ZDM – Mathematics Education
https://doi.org/10.1007/s11858-024-01601-y

ORIGINAL PAPER

Socio-political features shaping the mathematics teaching and professional identities of muslim minority teachers in Western Thrace

C. Sakonidis[1] · A. Klothou[1]

Accepted: 3 June 2024

Abstract
The importance of mathematics education in preparing students to become successful citizens raises continuous challenges for schools and educational systems, especially in minoritized settings. Research has identified practices and structures in the social and educational context that do not equally support all students' learning and all teachers' teaching and learning to teach. The study presented here employs a cultural-historical activity theory perspective to identify micro and macro practices and structures that challenge Muslim teachers' mathematics teaching and their professional development in Western Thrace, Greece. The data exploited in the study, which is part of a larger project on Muslim students' learning, were generated through interviews with in-service minority teachers. The results highlight micro and macro socio-political formations that interact to activate minoritization processes. These shape the practice of teaching mathematics as well as individual and collective identities connected to learning and teaching mathematics.

Keywords Mathematics teaching activity · Minority teachers · Teacher identity formation · Socio-political features

1 Introduction

Despite significant advances in the field of mathematics education, its school practice remains a microworld of schooling that still classifies students (Domina et al., 2017). Research has identified practices and structures in the social and educational contexts that systematically privilege some students and disenfranchise others. Chen and Horn (2022) noticed that these practices and structures, while enacted on a micro level, produce inequality at a macro level. Planas and Valero (2016) argued that these two levels are complementary and connect dialectically local and systemic forces in mathematics education settings. Although power is a critical feature at both levels, the micro level of the classroom discourse interacts with the macro level of broader social and political structures.

The socio-political understanding of mathematics education brought to the fore the study of exclusion of particular groups of students from access to and full participation in school mathematics. The relevant research adopted predominately sociological and political lenses to examine marginality in mathematics education (Planas & Valero, 2016). Sociological approaches inquire into structures and societal rules that nurture marginality. In contrast, political perspectives examine distribution of resources and power and identify hegemonic ideologies as mechanisms for maintaining these distributions (e.g., Battey & Leyva, 2016). These two perspectives are not mutually exclusive. Understanding and addressing marginalization in mathematics education requires attending critically to structures and individuals to "document the implications of far-away policies [macro level] and up-close decisions by, for, and against the interests of privileged and marginalized youth" (Weis & Fine, 2012, p. 196).

A considerable body of mathematics education research has addressed issues of marginality experienced by some groups of students in their learning and being in the mathematics classroom (Chen & Horn, 2022). This contrasts with the scarce body of mathematics education research on issues of marginality experienced by some groups of teachers in their professional learning (Gaudreault & Woods, 2012).

This paper sets out to report on micro and macro practices and structures that constrain Muslim teachers' mathematics

✉ C. Sakonidis
xsakonid@eled.duth.gr

A. Klothou
aklothou@eled.duth.gr

[1] Democritus University of Thrace, Thrace, Greece

Published online: 10 June 2024

teaching and their respective professional development (PD). We use the term "minoritization" to refer to processes that enact these practices and structures in ways that lead to social exclusion. We believe that this term makes visible the power dynamics and the systemic structures via which communities like the one under consideration are underserved and hence marginalized within the larger society.

Aiming to attend critically to both individuals and structures, we employ the theoretical lens of cultural-historical activity theory (CHAT) and the notions of boundary crossing and professional identity to study the minoritization processes that the Muslim minority teachers experience across mathematics learning for school teaching and teaching sites.

2 Theoretical framing

We first briefly discuss CHAT and then the two focal concepts employed for the study purposes, boundary crossing and professional identity.

Cultural-historical activity theory Vygotsky's original model to study human activity at the individual level (subject, object, and tools) has been further developed over time. Leont'ev (1974) identified three hierarchical levels of activity (operations, actions, and activity) and emphasized the importance of the object of an activity related to participants' goals and motives. Engeström (1987) introduced additional elements (mediators) of the human activity (rules, community, and division of labor). The relationships between these are constantly mediated.

Community is a key element in Engeström's (2001) third generation framework of CHAT concerned with interacting activity systems (Fig. 1). Here, the subject is seen to achieve the object of the activity system through the mediation of an instrument or artefact. Besides the mediation of tools (textbooks, digital tools, or mathematical symbols), rules, community, and division of labor are also important mediators in an activity system. Objects in the two interacting activity systems move from an un-reflected and situationally given goal, to a collectively meaningful object, and to a potentially shared or jointly constructed object (Engeström, 2001). This process is characterized by contradictions seen as historically accumulating structural tensions within and between activity systems. A new element adopted by an activity system often leads to a contradiction. This generates disturbances and conflicts and fuels attempts to change the activity.

The minority teachers who are the subjects of the present study engage in professional activities situated within two activity systems: teaching mathematics to serve the official purposes of the Greek State and teaching mathematics to serve the needs of the minority children. Within the CHAT perspective, the activity system and its elements in the two distinct contexts are:

- the activity under consideration is the minority teachers' (the subjects) teaching of mathematics (the object);
- the official documents, the educational materials and resources, and the language of teaching mathematics can be seen as the cultural-historical influences on the teachers' pedagogy development (mediating artefacts);

