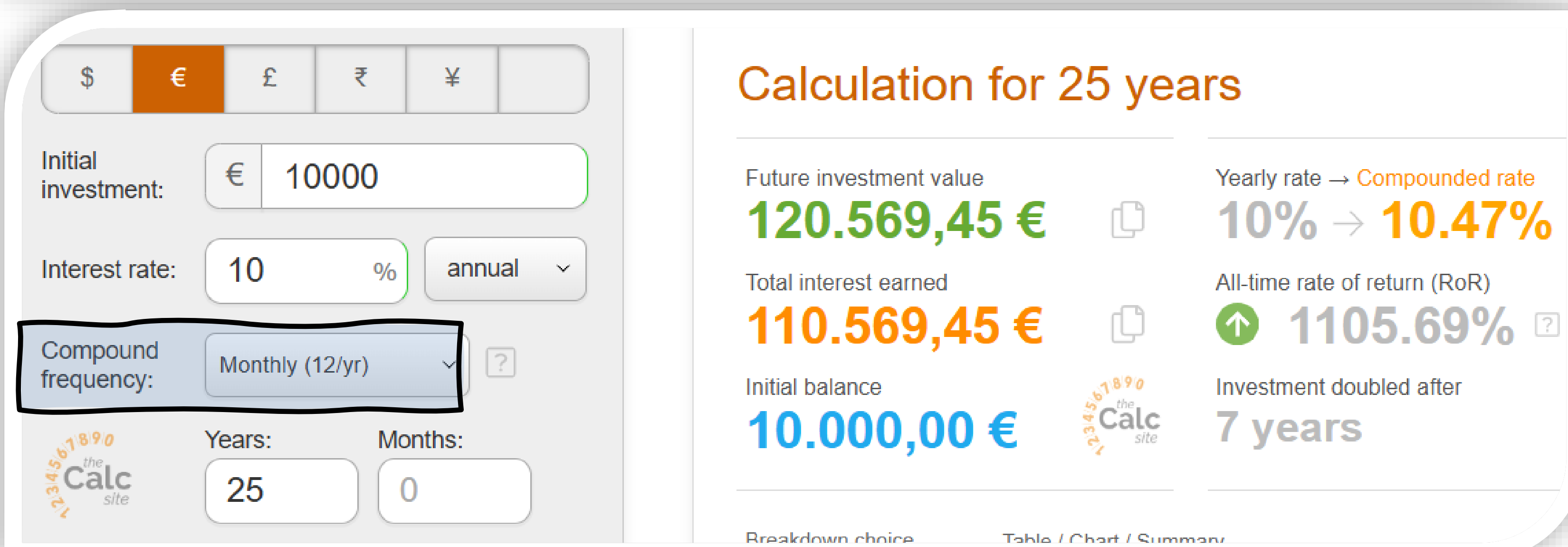


Power of Compounding Frequency

A



B



https://www.thecalculatorsite.com/finance/calculators/compoundinterestcalculator.php

Activity

Future Value → Present Value

Ένας πλούσιος θείος/α σας σας κάνει δώρο ένα από τα παρακάτω:

1. **Σήμερα €10,000**
2. **Σε 5 χρόνια €15,000**

Τι θα επιλέγατε αν τα επιτόκια είναι 8%;

Future Value → Present Value

- Παρούσα αξία **μίας μελλοντικής** **πληρωμής**

$$PV = \frac{FV}{(1+i)^t}$$

- Where:
  - FV = future cash flow
  - i = interest rate
  - t = number of periods ahead

