

Επιστημονικοί Υπολογισμοί

Αριθμητική Επίλυση συστημάτων μη-γραμμικών εξισώσεων - Μη γραμμικές μέθοδοι

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https://scholar.google.gr/citations?user=mht7W_YAAAAJ&hl=el
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Τμήμα Ηλεκτρολόγων Μηχανικών & Μηχανικών Υπολογιστών
Δημοκρίτειο Πανεπιστήμιο Θράκης
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Περιεχόμενα I

- 1 Γενική επαναληπτική
Γεωμετρική ερμηνεία
Κριτήρια τερματισμού
Αλγόριθμος
- 2 Μη γραμμική SOR
Κατασκευή
Γεωμετρική ερμηνεία
Κριτήρια τερματισμού
Αλγόριθμος
- 3 Μη γραμμική Jacobi
Κατασκευή
Γεωμετρική ερμηνεία
Κριτήρια τερματισμού
Αλγόριθμος
- 4 Βιβλιογραφία



ep̄lush ex̄swsh̄c e[∞]resh stajer_n shme[∞]wn

To p̄iblhma thc ep̄lushc exis_n sewn sqet̄zetai me to p̄iblhma tou upologismo[∞] stajer_n shme[∞]wn miac sun[∞]rthshc.

ep̄lush ex̄swsh̄c eōresh stajer, n shmēDwn

To p̄rblhma thc ep̄lushc exis, sewn sqet̄Dzetai me to p̄rblhma tou upologismoō̄ stajer, n shmēDwn miac sun̄rthshc.

To p̄rblhma upologismoō̄ stajer, n shmēDwn

Dedom̄nhc thcf : $[a; b] \quad \mathbb{R} \rightarrow \mathbb{R}$

na brejēD k̄poio shmēDσ 2 (a; b) t̄etoio ,ste $f(r) = r$

ep̄lush ex̄swshç eÔresh stajer,n shmeĐwn

To p̄blhma thç ep̄lushç exis,sewn sqetĐzetai me to p̄blhma tou upologismoÔ stajer,n shmeĐwn miac sun^rthshç.

To p̄blhma upologismoÔ stajer,n shmeĐwn

Dedomènhc thçf : [a; b] R ! R

na brejeĐ k^poio shmeĐσ 2 (a; b) tètioio ,ste f(r) = r

ep̄lush ex̄swshç eÔresh stajer,n shmeĐwn

ep̄dlush ex̄dswshç eÔresh stajer,n shmeĐwn

To p̄iblhma thç ep̄dlushç exis,sewn sqetĐzetai me to p̄iblhma tou upologismoÔ stajer,n shmeĐwn miac sun^rthshç.

To p̄iblhma upologismoÔ stajer,n shmeĐwn

Dedomènhc thçf : $[a; b] \rightarrow \mathbb{R} \rightarrow \mathbb{R}$

na brejeĐ k^poio shmeĐσ 2 (a; b) tètioio ,ste $f(r) = r$

ep̄dlush ex̄dswshç eÔresh stajer,n shmeĐwn

me k^poia arijmhtik mèjodo jèloume na epilÔsoume thn

$$f(x) = 0$$

επιλυση εξισωσης ε̂resh stajr,n shmeDwn

To p̂rblhma thc epilushc exis,sewn sqet̂zetai me to p̂rblhma tou upologismô stajr,n shmeDwn miac sun̂rthshc.

To p̂rblhma upologismô stajr,n shmeDwn

Dedom̂nhc thcf : $[a; b] \subset \mathbb{R} \rightarrow \mathbb{R}$

na breĵD k̂poio shmeD̂ $\sigma \in (a; b)$ t̂toio ,ste $f(\sigma) = 0$

επιλυση εξισωσης̂ ε̂resh stajr,n shmeDwn

me k̂poia arijmhtik̂ m̂jodo ĵlouse na epil̂soume thn

$$f(x) = 0$$

epil̂soume thrf $f(x) = g(x)$ x kai br̂skoume th l̂shr

επιβολή της συνέχειας στην απόλυτη συνέχεια

Το πρόβλημα της συνέχειας είναι σχετικό με το πρόβλημα του απόλυτου στην απόλυτη συνέχεια.

Το πρόβλημα απόλυτου στην απόλυτη συνέχεια

Δεδομένης της $f : [a; b] \rightarrow \mathbb{R} \rightarrow \mathbb{R}$

να βρεθεί κάποια συνέχεια f στο $(a; b)$ τέτοια ώστε $f(r) = r$

επιβολή της συνέχειας στην απόλυτη συνέχεια

με κάποια αριθμητική μέθοδο βρούμε να επιλύσουμε την

$$f(x) = 0$$

επιλύουμε την $f(x) = g(x)$ και βρούμε τη λύση

$$\text{επειδή } f(r) = 0$$

επιβολή εξίσωσης εὐρέως σταθερῆς συνάρτησης

Το πρόβλημα της επιβολής εξίσωσης συνάρτησης με το πρόβλημα του υπολογισμού σταθερῆς συνάρτησης.