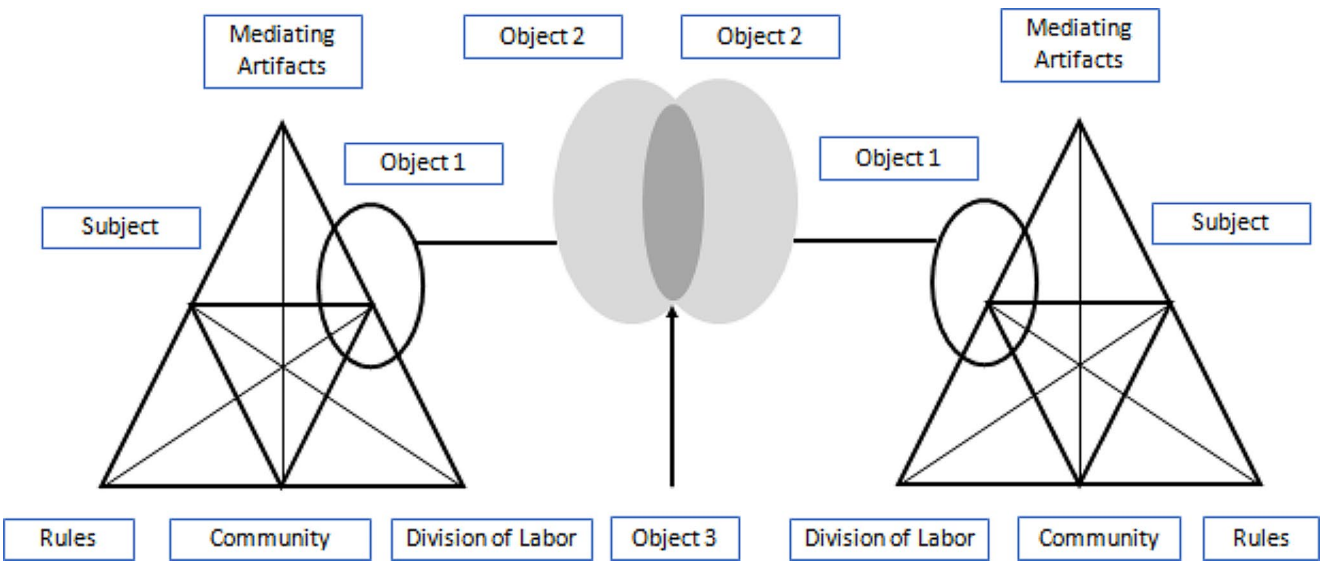

Fig. 1 Interacting activity systems (Engeström, 2001)

- the norms and the conventions that regulate the teaching and learning of mathematics, and explicit school practices and policies provide the rules of the activity;
- minority teachers, students and their parents, and other stakeholders (e.g., local education authority members), form the community, while the way mathematics teaching is managed in schools explicates the division of labor.

Boundary crossing When activity systems' participants are collaborating, they bring their communities' practices and discourses together and enable continuity across the corresponding activity systems at their boundaries defined as "socio-cultural differences leading to discontinuity in action or interaction" (Akkerman & Bakker, 2011, p. 133). These are dynamic constructions denoting co-location of practices and co-existence of competing discourses. Efforts to restore continuity in action or interaction across practices trigger dialogical engagement and collective reflection, compelling people to reconsider their assumptions and look beyond what is known and familiar.

Boundaries are "spaces" of potential learning. Through collaboration at boundaries between activity systems, new and hybridized ideas and practices emerge, where mutual understanding of shared tasks and problems develops (Edwards & Fowler, 2007). Described as boundary crossing (Engeström et al., 1995), this process involves moving into unfamiliar territories and requires cognitive retooling. New elements are introduced from one community of practice to another via boundary crossers or "brokers": people who are simultaneously members of multiple communities or in transition from one to another. These elements, which are referred to as boundary objects (Star, 1989), often lead to the creation of new tools and artifacts. Thus, boundary space is a site for new understandings, identity development, change of practices, and institutional development.

The minority teachers of the study interact daily with their majority colleagues in the minority school. This working condition inevitably creates potential spaces of mutual learning at the boundaries of the activity systems in which they respectively operate. In these spaces, educational artifacts such as textbooks and syllabi are imported from one activity system into the other and become the object of inquiry (boundary crossing and boundary objects), often by participants who operate in both systems, such as minority teachers with university and professional studies beyond those of their basic degree (brokers).

Professional identity Becoming a teacher is a matter of developing an identity that is legitimized through participation in the respective community of practice. Within a CHAT perspective, teachers' professional community of practice is one of the communities they are part of (Quinn & Hobbs, 2024). Studying this community with CHAT lenses offers a fundamental view of it in cultural and historical contexts.

CHAT views identity as an ongoing process of development through which the subjects of the activity are continually reshaped in relation to the social practices and shared cultural experiences. This process guides subjects in developing a sense of their role within the activity system (Cross, 2020). Similarly, Wenger (1998) argued that identity is a process of "constant becoming", which defines who we are by "the ways we participate and reify ourselves; our community membership; our learning trajectories…; reconciling our membership in a number of communities into one identity; and negotiating local ways of belonging with broader, more global discourse communities" (p. 149).

Learning to be a teacher within a community of practice requires alignment with the norms of the practice (Wenger, 1998). If alignment is critical, then inquiry develops as one of the norms of the practice and individual identity develops through reflective inquiry (Jaworski, 2006). To bring about development and growth, such reflection needs to be carried out collaboratively (Chapman, 2009). Collaborative environments constitute complex settings characterized by tensions, contradictions, and constraints, which create learning opportunities for all participants.

For purposes of the present study, we view identity as described above but specified for mathematics teaching as suggested by Ntow and Adler (2019): "a relationship with specific practices, where the process of identification involves an interaction between the person and the set of resources made available in the learning community and contexts of practice" (p. 420–421). These researchers distinguish resources into ideational (ideas about oneself, one's place in the practice and the world, and what is valued), material (ways in which one's environment, artefacts, and organization support the connection to the practice), and relational (relationship with others that can increase connection to the practice).

The professional identities of the minority teachers involved in the study reported here are seen as dynamic processes of learning to teach mathematics through reconciling their membership in the Muslim teachers' community and in the Greek teachers' community. These processes are determined by the interaction ensured between these teachers and the resources of each community made available to them.

3 The study

The research reported here is part of a larger project aiming to improve the quality of the education offered in the Muslim minority schools of Western Thrace. Below, we describe first the overall research site and then the data generation processes and means.

3.1 The research site

Western Thrace in the northeast part of Greece is home to the only officially recognized minority in the country, a politically significant population of about 120,000 Muslims. The community comprises individuals of Turkish origin, Roma, and Pomaks who live in the region with a Christian Orthodox majority. Most of the minority population belongs to the lower social strata. About half the population is employed in the agricultural sector (the national average is only 19%). Almost 80% of the minority members are only primary school graduates. Only 2.6% of men and 0.2% of women hold a university degree (Askouni, 2006).

The education of the Muslim minority is framed by the Treaty of Lausanne and two educational bilateral agreements signed in 1951 and in 1968 between Greece and Turkey. The treaty provides for a bilingual minority education system, where students are taught in both Turkish and Greek. The use of the Turkish language for all minority children ignores the existence of other native languages in Western Thrace (e.g., Pomak) (Boussiakou, 2007).

