EMPIRICAL TESTS OF PPP

UNIT ROOTS, COINTEGRATION

Empi Tests

1. Bivariate OLS: $s_t = a + p_t - p_t^* \Rightarrow$

$s_t = a + \beta(p_t - p_t^*) + \varepsilon_t$, test: $H_0: \beta = 1$

2. Multivariate OLS: $s_t = a + \beta_1 p_t + \beta_2 p_t^* + \varepsilon_t$, $H_0: \beta_1 = 1$, $\beta_2 = -1$

Empirical Problem in estimation: Non-stationarity, Unit Roots, $I(1)$

Definition: AR(1) $y_t = a_1 y_{t-1} + \varepsilon_t$ is $I(1)$ if $a_1 = 1$

$\Rightarrow y_t = y_{t-1} + \varepsilon_t$, $E(\varepsilon) = 0$, $E^2(\varepsilon) = \sigma_\varepsilon^2$, then $y_1 = y_0 + \varepsilon_1 \Rightarrow$

$y_2 = y_1 + \varepsilon_2 = y_0 + \varepsilon_1 + \varepsilon_2 \Rightarrow y_3 = y_0 + \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \Rightarrow$

$y_t = y_0 + \sum_{j=1}^{t} \varepsilon_j$ repeated substitution (1)

Mean: $Ey_t = Ey_0 + \cancel{E\varepsilon_0} + \dots + \cancel{E\varepsilon_t} = y_0$ Constant (1)

Variance: $Var(y_t) = Var(y_0) + Var(\varepsilon_0) + \dots + Var(\varepsilon_t) = t \cdot \sigma_\varepsilon^2$

Non-stationary

(1) + (2) $\Rightarrow$ Will cross the mean infrequently
Long positive and negative fluctuations

In general $\forall a$: an $y_t = a y_{t-1} + \varepsilon_t \Rightarrow y_1 = a \cdot y_0 +$

$\Rightarrow y_2 = a y_1 + \varepsilon_2 = a(a y_0 + \varepsilon_1) + \varepsilon_2 = a^2 y_0 + a\varepsilon_1 + \varepsilon_2$

$\Rightarrow y_t = a^t \cdot y_0 + \varepsilon_t + a \cdot \varepsilon_{t-1} + a^2 \varepsilon_{t-2} + \dots + a^t \cdot \varepsilon_0$

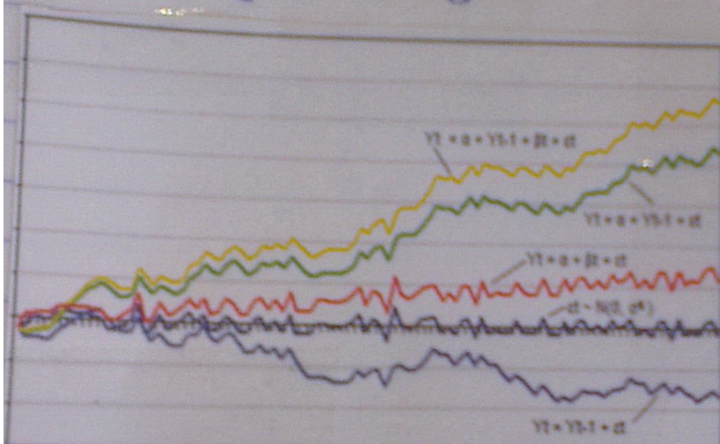
3 Cases:

1. $a < 1 \Rightarrow a^t \to 0$ as $t \to \infty$: shocks gradually die
2. $a = 1 \Rightarrow a^t = 1 \ \forall t$: persistent shocks get (1)
3. $a > 1 \Rightarrow a^t > a^{t-1} > a^{t-2} > \ldots > 1$: shocks are more influential with time (t).

Problems with I(1) variables

- Spurious Regression (Granger & Newbold, 1976):

High R^2, high t-ratios but no economic meaning.



- Coeff. biased downwards
- distr. of t-statistics are not t or close to normal.