Το πρόβλημα υπολογισμού σταθερῆς συνάρτησης

Δεδομένης τῆς $f : [a; b] \rightarrow \mathbb{R}$

να βρεθῆ κῆποι $r \in (a; b)$ τέτοιοι, ὅστε $f(r) = r$

επιβολή εξίσωσης εὐρέως σταθερῆς συνάρτησης

με κῆποια αριθμητικὴ μέθοδο βρούμε να επιβούμε τὴν

$$f(x) = 0$$

επιβούμε τῆς $f(x) = g(x) - x$ καὶ βρούμε τὴν ἰσο-

$$\text{επίπεδο } f(r) = 0 \iff g(r) - r = 0$$

επιβολή εξίσωσης εὐρέως σταθερῆς τιμῆς

Το πρόβλημα της επιβολῆς εξίσωσης σε σχέση με το πρόβλημα του υπολογισμοῦ σταθερῆς τιμῆς μιὰς συνάρτησης.

Το πρόβλημα υπολογισμοῦ σταθερῆς τιμῆς

Δεδομένης τῆς $f : [a; b] \rightarrow \mathbb{R}$

να βρεθῆ κῆποις $x \in [a; b]$ τέτοιος ὥστε $f(x) = x$

επιβολή εξίσωσης εὐρέως σταθερῆς τιμῆς

με κῆποις αριθμητικῆς μέθοδοις ἐπιλύουμε τὴν

$$f(x) - x = 0$$

επιλύουμε τὴν $f(x) - x = 0$ καὶ βρῶμε τὴν ἰσορροπία

$$\text{επειδὴ } f(x) - x = 0 \iff f(x) = x$$

ep̄loush ex̄dswshç eÔresh stajer,n shmeĐwn

To p̄rblhma thç ep̄loushç exis,sewn sqetĐzetai me to p̄rblhma tou upologismoÔ stajer,n shmeĐwn miac sun^rthshç.

To p̄rblhma upologismoÔ stajer,n shmeĐwn

Dedomènhc thçf : [a; b] R ! R

na brejeĐ k^poio shmeĐσ 2 (a; b) tètioio ,ste f(r) = r

ep̄loush ex̄dswshç eÔresh stajer,n shmeĐwn

me k^poia arijmhtik mètudo jèloume na epilÔsoume thn

$$f(x) = 0$$

epilÔsoume thrf (x) = g(x) x kai brĐskoume th lÔshr

$$\text{epeid } f(r) = 0 \text{) } g(r) \text{ } r = 0 \text{) } g(r) = r$$

to r eĐnai stajerì shmeĐo thçg(x)

ep̄lush ex̄swshç eÔresh stajer,n shmeĐwn

To p̄blhma thç ep̄lushç exis,sewn sqetĐzetai me to p̄blhma tou upologismoÔ stajer,n shmeĐwn miac sun^rthshç.

To p̄blhma upologismoÔ stajer,n shmeĐwn

Dedomènhc thçf : [a; b] R ! R

na brejeĐ k^poio shmeĐσ 2 (a; b) tètioio ,ste f(r) = r

ep̄lush ex̄swshç eÔresh stajer,n shmeĐwn

me k^poia arijmhtik mējodo jèloume na epilÔsoume thn

$$f(x) = 0$$

epilÔsoume thrf(x) = g(x) x kai brĐskoume th lÔshr

$$\text{epeid } f(r) = 0 \text{) } g(r) \text{ } r = 0 \text{) } g(r) = r$$

to r eĐnai stajerì shmeĐo thçg(x)

eÔresh stajer,n shmeĐwn) ep̄lush ex̄swshç

ep̄lush ex̄swshç eÔresh stajer, n shmeĐwn

To p̄blhma thç ep̄lushç exis, sewn sqetĐzetai me to p̄blhma tou upologismoÔ stajer, n shmeĐwn miac sun^rthshç.

To p̄blhma upologismoÔ stajer, n shmeĐwn

Dedomènhc thçf : [a; b] R ! R

na brejē k̄poio shmeĐσ 2 (a; b) tētoio ,ste f(r) = r

ep̄lush ex̄swshç eÔresh stajer, n shmeĐwn

me k̄poia arijmhtik mējodo jēloume na epilÔsoume thn

$$f(x) = 0$$

epilÔsoume thrf (x) = g(x) x kai br̄skoume th IÔshr

$$\text{epeid } f(r) = 0 \text{) } g(r) \text{ } r = 0 \text{) } g(r) = r$$

to r eĐnai stajerì shmeĐo thçg(x)

eÔresh stajer, n shmeĐwn) ep̄lush ex̄swshç

me k̄poia arijmhtik mējodo jēloume na broÔme èna stajerì shmeĐo gia thrg(x)

επιλύσιμη εξίσωση εὐκλείδειου σταθμίου σημείου

Το πρόβλημα της επιλύσιμης εξίσωσης σε σχέση με το πρόβλημα του υπολογισμού σταθμίου σημείου είναι συνάρτησης.