The Lausanne Treaty referred only to primary schools, so pre-school education is provided at monolingual Greek mainstream schools. Most minority primary schools are in remote villages and their student numbers are declining. The student/teacher ratio varies widely in minority schools, with the standard type of primary school with six separate classrooms (one for each grade) being the exception. In most of these schools, students are grouped into two classes where different age groups are taught together (Dragonas & Fragoudaki, 2006).

Half of the courses in the primary minority schools are taught in Turkish (e.g., Turkish language, mathematics, and science) and the other half in Greek (e.g., Greek language and history). For secondary education, minority children can choose between the only two bilingual minority schools, the two religious schools (*medresse*), monolingual Greek schools, or schools in Turkey. There is no provision for first language instruction in the monolingual Greek secondary or primary school. This, given that Muslim students are formally exposed to the Greek language for the first time in primary school, suggests limited opportunities to acquire proficiency in the Greek language, which is a barrier to graduating from high school and continuing to higher education. Almost no minority students enrolled in a Greek university prior to the positive discrimination measure of 1996 (see below). This situation further hinders the integration of the members of the Muslim minority into Greek society (Dragonas & Fragoudaki, 2006).

The Greek part of the curriculum is taught exclusively by Greek teachers trained like all teachers in the country, with no special knowledge in matters of minority education. The courses in Turkish are taught by graduates of the Special Pedagogical Academy of Thessaloniki (EPATH) established in 1969. EPATH students were mainly graduates of medresse and recruited from poor Pomak villages. EPATH graduates are criticized as being not properly trained to teach the minority children, among other things, because of their poor command of Turkish. The EPATH teachers are Greek civil servants (public employees) who are expected to serve the educational needs of the Muslim children as required by the Greek authorities but also meet the Muslim minority community's expectations. Thus, a kind of institutional gap can be identified between the Christian and Muslim teachers as they each receive education in different institutions (Dragonas & Fragoudaki, 2006).

In the last 30 years, the Greek State has taken some important steps to improve the quality of minority education. A key one was introducing a 0.5% quota for minority students to enter Greek universities. Thus, minority students had to compete only with each other for the available places. In addition, with financial support from the EU, a specific educational project, "Program of the Education of Muslim Children" (PEM), was implemented between 1997 and 2017. It aimed to enhance the quality of educational outcomes for students attending minority schools by improving their Greek language skills. The program included development of educational materials and providing teacher training to support the adopted pedagogical approach. Another important development was EPATH's closure in 2010. Now, those who wish to teach the minority curriculum must enroll in the specialized program for minority education at the Primary Education Department of an urban university (Aristotle University of Thessaloniki). Finally, in 2015 a new Parliamentary Act introduced for the first time a voluntary PD program to be implemented by the Primary Education Department of the local Democritus University of Thrace. It aimed at upgrading EPATH studies via the successful completion of 1-to-2-year courses designed to fulfill teachers' professional needs.

3.2 Research questions

We seek to identify socio-political structures and practices woven into the Muslim community, its schools and classrooms, ultimately shaping individual Muslim teachers'

mathematics teaching and their respective professional identities.

Specifically, our study addresses the following research questions:

RQ1: How do socio-political features shape the mathematics teaching activity of the Muslim minority teachers in Western Thrace?

RQ2: How do socio-political features shape the development of professional identities regarding mathematics and mathematics teaching by the Muslim minority teachers?

Using the CHAT perspective allows us to identify micro and macro-practices and structures of socio-political character in the two activity systems where the Muslim minority teachers operate. These systems interact, leading to contradictions and tensions that shape the mathematics teaching of these teachers (RQ1) and to potential spaces of mutual learning at the boundaries of the two activity systems. This interaction also makes diverse learning resources available to the Muslim minority teachers, which shapes their own learning to teach mathematics, and thus their professional identities (RQ2).

3.3 Research methodology

We employed a multiple case methodology involving two research sites. The cases of the study are Muslim minority teachers participating in two programs, the Program of Education of Muslim children (PEM), and the program of Upgrading Muslim minority Teachers' EPATH studies (UPEPATH).

The PEM interventions (site A) related to mathematics aimed at creating a learning environment for minority compulsory education students that encourages active learning, participatory work, and diversity of practices. For this purpose, relevant educational material was developed. The mathematics lessons were voluntary and taught outside normal school hours by the teachers of the morning classes and teachers employed by the project as needed. Some 700 students and almost 50 teachers were involved in these lessons. Mathematics lessons were also offered in PEM's support centers (KESPEM) outside school hours for primary and secondary minority students who wished to attend. In addition, PD activities were organized in these centers and attended by all teachers involved in the mathematics education component of PEM.

The UPEPATH program of study (site B) consists of three main components: (a) attendance of 11 to 22 courses, depending on the years of in-service experience, which included mathematics and mathematics education courses, (b) an undergraduate research project, and (c) practicum for those with no previous educational experience. Almost 200 EPATH graduates, both in-service and pre-service minority primary teachers (more than half of the population), have enrolled in the upgrade program since its first academic year of establishment (2015–2016). About two-thirds of them have already completed the course.

The data utilized were generated via two semi-structured interviews, one with selected participants in site A incorporating ethnographic elements and one with selected participants in site B of a narrative type. The site A interviews aimed to provide data for both research questions triggered by PEM intervention and hence, from within a teaching practice perspective. These interviews, flexibly structured and conducted informally, were organized along three axes. Axis 1 focused on critical features of the participants' personal pathway to teaching (PPT), axis 2 regarded issues of *becoming* a teacher of mathematics and of *being* a teacher of mathematics, while axis 3 included questions on PD.

The narrative type interviews (site B) sought to generate data again for both research questions but this time from a PD perspective. These interviews aimed to elicit retelling experiences of events as they had happened (Avidan, 2017) and were organized along three axes analogous to those of site A in terms of content but less flexible with respect to structure. Here, axis 1 regarded critical features of the participants' personal pathway to teaching (how and why they chose to become teachers, how satisfied they were). Axis 2 focused on issues of becoming a teacher of mathematics (experiences of school mathematics, undergraduate studies related to mathematics/learning, and teaching mathematics), and of being a mathematics teacher (how do they teach mathematics, pros and cons of the mathematics teaching practice, difficulties, and their vision of being a minority teacher in mathematics). Finally, axis 3 of the narrative interviews included questions on PD (pros and cons of the PD course and vision of its future pathways).