Present Value

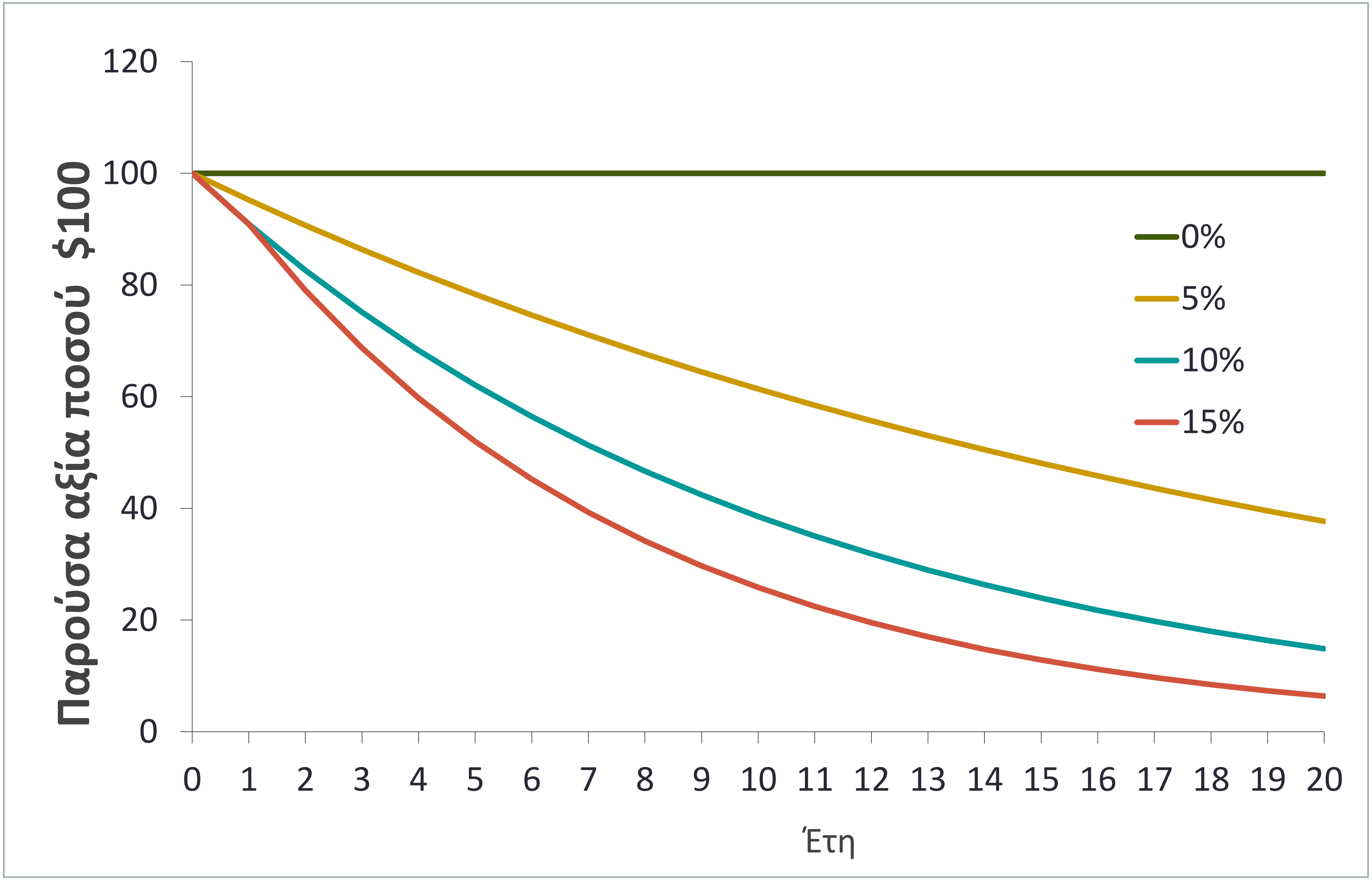
- Παρούσα αξία **πολλών** μελλοντικών **πληρωμών**

$$PV = \frac{C_1}{(1+i)^1} + \frac{C_2}{(1+i)^2} + \frac{C_3}{(1+i)^3} + \ldots + \frac{C_n}{(1+i)^n}$$

