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Why and How to Differentiate Math Instruction

STUDENTS IN ANY CLASSROOM differ in many ways, only some of which the teacher can reasonably attend to in developing instructional plans. Some differences will be cognitive—for example, what previous concepts and skills students can call upon. Some differences will be more about learning style and preferences, e.g., whether the student learns better through auditory, visual, or kinesthetic approaches. Other differences will be more about preferences, including behaviors such as persistence or inquisitiveness or the lack thereof and personal interests.

THE CHALLENGE IN MATH CLASSROOMS

Although teachers in other subject areas sometimes allow students to work on alternative projects, it is much less likely that teachers vary the material they ask their students to work with in mathematics. The math teacher will more frequently teach all students based on a fairly narrow curriculum goal presented in a textbook. The teacher will recognize that some students need additional help and will provide as much support as possible to those students while the other students are working independently. Perhaps this occurs because **differentiating instruction** in mathematics is a relatively new idea. Perhaps it is because teachers may never have been trained to really understand how students differ mathematically. However, students in the same math classroom clearly do differ mathematically in significant ways. Teachers want to be successful in their instruction of all students. Understanding differences and differentiating instruction are important processes for achievement of that goal.

The National Council of Teachers of Mathematics (NCTM), the professional organization whose mission it is to promote, articulate, and support the best possible teaching and learning in mathematics, recognizes the need for differentiation. The first principle of the NCTM *Principles and Standards for School Mathematics* reads, “Excellence in mathematics education requires equity—high expectations and strong support for all students” (NCTM, 2000, p. 12).

In particular, NCTM recognizes the need for accommodating differences among students, taking into account both their readiness and their level of mathematical talent/interest/confidence, to ensure that each student can learn important mathematics. "Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students" (NCTM, 2000, p. 12).

THE PARTICULAR CHALLENGE IN GRADES 6–12

The challenge for teachers of grades 6–12 is even greater than in the earlier grades, particularly in situations where students are not streamed. Although there is much evidence of the value, particularly for the struggling student, of being in heterogeneous classrooms, the teacher in those rooms must deal with significant student differences in mathematical level. While some students are still struggling with their multiplication facts or addition and subtraction with decimals, others are comfortable with complex reasoning and problem solving involving fractions, decimals, and percents. The differences between students' mathematical levels, beginning as far back as kindergarten or grade 1, continue to be an issue teachers must face all through the grades.

Where some see the answer as streaming, many believe that the answer is a differentiated instruction environment in a destreamed classroom.

WHAT IT MEANS TO MEET STUDENT NEEDS

One approach to meeting each student's needs is to provide tasks within each student's **zone of proximal development** and to ensure that each student in the class has the opportunity to make a meaningful contribution to the class community of learners. Zone of proximal development is a term used to describe the "distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 86).

Instruction within the zone of proximal development allows students, whether with guidance from the teacher or by working with other students, to access new ideas that are beyond what the students know but within their reach. Teachers are not using educational time wisely if they either are teaching beyond the student's zone of proximal development or are providing instruction on material the student already can handle independently. Although other students in the classroom may be progressing, the student operating outside his or her zone of proximal development is often not benefiting from the instruction.

For example, a teacher might be planning a lesson on calculating the whole when a percent that is greater than 100% of the whole is known, using a problem such as asking students to determine what number 30 is 210% of. Although the skill that the teacher might emphasize is solving a proportion such as

$$\frac{210}{100} = \frac{30}{x}$$

the more fundamental objective is getting students to recognize that solving a percent problem is always about determining a ratio equivalent to one where the second term is 100.

Although the planned lesson is likely to depend on the facts that students can work algebraically with two fractions, one involving a variable, and that they understand the concept of a percent greater than 100%, a teacher could effectively teach a meaningful lesson on what percent is all about even to students who do not have those abilities. The teacher could allow the less developed students to explore the idea of determining equivalent ratios to solve problems using percents less than 100% with ratio tables or other more informal strategies (rather than formal proportions) while the more advanced students are using percents greater than 100% and more formal methods. Only when the teacher felt that the use of percents greater than 100% and algebraic techniques were in an individual student's zone of proximal development would the teacher ask that student to work with those sorts of values and strategies. Thus, by making this adjustment, the teacher differentiates the task to locate it within each student's zone of proximal development.

ASSESSING STUDENTS' NEEDS

For a teacher to teach to a student's zone of proximal development, first the teacher must determine what that zone is. This can be accomplished by using prior assessment information in conjunction with a teacher's own analysis to ascertain a student's mathematical developmental level. For example, to determine an 8th-grade student's developmental level in working with percents, a teacher might use a diagnostic to find out whether the student interprets percents as ratios with a second term of 100, relates percents to equivalent fractions and/or decimals, can represent a percent up to 100% visually, can explain what 150% means, and recognizes that solving a percent problem involves determining an equivalent ratio.