Four in-service minority teachers provided data for the study: two in site A (Omer[1] and Atnan) and two in site B (Idris and Nermin). Their selection was based on distinct social, cultural, and educational characteristics. Omer and Atnan are graduates of EPATH and they are of Pomak origin. Idris graduated from a religious school and UPEPATH, while Nermin is a graduate of EPATH and UPEPATH. Both are of Turkish origin. Atnan, Omer, and Nermin's teaching experience in minority primary schools ranged from 10 to 30 years, whereas Idris had less than five years of teaching experience in primary schools but about 10 years in secondary schools as a religion teacher.

[1] The participant names are pseudonyms.

Omer has served as a Muslim minority primary school consultant (MMPSC) for the Turkish language program in Eastern Macedonia-Thrace. He comes from a mountain village in the region and holds a degree in Greek language and literature and a postgraduate master's degree in linguistics. He often speaks publicly (e.g., as an elected representative of the local Muslim minority primary teachers' association,) about the quality of the Muslim minority education.

Atnan has been a Muslim minority primary schools' consultant for the Turkish language program in Eastern Macedonia-Thrace since the introduction of this institutional role in 2014. He comes from a mountainous region and is the author of two books on Turkish language and minority education. He has taught Turkish as a second language in the local state-run university and been involved with various research projects related to the Muslim minority education.

Idris comes from a semi-mountainous village in Thrace. He is a religion teacher and a primary school teacher. Due to his dual capacity, he has taught mathematics in primary schools and religious studies in secondary schools attended by Muslim students. Thus, Idris understands how Muslim minority students cope with secondary education learning sites.

Finally, Nermin comes from a semi-urban area and graduated from EPATH. She has taught in various Muslim minority primary schools in Thrace. A main factor for selecting Nermin was that she is one of the few cases of a female EPATH graduate who teaches in minority primary schools.

Both interview types lasted an average of two hours for each participant and were carried out in two meetings. Site A interviews took place in the participants' schools at convenient hours, whereas site B interviews were conducted at the University campus, at hours participants suggested. During the interviews, introductory and transitional questions were asked and additional and clarifying questions were provided to facilitate responses and further explore participants' statements.

Three researchers were involved in generating data at site A. Two of them, the authors of this paper, are members of the teaching staff in the Primary Education Department of the local state-run university, were responsible for the component of PEM related to mathematics education and also involved in implementing UPEPATH.

A "broad data" grounded approach was adopted for the data analysis (Corbin & Strauss, 2015). Specifically, after carefully reading the data, we identified the following focal concept-categories as dominating the participants' responses regarding mathematics teaching and learning to teach mathematics (PD): "personal pathway to teaching" (PPT), "becoming a teacher of mathematics" and "being a mathematics teacher." Following this, we employed two levels of further data analysis. First, we carefully and repeatedly read the transcribed interviews to locate excerpts relevant to each focal concept-category. We then coded these excerpts and grouped together and labelled those that indicated a discrete practice or structure of socio-political character.

Table 1 presents the second-level analysis of the data from Nermin's interview regarding the first focal concept-category, her personal pathway to teaching. Nermin's excerpts that were considered to fall within PPT are in the 2nd column. The 3rd column records the codes used to indicate each of these excerpts. These codes included the initials of the focal category to which the excerpt matches (the PPT), the initial of the teacher to whom the formulation belonged as an index, and a number indicating the serial number of the excerpt in the interview transcript (e.g., PPT_N27). The 4th column presents the grouped labels/codes that suggest a practice or structure of socio-political character, while the 5th one provides the label of this practice or structure. Some excerpts may appear in more than one x-level category because references were identified in the teacher's corresponding phrases that are relevant to two of the focal categories.

The 2nd level analysis results allowed us to identify micro and macro practices and structures of socio-political character in the two activity systems where the Muslim minority teachers operate that suggest marginalizing conditions likely to constrain Muslim minority teachers' teaching of mathematics (RQ1) and the development of their professional identity (RQ2).

Table 1 Second level analysis for Nermin's responses regarding the focal concept-category "participants' personal pathway to teaching."

1st level	Excerpt	Labels/ codes	Grouping	2nd level/ Practice or structure of socio-political character
Participants' personal pathway to teaching.	*I used tasks from the Greek textbooks for math and that was a problem because I didn't know the language or the mathematics.*	PPT_N25	$PPTA_N25$, PPP_N29, PPT_N50, PPP_N57, PPT_N98, PPP_N101, PPT_N128	Accessibility to teaching resources

4 Results

Following the data analysis procedure described above we tried to critically pay attention to the micro and macro practices and structures of socio-political character in the two activity systems present in the participants' discourse.

4.1 Mathematics teaching activity of the muslim minority teachers (RQ1)

Table 2 offers an overview of the elements of the two activity systems where the Muslim minority primary teachers (MMPTs) operate regarding the teaching of mathematics.

The activity under consideration in both systems is mathematics teaching to the Muslim minority pupils by MMPTs (the subjects) for academic success in the short term and active citizenship in the long term. However, the analysis of the data along the elements of the two activity systems reveals micro and macro practices and structures of socio-political character that appear to constrain the quality of teaching mathematics and, therefore, of the learning of mathematics by these children.

4.2 Object

Mathematics is taught in Turkish for three teaching sessions (45 min) per class per week (as a comparison, pupils in general primary schools in Greece receive four to five sessions per week). For most minority elementary school classes, the actual time dedicated to mathematics is reduced to 1.5 teaching sessions on average per week per grade. This is because most of these classes are comprised of children from at least two different age groups (and grades) and that religious holidays of both calendars are observed.

> *Altogether, these children are taught mathematics for less than 1–2 h per week during a school year that does not last more than 170 days* (Atnan, site A).[2]

The learning environment in the Muslim minority mathematics classrooms tends to be structured with a behavioral orientation.