→

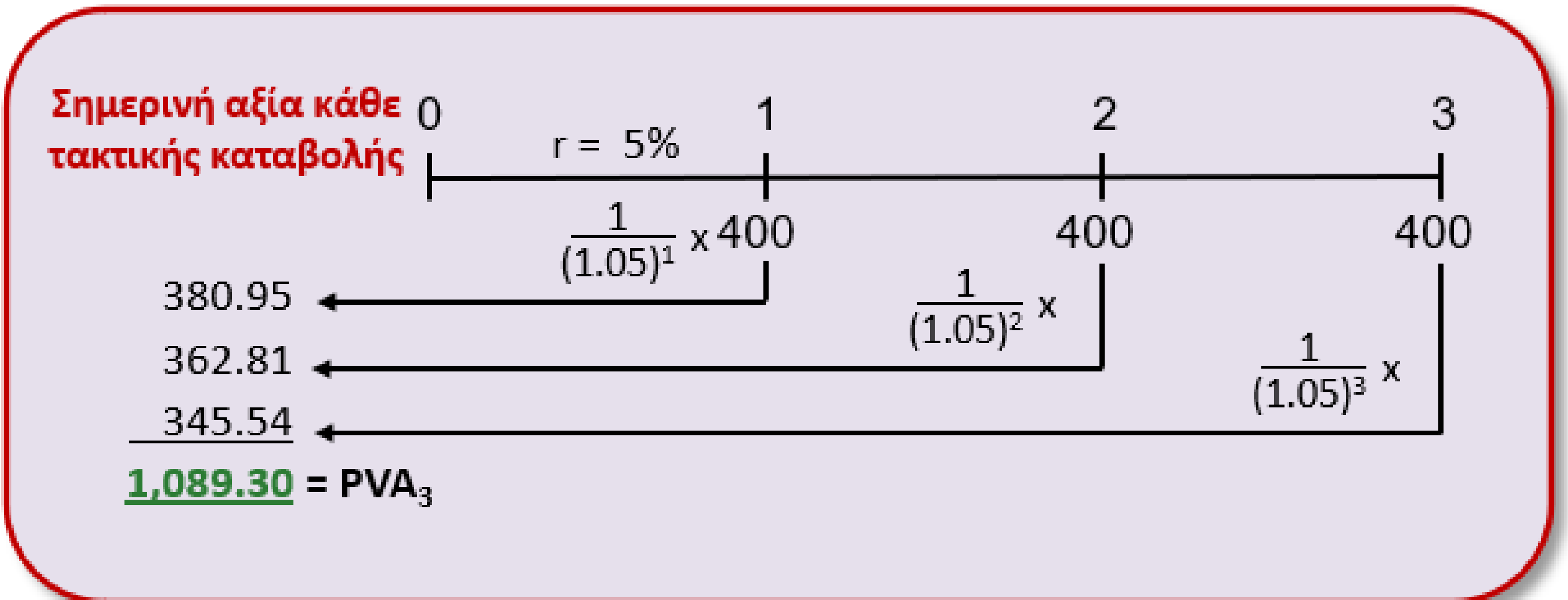
$$PV = \sum_{t=1}^{n} \frac{C_t}{(1+i)^t}$$

Ρόλος επιτοκίου – Παρούσα Αξία



More than 1 payments →Periodic Payments

What is the **PV** of a **3-year** schedule of constant payments of **$400 for r = 5%**?



Annuities
Ράντες

Annuities

What is an Annuity?
- **Fixed** periodic payment
- For a **given number** of **periods**
- A **given interest rate**
- **Compounded**

What is the value of an annuity?
- At the start – **Present Value**
- At the end – **Future Value**

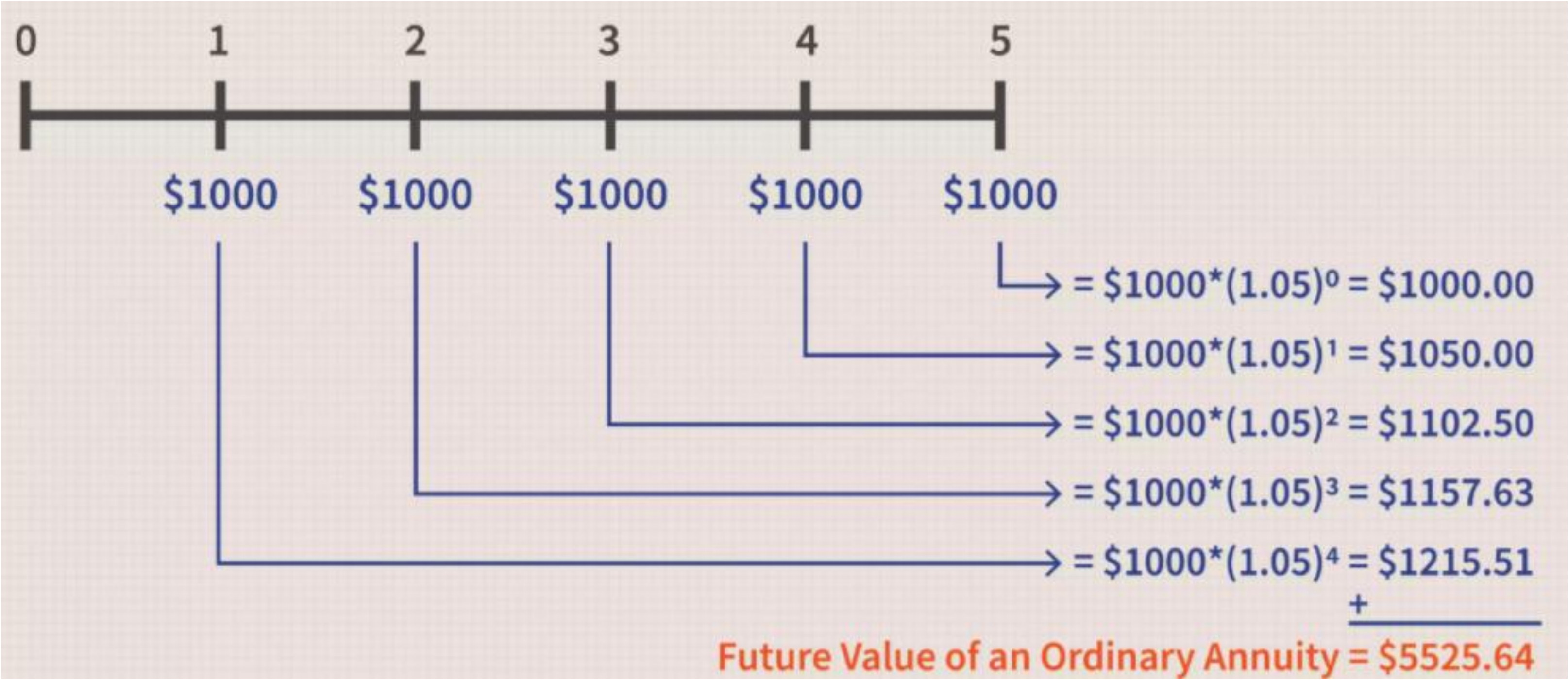
Examples of uses:
- Future Value of annuity
  Kid: $500, for 18 years, at 8%, compound – future value?
- Present Value of annuity
  Lottery: nominal win $1.8 billion over 30 years, $60 million per year, 5% - present value?

