

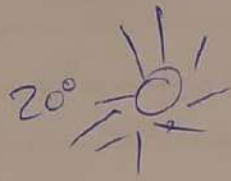
Chaos - Chaos → Deterministic

①

- Edward Lorenz MIT 1950s

"Sensitive Dependence on Initial Conditions"

18.72239
18.7223



25 km

Logistic Map

Robert May (1976)

$$X_{t+1} = r \cdot X_t (1 - X_t) \quad X_t \in [0, 1]$$

$r \rightarrow 0, 1 \rightarrow X=0$

$\rightarrow 1, 2 \rightarrow \frac{r-1}{r}$

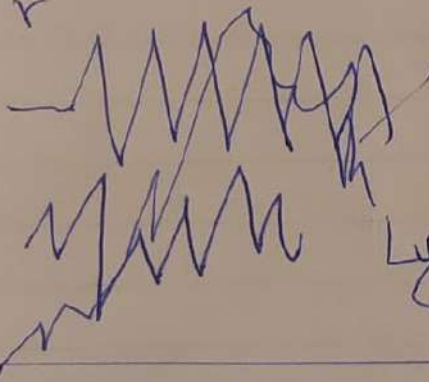
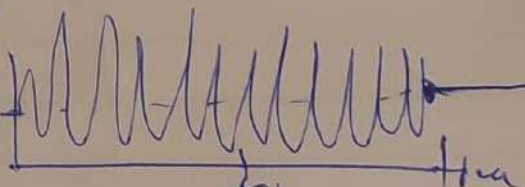
$\rightarrow 2, 3 \rightarrow \frac{r-1}{r}$

$\rightarrow 3, 1+\sqrt{6} \approx 3.45$

$X_0 = 0.46$

$r = 4.0000000123$

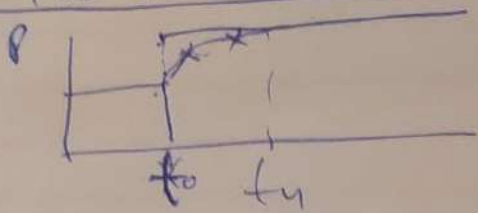
~~Chaos~~



Bifurcation
Lyapunov Exponent
 $\lambda > 0$

The Efficient Market Hypothesis (EMH) (2)

Fama-French (2012)

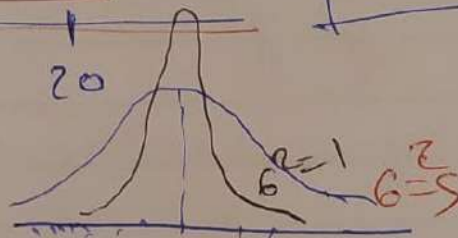
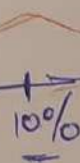
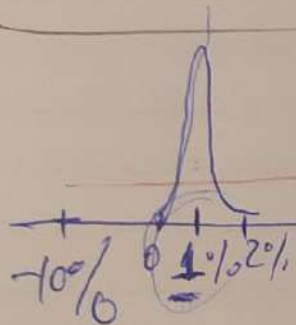
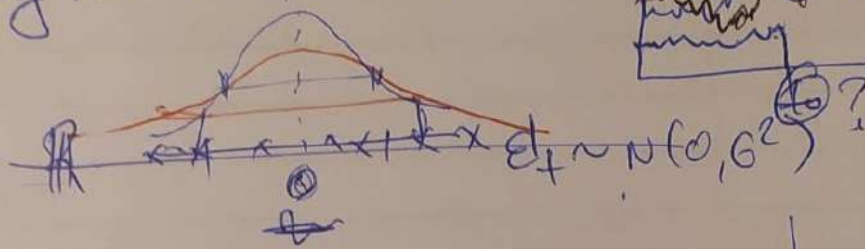


$$Z_{t+1} = X_{t+1} - E_t(X_{t+1} / I_t) > 0$$

EMH: $E_t(Z_{t+1} / I_t) = \phi$, $Cov(Z_t, Z_{t+k}) = 0$
 $\forall k$

(+5%) +8% +15%
 -8% -15%

Consistently beat the market



FX: USD/EUR \rightarrow RW

RWD?

$E_t(R)$



$$X_{t+1} = \mu + X_t + \epsilon_{t+1}$$

~~$\sigma^2 = 1$~~
 $\sigma^2 = 5$