> *Teachers followed a traditional way of teaching and learning, there were no materials, and all these things contribute to making a lesson boring. Asking you to learn…and if you don't learn it, you are constantly*

Table 2 Analysis of the two activity systems where Muslim minority primary teachers operate when teaching mathematics

Elements	Activity System 1	Activity System 2
Subjects	MMPTs teaching mathematics in Western Thrace minority primary schools	MMPTs teaching mathematics in primary State schools
Object	Teaching mathematics up to poorly defined standards by the Greek State	Teaching mathematics up to standards updated by the Greek State
Tools	Textbooks in Turkish approved by both the Greek and the Turkish States; long outdated mathematics curriculum; oral instructions provided by officially appointed Muslim minority primary school consultants (MMPSCs) on the basis of directives addressed to all State schools; official documents written only in Greek	Program of Study, textbooks common to all schools approved by IEP; Teachers' Guides; Written guidelines provided by IEP; Supplementary educational materials approved by IEP
Community	MMPTs, Christian teachers for the courses taught in Greek, minority students, parents, MMPSCs, local minority and majority agencies (e.g., the local association of minority scientists) and institutions (e.g., local state-run Democritus University of Thrace) related to minority education	Teachers, students, parents, Ministry of Education officials, local and national agencies and institutions (e.g., Hellenic Mathematics Society) and other stakeholders of primary education (e.g., publishers of educational materials)
Rules	Curriculum, assessment, and homework requirements set and pursued locally. Within classroom norms about teaching and learning mathematics informed by teaching experience framed exclusively in Western Thrace minority schools; limited PD paths attended locally and planned for non-minority schools.	Curriculum, assessment, and homework requirements set by the Ministry of Education which are communicated through official documents. Within classroom norms about teaching and learning mathematics shaped by national policies and local practices and informed by PD courses organized locally and nationally.
Division of labor	Students are invited to attend to the working agenda for mathematics teaching set by individual teachers; teachers are expected to base their mathematics teaching on the textbooks available; teachers are mainly locally accountable (to the minority community, the local MMPSCs and other education administration entities).	Teachers are expected to attend to a working agenda for mathematics teaching shaped by the State's guidelines. Teachers are accountable to a working agenda provided by the Ministry of Education and supervised by its "agents": the primary school consultants appointed by the Ministry.

[2] Note: We conducted all interviews orally in Greek and translated them to English for this paper and use numerals for numbers the interviewees referred to.

scolded, constantly yelled at, constantly given photocopies to solve exercises, constantly given exercises from booklets (Idris, site B).

Participants indicated that most of the minority primary school children demonstrate weaknesses and difficulties even in very basic mathematics.

I was very surprised that the children were having difficulty in operations such as 47 + 36 (Omer, site A).
Imagine, they are in the sixth grade, and they don't know the angle…the children have almost completed primary school and they don't know what an area is (Nermin, site B).

Language is a key factor in teachers' efforts to ensure academic success for minority students. The languages they speak at home (primarily Turkish or Pomak, but also some Arabic) differ from the languages of instruction at their school (Turkish, Greek, but also English). This situation undermines children's efforts to succeed academically, especially in mathematics, where home and school mathematical experiences differ substantially (Barwell et al., 2016).

When I went to (primary) school… everything was new to me…When I eventually found my way through in mathematics, everything I learnt, I knew it in Turkish. I knew division, but when they asked me, what division was in Greek, I didn't know how to say it in Greek (Atnan, site A).
Children are learning math in Turkish…Then they will go to a high school where they will do math in Greek. You ask the student, "What is addition?" "What is an angle?" They don't understand what you are saying… we are building mathematics on no bases (Idris, site B).

Low-level fluency of the Muslim minority students is reported for both Greek and Turkish languages (Tzevelekou et al., 2005; Huseyinoglu, 2012). This situation constitutes a serious barrier to their academic progress and significantly impedes their integration into Greek society.

Teachers saw that the child was following the syntax of one language to write in the other. Until the teachers realized this, it took a lot of time and they "blamed" the children (Nermin, site B).
I tell the student, "five by five," and they look at me, they don't know. To a Turkish-speaking student, "five by five" means nothing. It can be 55… it's two numbers… For them to understand that it's multiplication, you have to tell them "five times five"… In Turkish it's "bes kere bes." So, if they don't know this, the teacher labels them as inadequate (Atnan, site A).

4.3 Tools

Mathematics curriculum and textbooks can be seen as the central mediating artefacts for mathematics teaching in minority primary schools.

The mathematics curriculum (published in Greek but taught in Turkish) has not officially changed since 1958, is practice oriented, and is barely known to the teachers.

There is no curriculum except the one from 1958, which is so old that the textbooks don't match, and no one uses it or can find it in schools. This is why many times teachers follow the textbook faithfully (Omer, site A).
There is no curriculum, neither for the Turkish-language nor for the Greek-language program of the school. Teachers give the students photocopies of exercises, often irrelevant to the day's lesson, to keep parents happy (Atnan, site A).

The almost nonexistent guidelines for the didactical transformation of the curriculum's mandate are very general and outdated. There are no teachers' guides to the textbooks.

You teach as you were taught, that's what I came to realize… there are no guidelines…*teachers teach as they were taught. They read the lesson, a student reads it, what did you understand, okay move on, read on. This is not [good] teaching of course* (Atnan, site A).

Teaching practice is based almost exclusively on the unique textbook and is very traditional (content-centered), with an emphasis on procedural rather than conceptual thinking.

The problems [I was taught] *in mathematics had to do mainly with number operations, like additions and subtractions… dealing with the four operations… I don't remember any other kind of problems, it was all just about plain numbers* (Idris, site B).

Apart from the mathematics textbooks written in Turkish, MMPTs often draw from the mathematics textbooks distributed to all public schools in the country (written in Greek). However, the limited interaction between the two activity systems brings about contradictions which remain unresolved and does not enable boundary crossing processes to develop.

[Minority] teachers often borrow ideas and tasks from the mathematics textbooks of the public schools…This is sometimes problematic because the "philosophies" of the two textbooks are not often compatible *... our teachers are not aware of this… and they hesitate to ask their Christian colleagues* (Atnan, site A).

The absence of any PD courses and any type of support, guidance, or evaluation of the teaching practice until recently suggests that the mathematics education provided to the minority children is outdated.

Minority teachers do not attend any seminars, they are not up-to-date, they teach without any training… thus, often teach as they were taught mathematics in the minority school (Nermin, site B).

4.4 Community

The overwhelming majority of teachers in the primary minority schools are males (about 80%), graduates of EPATH, appointed by the Service Council of Minority Schools (which contrasts with the process of appointing Christian teachers in the same schools) and until recently, they had had virtually no in-service training.