Το πρόβλημα υπολογισμού σταθμίου σημείου

Δεδομένης τριγωνομετρίας : $[a; b] \subset \mathbb{R} \rightarrow \mathbb{R}$

να βρεθεί κάποιο σημείο $\alpha \in (a; b)$ τέτοιο, ώστε $f(\alpha) = 0$

επιλύσιμη εξίσωση εὐκλείδειου σταθμίου σημείου

με κάποια αριθμητική μέθοδο θέλουμε να επιλύσουμε την

$$f(x) = 0$$

επιλύουμε την $f(x) = g(x)$ x και βρίσκουμε τη λύση

$$\text{επειδή } f(x) = 0 \iff g(x) = 0 \iff g(x) = 0$$

τότε x είναι σταθμίο σημείο της $f(x)$

εὐκλείδειου σταθμίου σημείου) επιλύσιμη εξίσωση

με κάποια αριθμητική μέθοδο θέλουμε να βρούμε ένα σταθμίο

σημείο για την $f(x)$

$$\text{ψάχνουμε } g(x) = 0$$

επιλύση εξίσωσης εὐρέως σταθερῆς συνάρτησης

Το πρόβλημα της επιλύσης εξίσωσης συνάρτησης με το πρόβλημα του υπολογισμού σταθερῆς συνάρτησης.

Το πρόβλημα υπολογισμού σταθερῆς συνάρτησης

Δεδομένης τῆς $f : [a; b] \rightarrow \mathbb{R}$

να βρεθῆ κῆποιος $r \in (a; b)$ τέτοιος, ὅστε $f(r) = r$

επιλύση εξίσωσης εὐρέως σταθερῆς συνάρτησης

με κῆποια αριθμητικὴ μέθοδο θέλουμε να ἐπιτύχουμε τὴν

$$f(x) = 0$$

ἐπιτύχουμε τῆς $f(x) = g(x)$ x καὶ βρῶσκουμε τὴν ἰσο-

$$\text{επειδὴ } f(r) = 0 \iff g(r) = r = 0 \iff g(r) = r$$

το r εἶναι σταθερὸ σημείο τῆς $g(x)$

εὐρέως σταθερῆς συνάρτησης) ἐπιλύση εξίσωσης

με κῆποια αριθμητικὴ μέθοδο θέλουμε να βροῦμε ἕνα σταθερὸ

σημείο γὰρ τῆς $g(x)$

$$y \text{ ὅπου } g(x) = x$$

$$\text{επειδὴ } g(x) = x = 0,$$

ep̄lush ex̄swshç eÔresh stajer, n shmeĐwn

To p̄blhma thç ep̄lushç exis, sewn sqetĐzetai me to p̄blhma tou upologismoÔ stajer, n shmeĐwn miac sun^rthshç.

To p̄blhma upologismoÔ stajer, n shmeĐwn

Dedomènhc thçf : [a; b] R ! R

na brejeĐ k^poio shmeĐσ 2 (a; b) tètioio ,ste f(r) = r

ep̄lush ex̄swshç eÔresh stajer, n shmeĐwn

me k^poia arijmhtik mējodo jèloume na epilÔsoume thn

$$f(x) = 0$$

epilÔsoume thrf(x) = g(x) x kai brĐskoume th lÔshr

$$\text{epeid } f(r) = 0 \text{) } g(r) \text{ } r = 0 \text{) } g(r) = r$$

to r eĐnai stajerì shmeĐo thçg(x)

eÔresh stajer, n shmeĐwn) ep̄lush ex̄swshç

me k^poia arijmhtik mējodo jèloume na broÔme èna stajerì shmeĐo gia thrg(x)

$$y^qnoime g(x) = x$$

epeid g(x) x = 0, jètoume f(x) = g(x) x,

ep̄lush ex̄swshç eÔresh stajer, n shmeĐwn

To p̄blhma thç ep̄lushç exis, sewn sqet̄zetai me to p̄blhma tou upologismoÔ stajer, n shmeĐwn miac sun^rthshç.

To p̄blhma upologismoÔ stajer, n shmeĐwn

Dedomènhc thçf : [a; b] R ! R

na brejeĐ k^poio shmeĐσ 2 (a; b) t̄toio ,ste f(r) = r

ep̄lush ex̄swshç eÔresh stajer, n shmeĐwn

me k^poia arijmhtik m̄jodo j̄lounge na epilÔsoume thn

$$f(x) = 0$$

epilÔsoume thrf(x) = g(x) x kai br̄skoume th lÔshr

$$\text{epeid } f(r) = 0 \Rightarrow g(r) = r = 0 \Rightarrow g(r) = r$$

to r eĐnai stajerì shmeĐo thçg(x)

eÔresh stajer, n shmeĐwn) ep̄lush ex̄swshç

me k^poia arijmhtik m̄jodo j̄lounge na broÔme èna stajerì shmeĐo gia thrg(x)

$$y^qnome g(x) = x$$

epeid g(x) x = 0, j̄tουμε f(x) = g(x) x, katal goume

$$f(x) = 0$$

Je_rhma ('Uparxh stajer_n shmeÐwn)

An h sun^rthsh $g : [a; b] \rightarrow \mathbb{R}$ eÐnai suneq c sto di^sthma $[a; b]$,
tite h g eÐqi èna stajerì shmeÐo sto $[a; b]$.