1. Ordinary Annuity

Ordinary Annuity – Future Value

Ordinary Annuity

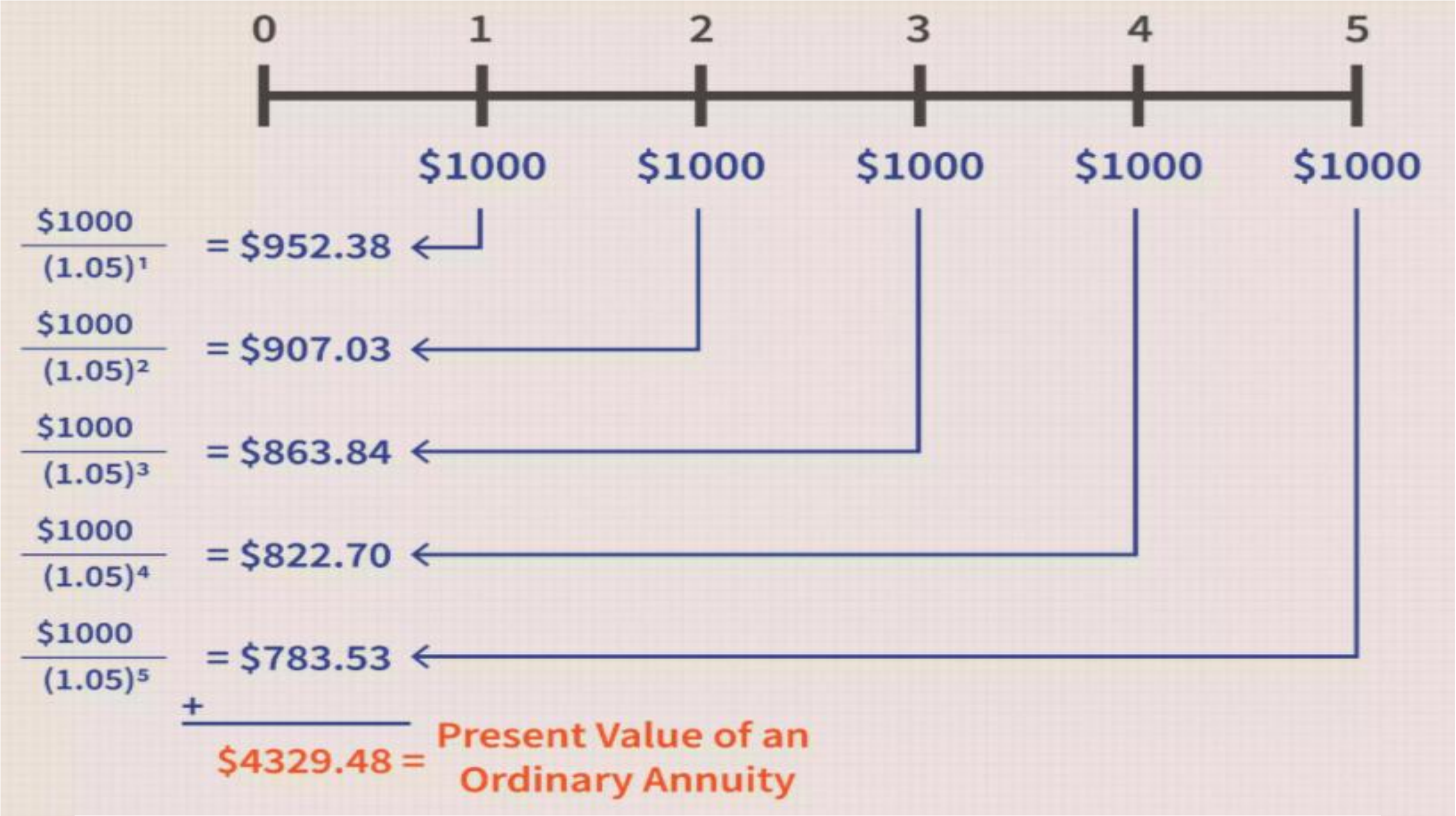
- Amount is deposited at the **end** of each period with i=5%
- In **Excel =FV(Rate,Periods,Amount,0,0)**, 0 is for end of period and is the default value.