Some tools to accomplish this sort of evaluation are tied to developmental continua that have been established to describe students' mathematical growth (Small, 2005a, 2005b, 2006, 2007, 2010). Teachers might also use locally or personally developed tools to learn about students' prior knowledge. Only after a teacher has determined a student's level of mathematical sophistication, can he or she meaningfully address that student's needs.

PRINCIPLES AND APPROACHES TO DIFFERENTIATING INSTRUCTION

Differentiating instruction is not a new idea, but the issue has been gaining an ever higher profile for mathematics teachers in recent years. More and more, educational systems and parents are expecting the teacher to be aware of what each individual student—whether a struggling student, an average student, or a gifted student—needs and to plan instruction to take those needs into account. In the past, this was less the case in mathematics than in other subject areas, but now the expectation is common in mathematics as well.

There is general agreement that to effectively differentiate instruction, the following elements are needed:

- ***Big Ideas.*** The focus of instruction must be on the **big ideas** being taught to ensure that they are addressed, no matter at what level (Small, 2009a; Small & Lin, 2010).
- ***Prior Assessment.*** Prior assessment is essential to determine what needs different students have (Gregory & Chapman, 2007; Murray & Jorgensen, 2007).
- ***Choice.*** There must be some aspect of choice for the student, whether in content, process, or product.

Teaching to Big Ideas

The Curriculum Principle of the NCTM *Principles and Standards for School Mathematics* states that "A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades" (NCTM, 2000, p. 14).

Curriculum coherence requires a focus on interconnections, or big ideas. Big ideas represent fundamental principles; they are the ideas that link the specifics. For example, the notion that benchmark numbers are a way to make sense of other numbers is equally useful for the 6th-grader who is trying to place –22 on a number line, the 8th-grader who relates π to the number 3.14, or the 10th-grader who is trying to estimate the sine of a 50° angle. If students in a classroom differ in their readiness, it is usually in terms of the specifics and not the big ideas. Although some students in a classroom where estimating the value of radicals is being taught may not be ready for that precise topic, they could still deal with the concept of estimating and why it is useful in simpler situations.

Big ideas can form a framework for thinking about "important mathematics" and supporting standards-driven instruction. Big ideas cut across grade bands. There may be differences in the complexity of their application, but the big ideas remain constant. Many teachers believe that curriculum requirements limit them to fairly narrow learning goals and feel that they must focus instruction on meeting those specific student outcomes. Differentiation requires a different approach, one that is facilitated by teaching to the big ideas. It is impossible to differentiate too narrow an idea, but it is always possible to differentiate instruction focused on a bigger idea.

Prior Assessment

To determine the instructional direction, a teacher needs to know how students in the classroom vary in their mathematical developmental level. This requires collecting data either formally or informally to determine what abilities or what deficiencies students have. Although many teachers feel they lack the time or tools to undertake prior assessment on a regular basis, the data derived from prior assessment should drive how instruction is differentiated.

Despite the importance of prior assessment, employing a highly structured approach or a standardized tool for conducting the assessment is not mandatory. Depending on the topic, a teacher might use a combination of written and oral questions and tasks to determine an appropriate starting point for each student or to determine what next steps the student requires. An example of one situation is described below, along with steps teachers might take, given the student responses.

Consider the task below:

> The Beep-Beep pager company charges $30 to set up a client with a pager along with a $7.50 monthly fee. The Don't Miss Them pager company charges $9 a month, but no set up fee. How long would you have to own a pager before the Beep-Beep deal is a better one?

Students might respond to the task in very different ways. Here are some examples:

- Joshua immediately raises his hand and just waits for the teacher to help him.
- Blossom says that 30 + 7.50 + 9 = 46.50, so it would take 46.50 months.
- Madison writes: $y = 30 + 7.50x$ and $y = 9x$, so $9x = 30 + 7.5x$. That means $1.5x = 30$, and $x = 30 \div 1.5$. Since $x = 20$, it would be 20 months.
- Lamont starts a table like this one, but forgets to add the terms to answer the question.

Beep-Beep	Don't Miss Them
37.50	9
7.50	9
7.50	9
7.50	9

- Hannah uses an appropriate table, and extends it until the Beep-Beep value is less and counts the number of entries.

Beep-Beep	Don't Miss Them
37.50	9
45	18
52.50	27
60	36
. . .	. . .
180	180
187.50	189

- Latoya reasons that the difference in price is $1.50 a month, so you just divide 30 by 1.50 to figure out how many months it would take to make up the extra cost at the start. She calculates the value to be 20 and then indicates that after 21 months (assuming whole numbers of months), the Beep-Beep plan is better.