Je_rhma ('Uparxh stajer_n shmeĐwn)

An h sun^rthsh $g : [a; b] \rightarrow \mathbb{R}$! $[a; b]$ eĐnai suneq c sto di^sthma $[a; b]$,
tite h g eĐnei eĐna stajer_i shmeĐo sto $[a; b]$.

Je_rhma (Monadikitha stajer_n shmeĐwn)

An up^rpei h pr_th par^gwgoc $g^0(x)$ thc sun^rthshc g sto $(a; b)$ kai
up^rpei mia stajer^ $p < 1$ tetoia ,ste na isq^e

$$g^0(x) = p; \text{ gia } \forall x \in (a; b)$$

tite to stajer_i shmeĐo pou an kei sto $[a; b]$ eĐnai monadiki.

Je_rhma ('Uparxh stajer_n shmeÐwn)

An h sun^rthsh $g : [a; b] \rightarrow \mathbb{R}$! $[a; b]$ eÐnai suneq c sto di^sthma $[a; b]$,
tite h g eÐpei èna stajerì shmeÐo sto $[a; b]$.

Je_rhma (Monadikithta stajer_n shmeÐwn)

An up^rpei h pr^th par^gwgoc $g^0(x)$ thc sun^rthshc g sto $(a; b)$ kai
up^rpei mia stajer^ $p < 1$ tètota ,ste na isq^Ðei

$$g^0(x) \leq p; \quad \text{gia } \forall x \in (a; b)$$

tite to stajerì shmeÐo pou an kei sto $[a; b]$ eÐnai monadikì.

Genik epanalhptik mèjodoc

Mporo^me na proseggÐsoume èna stajerì shmeÐo thc sun^rthshc $g(x)$

Je_rhma ('Uparxh stajer_n shmeĐwn)

An h sun^rthsh $g : [a; b] \rightarrow \mathbb{R}$! $[a; b]$ eĐnai suneq c sto di^sthma $[a; b]$,
tite h g eĐnei na stajeri shmeĐo sto $[a; b]$.

Je_rhma (Monadikithta stajer_n shmeĐwn)

An up^rqi h pr_th par^gwgoc $g'(x)$ thc sun^rthshc g sto $(a; b)$ kai
up^rqi mia stajer^ $p < 1$ tetoia ξ na isq^ei

$$g'(x) = p; \quad \text{gia } \xi \in (a; b)$$

tite to stajeri shmeĐo pou an kei sto $[a; b]$ eĐnai monadiki.

Genik epanalhptik m^ejodoc

Mporo^me na prosegg^soume na stajeri shmeĐo thc sun^rthshc $g(x)$
epil^gontac mia arqik pros^gish x_0

Je_rhma ('Uparxh stajer_n shmeĐwn)

An h sun^rthsh $g : [a; b] \rightarrow \mathbb{R} ! [a; b]$ eĐnai suneq c sto di^sthma $[a; b]$, tite h g eĐei òna stajerì shmeĐo sto $[a; b]$.

Je_rhma (Monadikithta stajer_n shmeĐwn)

An up^rpei h pr^th par^gwgoc $g^0(x)$ thc sun^rthshc g sto $(a; b)$ kai up^rpei mia stajer^ $p < 1$ tètòia ,ste na isq^Ōei

$$g^0(x) \leq p; \quad \text{gia } \forall x \in (a; b)$$

tite to stajerì shmeĐo pou an kei sto $[a; b]$ eĐnai monadikì.

Genik epanalhptik mèjodoc

MporoŌme na proseggĐsoume òna stajerì shmeĐo thc sun^rthshc $g(x)$ epilègontac mia arqik prosèggish x_0 dhmiourgeĐtai mia akoloujĐa ìrwn $f x_k g_{k=0}^1$ apì th sqèsh

$$x_{k+1} = g(x_k); \quad k = 0; 1; 2; \dots$$

Prosoq

O metasqhmatic thc exÐswshc $f(x) = 0$ sth morf
 $x = g(x)$ den eÐnai monadikc

Prosoq

O metasqhmatic thc exÐswsh $\epsilon(x) = 0$ sth morf
 $x = g(x)$ den eÐnai monadik

H genik epanalhptik mètjodoc de sugklÐnei gia
opoiad pote sun \hat{r} thsh $g(x)$

Prosoq

O metasqhmatic thc exÐswshc $f(x) = 0$ sth morf $x = g(x)$ den eÐnai monadikc

H genik epanalhptik mÐjodoc de sugklÐnei gia opoiad pote sunÐrthsh $g(x)$

AitÐec apotuqÐac sÐgklshc genik c epanalhptik c mejidou

† H sunÐrthsh $g(x)$ apeirÐzetai.