Stationary

- Mean reverting
- Finite σ^2
- Autocov ↓ as lags ↑

Non-Stationary

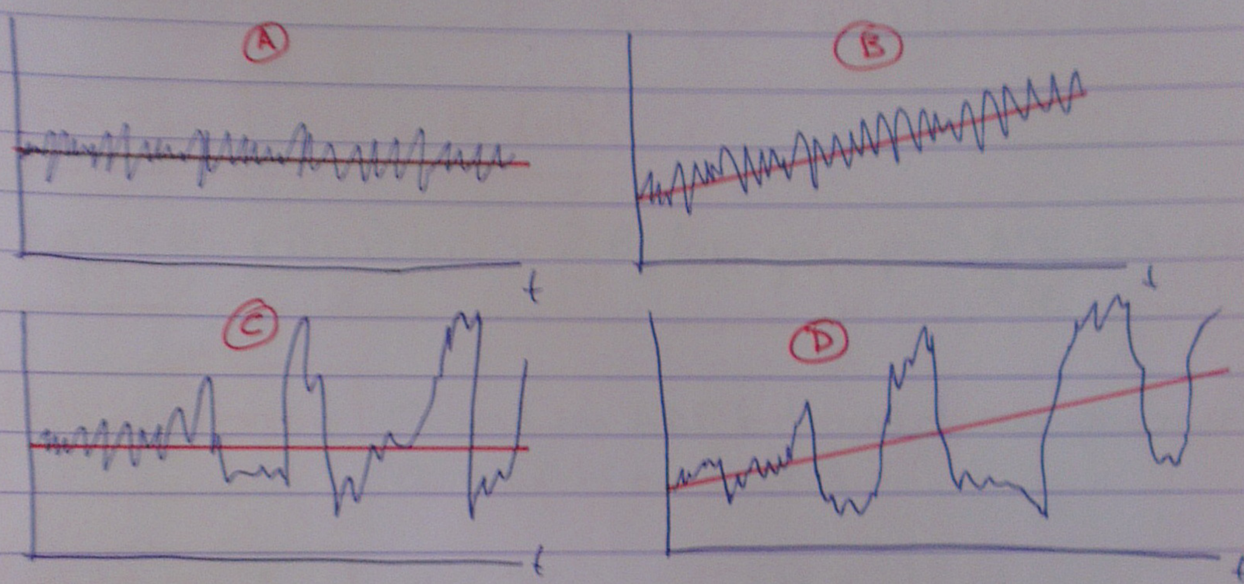
- Variance explodes : $t \cdot \sigma_\varepsilon^2$
- Stochastic trend
- Time trend (deterministic)
- Breakpoints: ,
  + Known break Chow test
  + Unknown " Quandt likelihood (finds largest Chow F-stat trimming 15% start/end obs for reliability

Trend Stationary Process

$y_t = a + \mu \cdot t + \varepsilon_t \ , \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$

$E(y_t) = a + \mu \cdot t$: varies with time

$Var(y_t) = V(a) + V(\mu t) + V(\varepsilon_t) = \sigma_\varepsilon^2$: constant



- A: White noise — $y_t = a + \varepsilon_t$
- B: Deterministic-Time trend — $y_t = a + \beta \cdot t + \varepsilon_t$
- C: Random walk — $y_t = y_{t-1} + \varepsilon_t$
- D: " " with drift — $y_t = a + y_{t-1} + \varepsilon_t$
- " " " " + time trend — $y_t = a + y_{t-1} + \beta \cdot t + \varepsilon_t$

④

TESTING FOR NON-STATIONARITY

① DF-Test — Random walk

$y_t = a y_{t-1} + \varepsilon_t$ for NS $H_0: a = 1$, $I(1)$; $H_1: a < 1$, $I(0)$] But under H_0 t is not distr. asy. as $N(\mu,\sigma)$

$y_t - y_{t-1} = a y_{t-1} - y_{t-1} + \varepsilon_t \Rightarrow$

$$\boxed{\Delta y_t = \underbrace{(a-1)}_{\gamma} y_{t-1} + \varepsilon_t}$$

$H_0: \gamma = 0, a = 1$; $H_1: \gamma < 0$ $a < 1$] t-test under Dickey-Fuller distr.

$+a_0$ drift

② ADF-Test — RW with drift

$$\boxed{\Delta y_t = \underbrace{a_0}_{\text{drift}} + \underbrace{(a-1)}_{\gamma} y_{t-1} + \sum_{j=1}^{p} \beta_j \Delta y_{t-j} + \varepsilon_t}$$

$p =$ AIC $+2$, $p < \sqrt{N}$
BIC

③ Phillips-Perron

Based on ADF but makes non-parametric correction on t-stat.

④ KPSS Test

$H_0: I(0)$ LM Test
$H_1: I(1)$

A: If y_t is $I(0)$ ok for regressions
B: " " $I(1)$ use first differences $\Delta y_t = y_t - y_{t-1}$

Test Δy_t → $I(0) \rightarrow y_t$ is $I(1)$
Test Δy_t → $I(1) \rightarrow y_t$ is $I(2)$

- In multivariate estimation if s_t $I(1)$, p_t^* $I(1)$ then we can use OLS if ε_t is $I(0)$