EPATH teachers are often blamed, especially by educated members of the Muslim minority, for insufficient knowledge of the Turkish language, poor pedagogical knowledge, and for acting as "state representatives."

They were skeptical about us because we had studied at EPATH in Greek and then we had to "say" what we learned in Turkish…they doubted that we could do it properly, they said we didn't know enough mathematics to be able to teach mathematics (Omer, site A).

Very little is known about the mathematics education courses offered in EPATH, except one course on basic mathematics (arithmetic, fundamental algebraic ideas, and elementary Euclidean geometry). Given the absence of PD courses for in-service MMPTs, non-existent day-to-day instructional support, few available educational resources, and limited interaction with other teacher communities, MMPTs' access to learning resources is very poor.

Some teachers are not trained, for me the main reason that minority mathematics education is not at a good level is the teachers themselves (Idris, site B).

4.5 Rules

There is very little interaction between the teaching activity of the courses in Greek and in Turkish due to poor communication between Muslim and Christian teachers. This can be attributed to the dissimilar pathways of education and PD they attend, the different procedures applied for their employment, and the climate of cautiousness often prevailing in minority schools' daily life, due to the uneasy relations between Greece and Turkey.

The Christian teachers do their own program and we do ours…I don't agree with that, and I try to cooperate, but I am one, one is not enough (Omer, site A).
Christian teachers attend a lot of training sessions organized by the Institute of Educational Policy and the universities, we [minority teachers] are far from all of them, with some exceptions of course (Idris, site B).

4.6 Division of labor

Mathematics is taught solely by MMPTs. Their teaching actions and choices are not really assessed by anyone but the Muslim minority primary school consultant. An MMPSC, though, can rarely focus on a teacher's mathematics teaching practice due to the large number of schools supervised and limited powers to intervene.

I have never been advised on mathematics teaching in the past…These [the MMPSCs] were busy people and they didn't know anything about the subjects taught in Turkish (Omer, site A).
The children understand that the teacher is struggling in math. I remember a teacher who had written a whole text on the blackboard for the younger students to copy. I asked him "What you have written, do you understand it?" And he replied, "I got it from a book." It's let's say the "hidden curriculum." The notebook is under the desk and the kids are writing rules (Atnan, site A).

4.7 Muslim minority teachers' professional identities regarding mathematics

The data analysis results presented in 4.1 imply that the only opportunities MMPTs have to develop their professional learning regarding the teaching of mathematics are: day-to-day teaching mathematics in minority primary schools; the very limited learning resources available in the activity

system where they mainly operate; and the few occasions where boundary spaces of potential learning are somehow enabled.

> *These [minority] teachers' mathematical knowledge is often very basic due to poor math experiences throughout their education… They try hard to survive while teaching mathematics… Since libraries with educational materials in Turkish are not allowed in minority schools, they often turn to the public-school textbooks [in Greek] for ideas. But they have troubles…due to their own low-level command of the Greek language and of mathematics* (Omer, site A).
> *When I do not like the tasks or the ideas suggested by the textbook in Turkish, I turn to the mathematics textbooks of the "other" schools and "borrow" or photocopy exercises… but I always wonder if I am doing the right thing… I rarely feel confident to seek advice from a minority or even a majority colleague who I trust to be more knowledgeable or better trained in teaching mathematics* (Nermin, site C).

Boundary crossing as a way to study teachers' development at the boundaries of multiple teaching practices has to date received scant focus. We sought boundary objects and brokers operating in the borders between the mathematics teaching practice that unfolded in the minority and the (majority) public primary schools. Limited activity was identified in the "boundaries" of the two activity systems due to the limited interactions of the two teachers' communities within and beyond minority schools. Yet, it is interesting to note that MMPTs and MMPSCs with university and professional studies beyond those of their basic degree (e.g. Omer and Atnan) sometimes acted as brokers. They used the mathematics textbooks and the respective teachers' guides of the public schools to draw ideas and suggest tasks, occasionally appropriating them for the needs of the minority schools.

> *I like the new mathematics curriculum introduced in 2022 for the public primary schools… I use it to talk with the [minority] teachers about what is important nowadays in mathematics education* (Atnan, site A).
> *I regularly discuss mathematics teaching practices with [minority] teachers… I try to convince them that they need to understand the theory behind the practice… But they insist that they only need to know how to do it… Despite my opposition, I am often forced to provide them with a list of "good" teaching practices to follow* (Omer, site A).
> *When [minority] teachers draw from educational materials in Greek, they are indeed "lost in translation," given their difficulties with both the Greek language and mathematics. To help them transfer meanings from one language to the other without stripping them of their epistemological and socio-cultural characteristics, I provide teachers with certain tools, e.g., a dictionary of math terms in Greek and Turkish, templates for organizing their math lesson, and so on.* (Atnan, site A).

Regarding professional learning achieved in practice by MMPTs, we searched for the type of resources that become available and accessible to them during this practice. Absence or poor quality of mathematics education resources for minority teachers and pupils alike was evident. These resources include outdated and essentially non-existent mathematics curriculum; textbooks that are old-fashioned or not tailored to the needs of the Muslim minority community; digital resources of questionable quality used inappropriately in the classroom; and a mix of languages to be orchestrated in the classroom, with Greek and Turkish claiming linguistic hegemony (Battey & Leyva, 2016).

> *Some minority teachers study and draw on educational and professional development materials published in Turkey to keep themselves updated… however, these materials do not fit to the needs of teaching mathematics in our minority schools* (Omer, site B).
> *The schools were given interactive whiteboards… when I asked if they were working on the interactive whiteboards, no one said "yes." They were not using them. They weren't using the digital sources. Sometimes they wrote on the whiteboard* (Atnan, site B).

The outcome of the limited availability of high standard educational and PD resources is poor quality mathematics teaching that is oriented to procedural rather than conceptual understanding, is tied to dubious quality educational material, and does not pursue clear daily objectives or serve any coherent mathematics education vision for the minority students (Weis & Fein, 2012). Consequently, the minority student rarely escapes from constructing a negative mathematics' learner identity.