$$FV_{\text{Ordinary Annuity}} = C \times \left[\frac{(1+i)^n - 1}{i} \right]$$

Ordinary Annuity – Present Value

Excel: PV(rate, nper, pmt, [fv], [type]) → type = 0

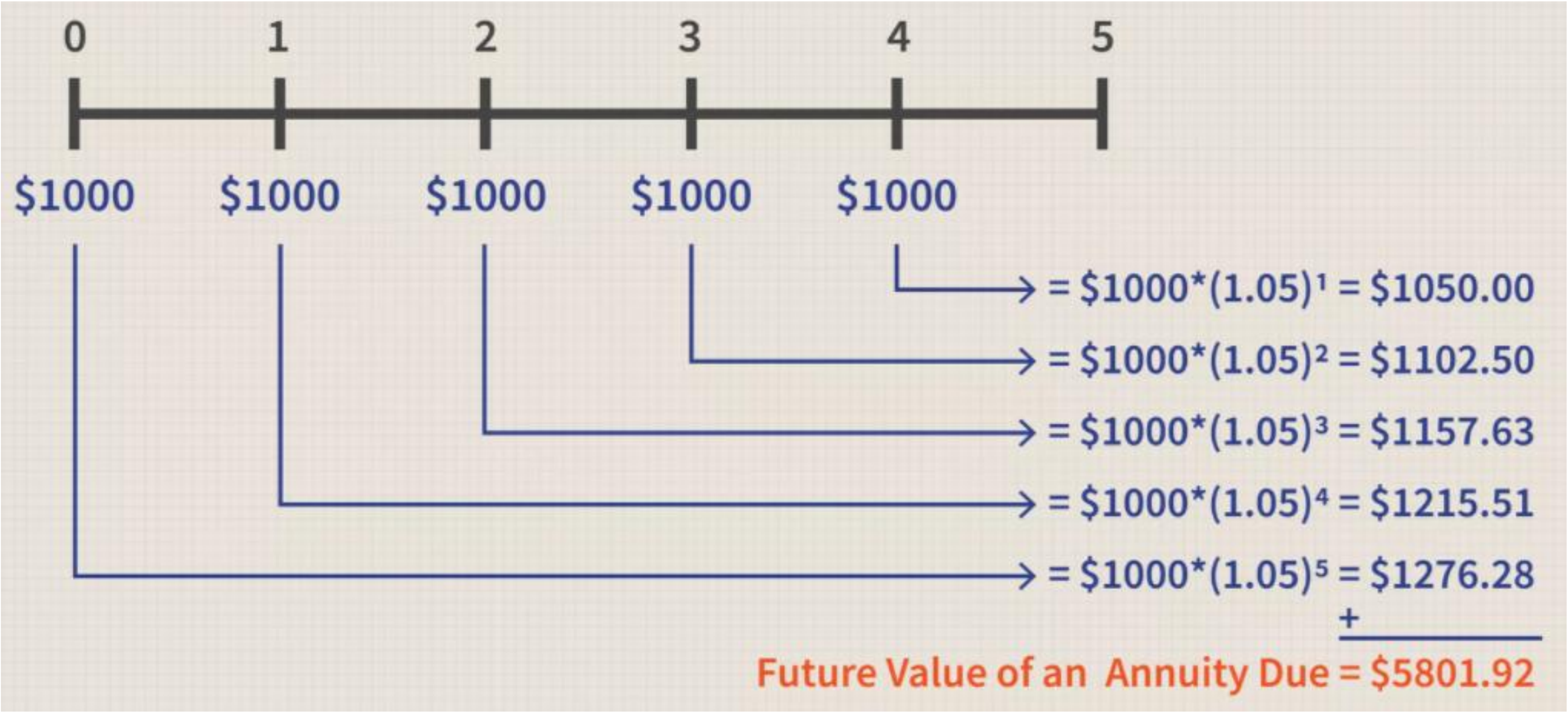


$$PV_{\text{Ordinary Annuity}} = C \times \left[\frac{1 - (1+i)^{-n}}{i} \right]$$

2. Annuity Due

Annuity Due – Future Value

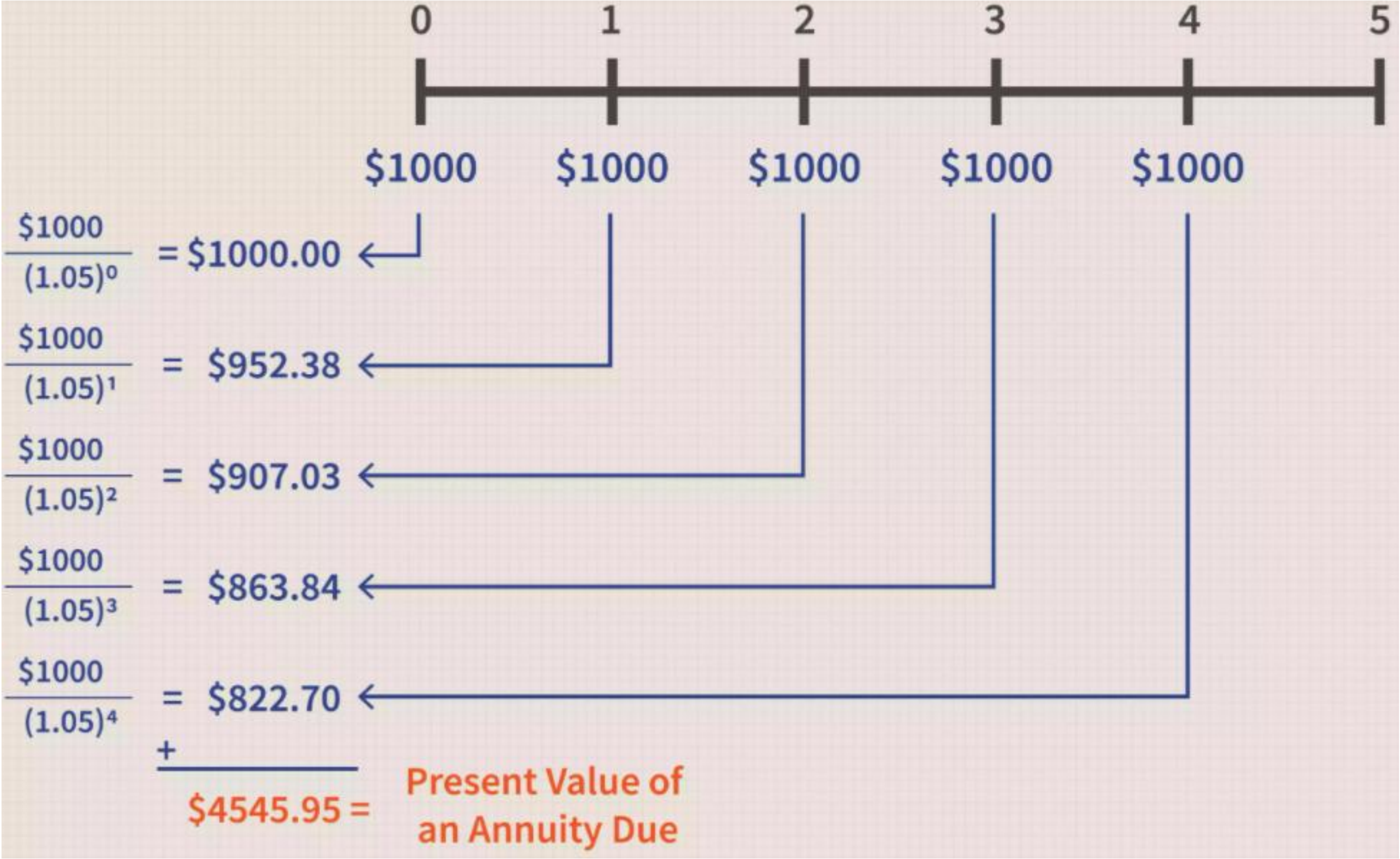
- Amount is deposited at the **start** of each period
- In **Excel = FV(Rate,Periods,Amount,0,1)**, 1 is for start of period.