The teacher needs to respond differently based on what has been learned about the students. For example, the teacher might wish to:

- Encourage Joshua to be more independent or set out an alternate related problem that is more suitable to his developmental level
- Help Blossom understand that just because there are three numbers in a problem, you don't automatically add, and emphasize the importance of reading carefully what the problem requests
- Encourage Madison's thoughtful approach to the problem, but help her see that she still hasn't really answered the question posed
- Ask Lamont to label his columns and tell what each represents, then ask him how the table might help him solve the problem
- Ask Hannah to think of a way she could have used her idea without having to show every single row in the table
- Ask Latoya, who clearly is thinking in a very sophisticated way, to create a different scenario where the Beep-Beep plan would not be better until, for example, 31 months

By knowing where the students are in their cognitive and mathematical development, the teacher is better able to get a feel for what groups of students might need in the way of learning and can set up a situation that challenges each individual while still encouraging each one to take risks and responsibility for learning (Karp & Howell, 2004).

Choice

Few math teachers are comfortable with the notion of student choice except in the rarest of circumstances. They worry that students will not make "appropriate" choices.

However, some teachers who are uncomfortable differentiating instruction in terms of the main lesson goal are willing to provide some choice in follow-up activities students use to practice the ideas they have been taught. Some of the strategies that have been suggested for differentiating practice include the use of menus from which students choose from an array of tasks, tiered lessons in which teachers teach to the whole group and vary the follow-up for different students, learning stations where different students attempt different tasks, or other approaches that allow for student choice, usually in pursuit of the same basic overall lesson goal (Tomlinson, 1999; Westphal, 2007).

For example, a teacher might present a lesson on using the exponent laws to simplify expressions to all students, and then vary the follow-up. Some students might work only with simple situations; these tasks are likely to involve simple multiplications of pairs of numbers with the same base, such as $2^5 \times 2^7$. Other students might be asked to work with situations where a variety of laws might be called on at once, such as simplifying $2^5 \times (2^7)^2 \div 2^5$. Some students might deal with even more challenging questions, such as determining two factors for 1 million where neither one involves a power of 10 (e.g., $10^6 = 2^6 \times 5^6$). By using prior assessment data, the teacher is in a better position to provide appropriate choices.

TWO CORE STRATEGIES FOR DIFFERENTIATING MATHEMATICS INSTRUCTION: OPEN QUESTIONS AND PARALLEL TASKS

It is not realistic for a teacher to try to create 30 different instructional paths for 30 students, or even 6 different paths for 6 groups of students. Because this is the perceived alternative to one-size-fits-all teaching, instruction in mathematics is often not differentiated. To differentiate instruction efficiently, teachers need manageable strategies that meet the needs of most of their students at the same time. Through the use of just two core strategies, teachers can effectively differentiate instruction to suit all students. These two core strategies are the central feature of this book:

- **Open questions**
- **Parallel tasks**

Open Questions

The ultimate goal of differentiation is to meet the needs of the varied students in a classroom during instruction. This becomes manageable if the teacher can create a single question or task that is inclusive not only in allowing for different students to approach it by using different processes or strategies but also in allowing for students at different stages of mathematical development to benefit and grow from attention to the task. In other words, the task is in the appropriate zone of proximal development for the entire class. In this way, each student becomes part of the larger learning conversation, an important and valued member of the learning community. Struggling students are less likely to be the passive learners they so often are (Lovin, Kyger, & Allsopp, 2004).

A question is open when it is framed in such a way that a variety of responses or approaches are possible. Consider, for example, these two questions, each of which might be asked of a whole class, and think about how the responses to each question would differ:

Question 1: Write the quadratic $y = 3x^2 - 12x + 17$ in vertex form.

Question 2: Draw a graph of $y = 3x^2 - 12x + 17$. Tell what you notice.

Question 1 is a fairly closed question. If the student does not know what vertex form is, there is no chance he or she will answer Question 1 correctly. In the case of Question 2, a much more open question, students simply create the graph and notice whatever it is that they happen to notice—whether that is the vertex, that the shape is parabolic, that it opens upward, and so on.

Strategies for Creating Open Questions. This book illustrates a variety of styles of open questions. Some common strategies that can be used to construct open questions are described below:

- Turning around a question
- Asking for similarities and differences
- Replacing a number, shape, measurement unit, and so forth with a blank
- Asking for a number sentence

Turning Around a Question. For the turn-around strategy, instead of giving the question, the teacher gives the answer and asks for the question. For example:

What is 75% of 20?	→	15 is a percent of a number. What percent of what number is it?
What is the hypotenuse of a right triangle if the legs are 3 units and 4 units long?	→	One side of a right triangle is 5 units long. What could the other side lengths be?

Asking for Similarities and Differences. The teacher chooses two items—two numbers, two shapes, two graphs, two probabilities, two measurements, and so forth—and asks students how they are alike and how they are different. Inevitably, there will be many good answers. For example, the teacher could ask how the number $\sqrt{2}$ is like the number $\sqrt{5}$ and how it is different. A student might realize that both are irrational numbers, both are less than 3, both are greater than 1, and both are side lengths of squares with a whole number of units of area.