5 Discussion and conclusions

Our study results suggest that the system of minority education in Western Thrace does not provide an adequate level of mathematics teaching to the Muslim students. This can be attributed to practices and structures of socio-political character operating locally and nationally, which shape the activity of teaching mathematics in the Muslim minority

primary schools (RQ1). Our interviews with the participating teachers revealed critical aspects of this operation discussed below.

Minority teachers are poorly educated, especially in matters related to mathematics education. Due to the highly centralized education system in Greece, teaching mathematics in Muslim minority primary schools is completely dependent on the Ministry of Education regarding the vision of mathematics education served — and the means and the human and material resources by which this vision can be pursued. Poor everyday interaction with the teachers of the subjects taught in Greek does not allow the emergence of a culture of inquiring and reflecting on everyday mathematics teaching (Jaworski, 2006) that could compensate for the lack of officially organized PD activities. Moreover, the Greek State and the majority and minority communities appear to treat these teachers cautiously or even suspiciously for reasons related to intersecting identities of education, religion, ideology, and loyalty. Finally, local MMPSCs and local educational authorities can seemingly do very little to support MMPTs' teaching practice in mathematics. Reasons for this could be because MMPSCs serve an institutional role that has recently been introduced, whereas the members of the local educational authorities are often trapped between explicit and "unofficial" aspirations, obsessions, and suspicions of the majority and minority communities (Dragonas & Fragoudaki, 2006).

The above national and local socio-political features contribute in complex ways to the low quality of mathematics teaching and thus, education provided to the Muslim minority children and to its perpetual reproduction. The critical characteristics of the resulting minoritization process are poor resources, inexplicit distribution of power, the linguistic hegemony struggle (Civil, 2014), mismatch between home and school mathematical experiences (Barwell et al., 2016) and deficient learning and teaching of mathematics (Sakonidis et al., 2017). Situated within the difficult relationship between Greece and Turkey, these characteristics aid understanding of how socio-economic inequalities between the members of the Muslim minority and the Christian majority are (re)produced.

We now turn to our RQ2 on the local and national socio-political features that shape MMPTs' professional identities regarding mathematics education. Our results suggest that Muslim minority teachers tend to be defensive regarding their adequacy to teach mathematics properly. This is very much connected to; (a) the criticism openly or covertly expressed for the criteria of their selection to study in EPATH; (b) the quality of their preparation to become teachers to teach in the highly demanding minority schools; and (c) their presumed role of acting as agents of the Greek State in the Muslim minority schools. This defensive positioning regarding their mathematics teaching practice and the expectations placed on them by the educational system and their own community lead to feelings of inadequacy and low self-esteem. These factors can ultimately impact the quality of teaching mathematics and the development of a powerful professional identity regarding mathematics teaching by these teachers. This impact is exacerbated by the institutional barriers that minority teachers often face, such as inadequate resources and lack of support from the education administration, experiences that were traced in the day-to-day practice of the EPATH teachers of our study (Dragonas & Fragoudaki, 2006).

MMPTs' can be seen to belong to a distinct community of practice situated within two activity systems. One system concerns mathematics teaching to serve the purposes of the official Greek State, while the other system relates to mathematics teaching to satisfy the needs of the Muslim minority community. Our study's results suggest that when teaching mathematics to minority schools, the Muslim minority teachers occasionally and uncertainly employ practices and structures aligned with those the Greek State officially promotes for public schools. This hesitantly might enable continuity between the two activity systems in their boundaries (Akkerman & Bakker, 2011).

According to the relevant literature, it is through collaboration within the boundary space of two interacting activity systems — via boundary crossing (Engeström et al., 1995) — that new and hybridized ideas emerge, and practices and mutual understanding of shared tasks and problems develop (Edwards & Fowler, 2007). However, the participating Muslim minority teachers in our study have had very few opportunities to collaborate and thus to elaborate, inquire or reflect on their hybridized ideas and practices with other teachers. This lack of peer interaction applies to Muslim and Christian colleagues, both in and outside of school, and with far less interaction occurring with Christian colleagues. Additional impediments include the absence until recently of any advisory mechanism and the limited access to scientific literature — particularly on mathematics. The lack of being able to further develop their professional knowledge is especially problematic for these teachers. MMPTs' already view themselves as being poorly equipped to teach mathematics. Yet it is this subject that has widely recognized value for minority students' futures. Thus, minority teachers experience isolation and helplessness when developing a robust mathematics teacher's identity.

Finally, through boundary crossing processes, new elements (boundary objects/new tools or artefacts) are introduced from one community of practice to another. This happens via brokers — people who are simultaneously members of multiple communities or in transition from one community/system to another (Star, 1989). Our results

suggest that the participating minority teachers, especially those with professional studies that exceed minimum requirements, may have acted, even unintentionally, as brokers at some stage of their careers. This is because the under-resourcing of the minority schools regarding mathematics education "forced" them to turn to pools of ideas and educational material (e.g. textbooks) available to mainstream Greek schools (in Greek) or from other mathematics education systems. Yet, such unintentional brokering is usually non-collaborative and hampered by limited background knowledge (e.g., of mathematics/mathematics teaching practice). Thus, these "brokering acts," lack the dynamics that lead to creating spaces for new understanding, changes in practice, or identity development.

Our results point to a series of micro and macro practices and structures of socio-political character that interact at local and national levels. This interaction shapes the teaching of mathematics in the minority primary schools of Western Thrace and the professional identities the minority teachers develop with respect to teaching mathematics. It is evident that we are only now beginning to trace this extremely complex mathematics education reality. Empowering the development of minority teachers' teaching and professional identities with regard to mathematics is crucial for a state to ensure it provides democratic, inclusive, and nondiscriminatory education. Thus, field studies are needed that will enable careful identification and deep understanding of the origins and operation of micro and macro practices and structures of socio-political character (e.g., national sovereignty issues, social cohesion). Such practices and structures are established in both minority and majority contexts and undermine the Muslim minority's mathematics education, and hence contribute to its marginalization within the larger society.

Declarations

Compliance with ethical standards No financial or non-financial competing interests. Informed consent of involved human participants.

References

Akkerman, S. F., & Bakker, A. (2011). Boundary crossing and boundary objects. *Review of Educational Research*, *81*, 132–169. https://doi.org/10.3102/0034654311404435.

Askouni, N. (2006). *Minority Education in Thrace*. Athens: Alexandria (in Greek).