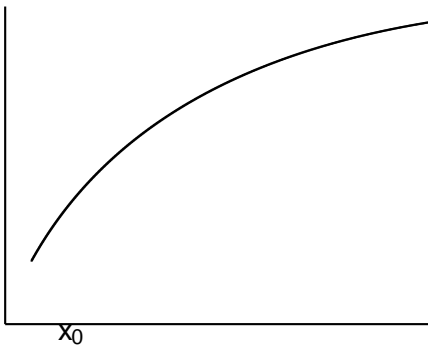
Prosoq

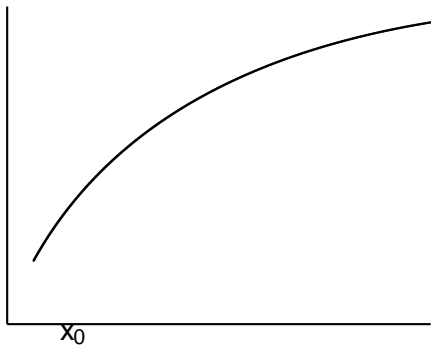
O metasqhmatic thc exÐswshc $f(x) = 0$ sth morf $x = g(x)$ den eÐnai monadikc

H genik epanalhptik mÐjodoc de sugklÐnei gia opoiad pote sunÐrthsh $g(x)$

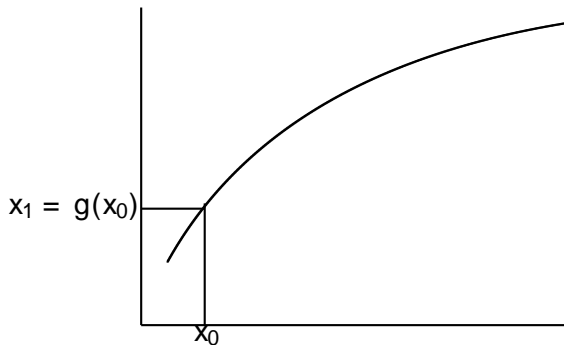
AitÐec apotuqÐac sÐgklshc genik c epanalhptik c mejidou

- 1 H sunÐrthsh $g(x)$ apeirÐzetai.
- 2 H sunÐrthsh $g(x)$ talant,netai.



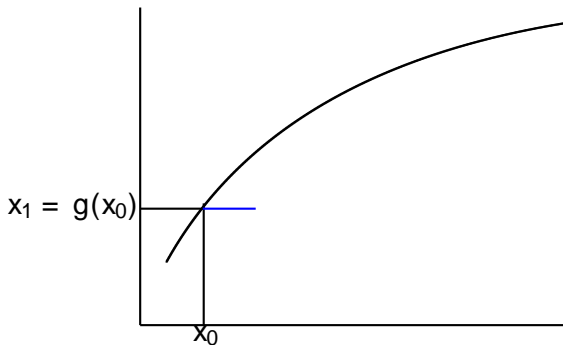


Fèroume thny = x



Fèrnoume thny = x

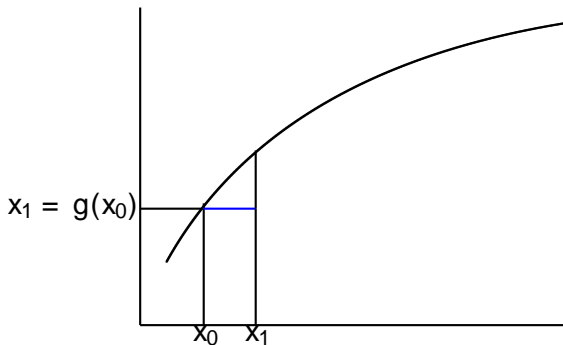
BrÐskoume ~~to~~ $g(x_0)$



Fèrnoume thny = x

BrÐskoume to $g(x_0)$

$y = x_1$ Tom $y = x$

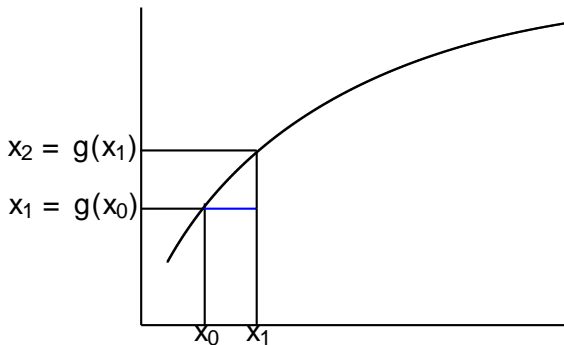


Fèroume thny = x

Brðskoume tog (x_0)