Replacing a Number with a Blank. Open questions can be created by replacing a number (or numbers) with a blank and allowing the students to choose the number(s) to use. For example, instead of asking for the surface area of a cone with radius 4" and height 15", the teacher could ask students to choose numbers for the radius and height and then determine the surface area. By allowing choice, the question clearly can go in many directions. Most importantly, students can choose values in such a way that their ability to demonstrate understanding of the concept being learned is not compromised by extraneous factors such as the complexity of the calculations required of them.

Asking for a Sentence. Students can be asked to create a sentence that includes certain words and numbers. For example, a teacher could ask students to create a sentence that includes the number 0.5 along with the words "sine," "rational," and "amplitude," or a sentence that includes the words "linear" and "increasing" as well as the numbers 4 and 9. The variety of sentences students come up with will often surprise teachers. For example, for the second situation, a student might produce any of the sentences below and many more:

- *An increasing linear pattern could include the numbers 4 and 9.*
- *In a linear pattern starting at 4 and increasing by 9, the tenth number will be 85.*
- *A linear pattern that is increasing by 9 grows faster than one that is increasing by 4.*

Shortcut for Creating Open Questions. A teacher can sometimes create an open question by beginning with a question already available, such as a question from a text resource. Here are a few examples:

Graph and solve this linear system of equations: $0.5x + 0.6y = 5.4$ $-x + y = 9$	→	Write two equations involving both *x* and *y*. Determine values for *x* and *y* that make both of them true.
Solve for *m*: $\frac{4m}{5} - \frac{1}{2} = \frac{-25}{2}$	→	The solution to an equation is $m = -15$. The equation involves a fraction. What might the equation be?
Matthew has 20 ounces of a 40% salt solution. How much salt should he add to make it a 45% solution?	→	Matthew has 20 ounces of a 40% salt solution. He wants a solution with a greater percentage of salt. Decide on the percentage of salt you want. Tell how much salt to add.

What to Avoid in an Open Question. An open question should be mathematically meaningful. There is nothing wrong with an occasional question such as *What do you like about algebra?* but questions that are focused more directly on big ideas or on curricular goals are likely to accomplish more in terms of helping students progress satisfactorily in math.

Open questions need just the right amount of ambiguity. They may seem vague, and that may initially bother students, but the vagueness is critical to ensuring that the question is broad enough to meet the needs of all students.

On the other hand, teachers must be careful about making questions so vague that they deter thinking. Compare, for example, a question like *What is infinity?* with a question like *How do you know that there are an infinite number of decimals* *between 0 and 1?* In the first case, a student does not know whether what is desired is a definition for the word, something philosophical, or the symbol ∞. The student will most likely be uncomfortable proceeding without further direction. In the second case, there is still ambiguity. Some students may wonder if a particular approach is desired, but many students will be comfortable proceeding by using their own strategies.

The reason for a little ambiguity is to allow for the differentiation that is the goal in the use of open questions. Any question that is too specific may target a narrow level of understanding and not allow students who are not at that level to engage with the question and experience success.

A Different Kind of Classroom Conversation. Not only will the mathematical conversation be richer when open questions are used, but almost any student will be able to find something appropriate to contribute.

The important point to notice is that the teacher can put the same question to the entire class, but the question is designed to allow for differentiation of response based on each student's understanding. All students can participate fully and gain from the discussion in the classroom learning community.

This approach differs, in an important way, from asking a question, observing students who do not understand, and then asking a simpler question to which they can respond. By using the open question, students gain confidence; they can answer the teacher's question right from the start. Psychologically, this is a much more positive situation.

Multiple Benefits. There is another benefit to open questions. Many students and many adults view mathematics as a difficult, unwelcoming subject because they see it as black and white. Unlike subjects where students are asked their opinions or where they might be encouraged to express different points of views, math is viewed as a subject where either you get it or you don't. This view of mathematics inhibits many students from even trying. Once they falter, they lose confidence and assume they will continue to falter—they may simply shut down.

It is the responsibility of teachers to help students see that mathematics is multifaceted. Any mathematical concept can be considered from a variety of perspectives, and those multiple perspectives actually enrich its study. Open questions provide the opportunity to demonstrate this.

Fostering Effective Follow-Up Discussion. Follow-up discussions play a significant role in cementing learning and building confidence in students. Thus, it is important for teachers to employ strategies that will optimize the effectiveness of follow-up discussions to benefit students at all developmental levels.

To build success for all students, it is important to make sure that those who are more likely to have simple answers are called on first. By doing so, the teacher will increase the chances that these students' answers have not been "used up" by the time they are called on.