$$FV_{\text{Annuity Due}} = C \times \left[\frac{(1+i)^n - 1}{i} \right] \times (1+i)$$

Annuity Due – Present Value

Excel: PV(rate, nper, pmt, [fv], [type]) → type = 1



$$\mathrm{PV}_{\text{Annuity Due}} = \mathrm{C} \times \left[\frac{1-(1+i)^{-n}}{i}\right] \times (1+i)$$

Annuity Due – Present Value

Examples using the templates

	Examples
	Future Value
	Kid
Amount	1,000
Periods	18
Interest Rate	8%
Ordinary	**37,450.24**
Due	**40,446.26**

Annuity Due – Present Value

Examples using the templates

	Examples	
	Future Value	**Present Value**
	Kid	**Lottery**
Amount	1,000	60,000,000
Periods	18	30
Interest Rate	8%	5%
Ordinary	**37,450.24**	**922,347,061.61**
Due	**40,446.26**	**968,464,414.69**

3. Perpetual Annuity

Perpetual Annuity

$$PV = \frac{C}{(1+r)^1} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} \cdots = \frac{C}{r}$$

where:

PV = present value

C = cash flow

r = discount rate

- Sequence – ακολουθία
- What is the **Future Value**?
- Grant, scholarship of €5,000 per year? How much do I pay?
- PV = 5,000/0,07 = 71,428.57

Annuity Due – Present Value

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Examples using the templates

	Examples			Perpetual	
	Future Value	Present Value			
	Kid	Lottery		Scholarship	
Amount	1,000	60,000,000		5,000	
Periods	18	30		100	
Interest Rate	8%	5%		7%	
Ordinary	**37,450.24**	**922,347,061.61**		**71,346.25**	A: Annuity
				71,428.57	B: formula
Due	**40,446.26**	**968,464,414.69**			

Financial Markets

DEPARTMENT OF ECONOMICS MBA

Stock Valuation

Stock Valuation

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Valuation of assets → pricing – how are assets priced?

- What is the **value** of **a company stock**?
- What are the cash inflows we expect from holding a stock?

$$P_0 = \frac{D_1}{(1+r)^1} + \frac{D_2}{(1+r)^2} + \ldots + \frac{D_\infty}{(1+r)^\infty} \qquad (1)$$

- But what if you want to **sell** at period 1 is this still **relevant**?
- Your value is: **PV** of **dividends** + **PV** of **selling price**

$$P_0 = \frac{D_1}{(1+r)^1} + \frac{P_1}{(1+r)^1} = \qquad (2)$$

$$\frac{P_1}{(1+r)^1} = \frac{\frac{D_2}{(1+r)^1}}{(1+r)^1} + \frac{\frac{D_3}{(1+r)^2}}{(1+r)^1} + \cdots + \frac{\frac{D_\infty}{(1+r)^\infty}}{(1+r)^1}$$

$$\frac{P_1}{(1+r)^1} = \frac{D_2}{(1+r)^2} + \frac{D_3}{(1+r)^3} + \cdots + \frac{D_\infty}{(1+r)^\infty} \qquad (3)$$

Stock Valuation

- Thus, **stock value** from (2) and (3) is: $$P_0 = \frac{D_1}{(1+r)^1} + \frac{D_2}{(1+r)^2} + \ldots + \frac{D_\infty}{(1+r)^\infty}$$

- If **constant dividend**, $D_1 = D_2 = \ldots$, then

$$P_0 = D_1 \times \sum_{t=1}^{\infty} \frac{1}{(1+r)^t} = D_1 \times \frac{1}{r} = \frac{D_1}{r} \quad \rightarrow \quad P_0 = \frac{D_1}{r}$$

Reminds you of something?