$y = x_1$ Tom $y = x$

k^jeth ston $x^0 x$



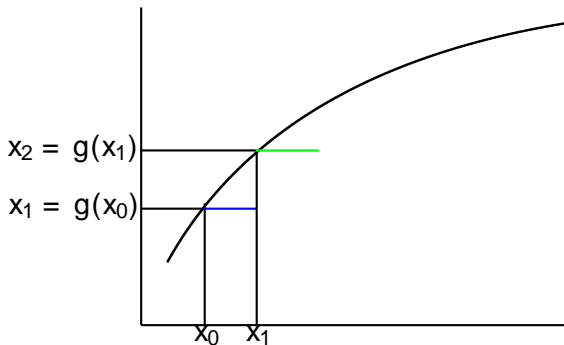
Fèrnoume thny = x

BrÐskoume to $g(x_0)$

BrÐskoume to $g(x_1)$

$y = x_1$ Tom $y = x$

k^jeth ston $x^0 x$



Fèrnoume thny = x

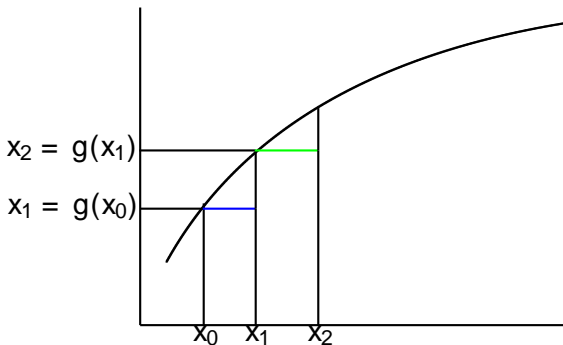
BrÐskoume to $g(x_0)$

$y = x_1$ Tom $y = x$

k[^]jeth ston $x^0 x$

BrÐskoume to $g(x_1)$

$y = x_2$ Tom $y = x$



Fèrnoume thny $y = x$

BrÐskoume $g(x_0)$

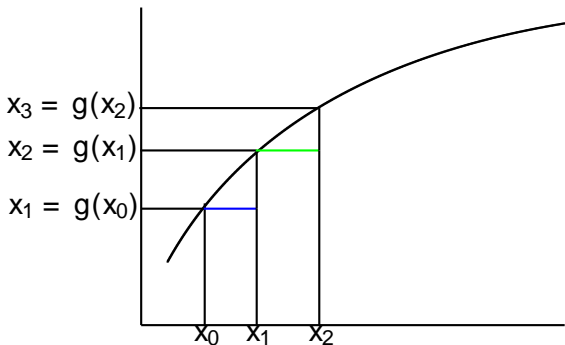
$y = x_1$ Tom $y = x$

k^jeth ston $x^0 x$

BrÐskoume $g(x_1)$

$y = x_2$ Tom $y = x$

k^jeth ston $x^0 x$



Fèrnoume thny $y = x$

BrÐskoume tog (x_0)

BrÐskoume tog (x_1)

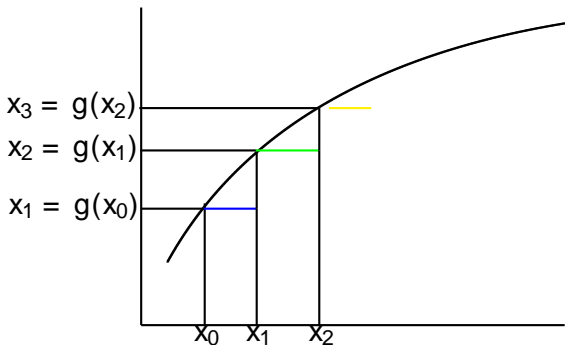
BrÐskoume tog (x_2)

$y = x_1$ Tom $y = x$

$y = x_2$ Tom $y = x$

k^jeth ston x^0x

k^jeth ston x^0x

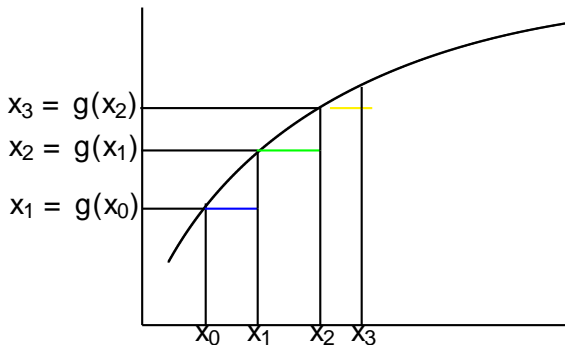


Fèrnoume thny $y = x$

BrÐskoume to $g(x_0)$
 $y = x_1$ Tom $y = x$
 k'jeth ston $x^0 x$

BrÐskoume to $g(x_1)$
 $y = x_2$ Tom $y = x$
 k'jeth ston $x^0 x$

BrÐskoume to $g(x_2)$
 $y = x_3$ Tom $y = x$



Fèrnoume thny $y = x$

BrÐskoume to $g(x_0)$
 $y = x_1$ Tom $y = x$
 k'jeth ston $x^0 x$

BrÐskoume to $g(x_1)$
 $y = x_2$ Tom $y = x$
 k'jeth ston $x^0 x$

BrÐskoume to $g(x_2)$
 $y = x_3$ Tom $y = x$
 k'jeth ston $x^0 x$

Mh sÔgklisch thc mejîdou

SÔgklisch thc mejîdou

Mh sÔgklisch thc mejîdou

H telik tim tou deĐkth epanal yewn, megalÔterh apì to dedomèno mègisto arijmì epanal yewn