Avidan, A. (2017). The role and contribution of narrative interviews in Educational Research. *American Journal of Educational Research*, *5*(4), 419–427. https://doi.org/10.12691/education-5-4-10.

Barwell, R., Clarkson, P., Halai, A., Kazima, M., Moschkovich, J., Planas, N., Phakeng, M. S., Valero, P., & Ubillús, M. V. (2016). *Mathematics education and language diversity: The 21st ICMI Study*. Springer Nature. https://doi.org/10.1007/978-3-319-14511-2.

Battey, D., & Leyva, L. A. (2016). A framework for understanding whiteness in mathematics education. *Journal of Urban Mathematics Education*, *9*(2), 49–80. https://doi.org/10.21423/jume-v9i2a294.

Boussiakou, I. K. (2007). The educational rights of the Muslim minority under Greek law. *Journal on Ethnopolitics and Minority issues in Europe*, 6(1), 1–25. PID: https://nbn-resolving.org/urn:nbn:de:0168-ssoar-61865.

Chapman, O. (2009). Educators reflecting on (researching) their own practice. In R. Even, & D. L. Ball (Eds.), *The professional education and development of teachers of mathematics* (pp. 121–126). Springer. https://doi.org/10.1007/978-0-387-09601-8_14.

Chen, G. A., & Horn, I. S. (2022). A call for critical bifocality: Research on marginalization in Mathematics Education. *Review of Educational Research*, *92*(5), 786–828. https://doi.org/10.3102/00346543211070050.

Civil, M. (2014). Why should mathematics educators learn from and about Latina/o students' in-school and out-of-school experiences? *Journal of Urban Mathematics Education*, *7*(2), 9–20. https://doi.org/10.21423/jume-v7i2a251.

Corbin, J., & Strauss, A. (2015). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (4rd ed.). Sage.

Cross, R. (2020). The 'subject' of Freeman & Johnson's reconceived knowledge base of second language teacher education. *Language Teaching Research*, *4*(1), 37–48. https://doi.org/10.1177/1362168818777521.

Domina, T., Penner, A., & Penner, E. (2017). Categorical inequality: Schools as sorting machines. *Annual Review of Sociology*, *43*, 311–330. 10.1146/ annurev-soc-060116-053354.

Dragonas, T., & Fragoudaki, A. (2006). Educating the muslim minority in Western Thrace. *Islam and Christian-Muslim Relations*, *17*(1), 21–41. https://doi.org/10.1080/09596410500399268.

Edwards, R., & Fowler, Z. (2007). Unsettling boundaries in making a space for research. *British Educational Research Journal*, *33*(1), 107–123. https://doi.org/10.1080/01411920601104565.

Engeström, Y. (1987). *Learning by expanding: An activity-theoretical approach to developmental research*. Orienta-Konsultit.

Engeström, Y. (2001). Expansive learning at work: Toward an activity theoretical reconceptualization. *Journal of Education and Work*, *14*, 133–156.

Engeström, Y., Engeström, R., & Kärkkäinen, M. (1995). Polycontextuality and boundary crossing in expert cognition: Learning and problem solving in complex work activities. *Learning and Instruction*, *5*, 319–336. https://doi.org/10.1016/0959-4752(95)00021-6.

Gaudreault, K. L., & Woods, A. M. (2012). The effects of achieved national board for professional teaching standards certification on the marginality of physical education teachers. *The Teacher Educator*, *47*, 283–301. https://doi.org/10.1080/08878730.2012.707760.

Huseyinoglu, A. (2012). *The development of minority education at the south-easternmost corner of the EU: the case of Muslim Turks in Western Thrace, Greece*. [PhD Thesis], University of Sussex.

Jaworski, B. (2006). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education*, *9*(2), 187–211. https://doi.org/10.1007/s10857-005-1223-z.

Leont'ev, A. N. (1974). The problem of activity in psychology. *Soviet Psychology*, *13*(2), 4–33. https://doi.org/10.2753/RPO1061-040513024.

Ntow, F., & Adler, J. (2019). Identity resources and mathematics teaching identity: An exploratory study. *Zdm Mathematics Education*, *51*, 419–432. https://doi.org/10.1007/s11858-019-01025-z.

Planas, N., & Valero, P. (2016). Tracing the Socio-Cultural-Political Axis in understanding Mathematics Education. In Á. Gutiérrez, G. C. Leder, & P. Boero (Eds.), *The 2nd Handbook of Research on the psychology of Mathematics Education*. Sense. https://doi.org/10.1007/978-94-6300-561-6_13.

Quinn, F., & Hobbs, L. (2024). I'm on my own and I'm not trained: A Cultural-historical activity theory analysis of Teaching Mathematics out-of-field in a Small School. *Int J of Sci and Math Educ*. https://doi.org/10.1007/s10763-024-10454-6.

Sakonidis, C., Klothou, A., Fakoudis, E., Margaritis, C., Antampoufis, N., Dramalidis, A., & Nizam, A. (2017). Aspects of mathematical literacy: the case of Thrace minority students (in Greek). In T. Zachariadis, D. Potari & G. Psycharis (Eds.) *Proceedings of the 7th Panhellenic Conference of the GARME* (pp 659–669). Athens, Greece: University of Athens.

Star, S. L. (1989). The structure of ill-structured solutions: Boundary objects and heterogeneous distributed problem-solving. In L. Gasser, & M. N. Huhns (Eds.), *Distributed artificial intelligence* (Vol. 2). Pitman / San Mateo, CA: Morgan Kaufmann.

Tzevelekou, M., Kantzou, V., Stamouli, S., Varlokosta, S., Papageorgakopoulos, G., Lytra, V., Iakovou, M., & Chondrogianni, V. (2005). Proficiency in Greek of the children attending Greek-Turkish bilingual minority schools of Western Thrace, *ALTE 2nd International Conference: Language Assessment in a Multilingual Context–Attaining Standards, Sustaining Diversity*, Berlin. http://www.museduc.gr/docs/ALTE-Berlin05.pdf.

Weis, L., & Fine, M. (2012). Critical bifocality and circuits of privilege: Expanding critical ethnographic theory and design. *Harvard Educational Review*, *82*(2), 173–201. https://doi.org/10.17763/haer.82.2.v1jx34n441532242.

Wenger, E. (1998). *Communities of practice. Learning, meaning and identity*. Cambridge University Press.