- If we have **constant dividend growth *g*:**

$$P_0 = \frac{D_0 \times (1+g)^1}{(1+r)^1} + \frac{D_0 \times (1+g)^2}{(1+r)^2} + \ldots + \frac{D_0 \times (1+g)^\infty}{(1+r)^\infty}$$

$$P_0 = D_0 \cdot \sum_{i=1}^{\infty} \left(\frac{1+g}{1+r} \right)^i = D_0 \cdot \frac{1+g}{r-g}$$

but $D_0(1+g) = D_1$

$$P_0 = \frac{D_1}{r-g}$$

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Variability of dividends →

Discounted Cash Flow Model – DCF

(the first one that we saw → more general

$$DCF = \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \frac{CF_3}{(1+r)^3} + \cdots \frac{CF_n}{(1+r)^n}$$

where:

CF_n = Cash flows in period n

Common Stock Valuation

- Cliff Inc., a drone company, paid the following per share dividends:
- What is the valuation of its stock?
- 2022 → Interest: 8%, Growth? 5%

$$P_0 = \frac{D_1}{r-g}$$

- 1.41 / (0.08-0.05) = **$47**

- **Approximate** with **variable** returns →
- Calculate average growth

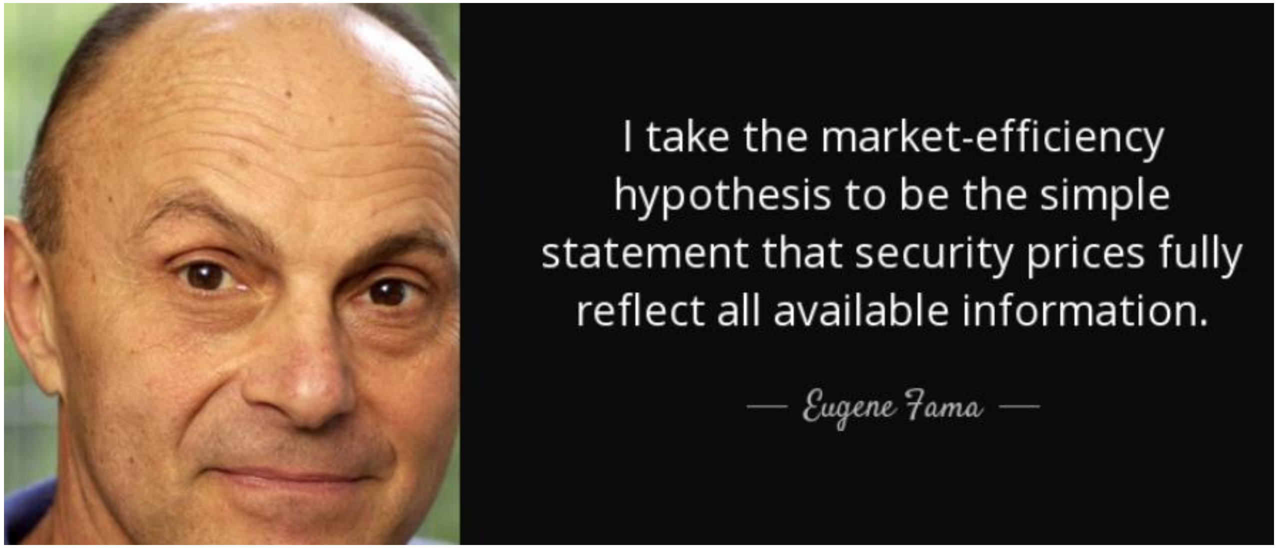
Year	Dividend	Growth
2016	$1.00	
2017	$1.05	0.05
2018	$1.10	0.05
2019	$1.16	0.05
2020	$1.22	0.05
2021	$1.28	0.05
2022	$1.34	0.05
2023	**$1.41**	

Year	Dividend	Growth
2016	$1.00	
2017	$1.07	7.00%
2018	$1.12	4.67%
2019	$1.16	3.57%
2020	$1.22	5.17%
2021	$1.28	4.92%
2022	$1.34	4.69%
2023	**$1.41**	**5.00%**

The
Efficient Market Hypothesis

Efficient Market Hypothesis

- Current stock price at t: $$P_0 = \frac{D_1}{(1+r)^1} + \frac{D_2}{(1+r)^2} + \ldots + \frac{D_\infty}{(1+r)^\infty}$$
- We **estimated** D_i based on I_t .
- We used **all available information** at t.
- When will the price of the stock **change**?
- When we have changes in I_t = **new information**, **news** on the firm.
- News are **by definition random**.
- Thus, **stock price changes** are **random**.



Efficient Market Hypothesis

LLMs and trading

Efficient Market Hypothesis