SÔgklisch thc mejîdou

Mh sÔgklîsh thc mejîdou

H telik tim tou deĐkth epanal yewn, megalÔterh apî to dedomèno mègisto arijmî epanal yewn

SÔgklîsh thc mejîdou

DÔo diadoqikèc proseggĐseic, kont[^] (akrĐbeia) h mĐa sthn[^]llh

$$jx_n \quad x_{n-1} <$$

Mh sŌgklisch thc mej̀idou

H telik tim tou deĐkth epanal yewn, megalŌterh apì to dedomèno mègisto arijmì epanal yewn

SŌgklisch thc mej̀idou

DŌo diadoqikèc proseggĐseic, kontˆ (akrĐbeia) h mĐa sthn
ˆllh

$$jx_n - x_{n-1}j <$$

H tim thc $f(x)$ kontˆ sto 0

$$jf(x_n)j <$$

Mh sÖgklisch thc mejìdou

H telik tim tou deÐkth epanal yewn, megalÔterh apì to dedomèno mègisto arijmì epanal yewn

SÖgklisch thc mejìdou

DÔo diadoqikèc proseggÐseic, kont^ (akrÐbeia) h mÐa sthn
^llh

$$|x_n - x_{n-1}| <$$

H tim thc $f(x)$ kont^ sto 0

$$|f(x_n)| <$$

Sqetikì sf^lma mikrìtero apì thn akrÐbeia

$$\frac{|x_n - x_{n-1}|}{|x_n|} < \epsilon, x_n \neq 0$$

Mh sÖgklisch thc mejìdou

H telik tim tou deÐkth epanal yewn, megalÔterh apì to dedomèno mègisto arijmì epanal yewn

SÖgklisch thc mejìdou

DÔo diadoqikèc proseggÐseic, kont^ (akrÐbeia) h mÐa sthn
^llh

$$|x_n - x_{n-1}| <$$

H tim thc $f(x)$ kont^ sto 0

$$|f(x_n)| <$$

Sqetikì sf^lma mikrìtero apì thn akrÐbeia

$$\frac{|x_n - x_{n-1}|}{|x_n|} < \epsilon, x_n \neq 0$$

Kajorismic eisidwn mejidou

Kajorismic eisidwn mejidou
g o tÔpoc thc sun^rthshc

Kajorismic eisidwn mejidou
g o tÔpoc thc sun^rthshc
x₀ h pr_,th prosèggish thc rÐzac

Kajorismic eisidwn mejidou

g o tÔpoc thc sun^rthshc

x₀ h pr,th prosèggish thc rÐzac

MIT to mègisto pl joc epanal yewn thc mejidou

Kajorismic eisidwn mejidou
g o tÔpoc thc sun^rthshc
x₀ h pr₁th prosèggish thc rÐzac
MIT to mègisto pl joc epanal yewn thc mejidou
h akrÐbeia pou jètoume

Kajorismic eisidwn mejidou

g o tÔpoc thc sun^rthshc

x₀ h pr,th prosèggish thc rÐzac

MIT to mègeisto pl joc epanal yewn thc mejidou

h akrÐbeia pou jètoume

Jètoume k = 1 kai phgaÐnoume sto epimeno b ma

Kajorismic eisidwn mejidou

g o tÔpoc thc sun^rthshc

x_0 h pr,th prosèggish thc rÐzac

MIT to mègisto pl joc epanal yewn thc mejidou

h akrÐbeia pou jètoume

Jètoume $k = 1$ kai phgaÐnoume sto epimeno b ma

Anane,noume tok ($k = k + 1$) kai phgaÐnoume sto epimeno
b ma

Kajorismic eisidwn mejidou

g o tÔpoc thc sun^rthshc

x_0 h pr,th prosèggish thc rÐzac

MIT to mègisto pl joc epanal yewn thc mejidou

h akrÐbeia pou jètoume

Jètoume $k = 1$ kai phgaÐnoume sto epimeno b ma

Anane,noume tok ($k = k + 1$) kai phgaÐnoume sto epimeno

b ma

Elègqoume $ank > MIT$

Kajorismic eisidwn mejidou

g o tÔpoc thc sun[^]rthshc

x_0 h pr₃th prosèggish thc rÐzac

MIT to mègisto pl joc epanal yewn thc mejidou

h akrÐbeia pou jètoume

Jètoume $k = 1$ kai phgaÐnoume sto epimeno b ma

Anane₃ noume tok ($k = k + 1$) kai phgaÐnoume sto epimeno
 b ma

Elèggoume $ank > MIT$

An den isqÔei, jètoume $x_{k+1} = g(x_k)$ kai suneqÐzoume sto
epimeno b ma

Kajorismic eisidwn mejidou

g o tÔpoc thc sun^rthshc

x_0 h pr,th prosèggish thc rÐzac

MIT to mègisto pl joc epanal yewn thc mejidou

h akrÐbeia pou jètoume

Jètoume $k = 1$ kai phgaÐnoume sto epimeno b ma

Anane,noume tok ($k = k + 1$) kai phgaÐnoume sto epimeno
 b ma

Elèggoume $ank > MIT$

An den isqÔei, jètoume $x_{k+1} = g(x_k)$ kai suneqÐzoume sto
epimeno b ma

An isqÔei phgaÐnoume sto b ma 6

Kajorismic eisidwn mejidou

g o t \hat{O} poc thc sun \hat{r} thshc

x_0 h pr \hat{t} h pros \hat{e} ggish thc r \hat{D} zac

MIT to m \hat{e} gisto pl joc epanal yewn thc mejidou

h akr \hat{D} beia pou j \hat{e} toume

J \hat{e} toume $k = 1$ kai phga \hat{D} noume sto ep \hat{i} meno b ma

Anane \hat{t} , noume tok ($k = k + 1$) kai phga \hat{D} noume sto ep \hat{i} meno
 b ma

El \hat{e} ggoume $ank > MIT$

An den isq \hat{O} ei, j \hat{e} toume $x_{k+1} = g(x_k)$ kai suneq \hat{D} zoume sto
ep \hat{i} meno b ma

An isq \hat{O} ei phga \hat{D} noume sto b ma 6

El \hat{e} ggoume \hat{e} na periss \hat{i} tera krit ria termatismo \hat{O} (\hat{e} stw
 $|x_{k+1} - x_k|$)

Kajorismic eisidwn mejidou

g o t \hat{O} poc thc sun \hat{r} thshc

x_0 h pr \hat{t} h pros \hat{e} ggish thc r \hat{D} zac

MIT to m \hat{e} gisto pl joc epanal yewn thc mejidou

h akr \hat{D} beia pou j \hat{e} toume

J \hat{e} toume $k = 1$ kai phga \hat{D} noume sto ep \hat{i} meno b ma

Anane \hat{t} , noume tok ($k = k + 1$) kai phga \hat{D} noume sto ep \hat{i} meno
b ma

El \hat{e} ggoume $ank > MIT$

An den isq \hat{O} ei, j \hat{e} toume $x_{k+1} = g(x_k)$ kai suneq \hat{D} zoume sto
ep \hat{i} meno b ma

An isq \hat{O} ei phga \hat{D} noume sto b ma 6

El \hat{e} ggoume \hat{e} na periss \hat{i} tera krit ria termatismo \hat{O} (\hat{e} stw
 $jx_{k+1} - x_kj$)

An den isq \hat{O} oun, suneq \hat{D} zoume sto b ma 3

Kajorismic eisidwn mejidou

g o t \hat{O} poc thc sun \hat{r} thshc

x_0 h pr $\hat{,}$ th pros \hat{e} ggish thc r \hat{D} zac

MIT to m \hat{e} gisto pl joc epanal yewn thc mejidou

h akr \hat{D} beia pou j \hat{e} toume

J \hat{e} toume $k = 1$ kai phga \hat{D} noume sto ep \hat{i} meno b ma

Anane $\hat{,}$ noume tok ($k = k + 1$) kai phga \hat{D} noume sto ep \hat{i} meno
b ma

El \hat{e} ggoume $ank > MIT$

An den isq \hat{O} ei, j \hat{e} toume $x_{k+1} = g(x_k)$ kai suneq \hat{D} zoume sto
ep \hat{i} meno b ma

An isq \hat{O} ei phga \hat{D} noume sto b ma 6

El \hat{e} ggoume \hat{e} na periss \hat{i} tera krit ria termatismo \hat{O} (\hat{e} stw
 $jx_{k+1} - x_kj$)

An den isq \hat{O} oun, suneq \hat{D} zoume sto b ma 3

An isq \hat{O} oun phga \hat{D} noume sto b ma 6

Kajorismic eisidwn mejidou

g o t \hat{O} poc thc sun \hat{r} thshc

x_0 h pr \hat{t} h pros \hat{e} ggish thc r \hat{D} zac

MIT to m \hat{e} gisto pl joc epanal yewn thc mejidou

h akr \hat{D} beia pou j \hat{e} toume

J \hat{e} toume $k = 1$ kai phga \hat{D} noume sto epimeno b ma

Anane \hat{t} , noume tok ($k = k + 1$) kai phga \hat{D} noume sto epimeno

b ma

El \hat{e} ggoume ank $>$ MIT

An den isq \hat{O} ei, j \hat{e} toume $x_{k+1} = g(x_k)$ kai suneq \hat{D} zoume sto epimeno b ma

An isq \hat{O} ei phga \hat{D} noume sto b ma 6

El \hat{e} ggoume ϵ na periss \hat{i} tera krit ria termatismo \hat{O} (ϵ stw

$|x_{k+1} - x_k|$)

An den isq \hat{O} oun, suneq \hat{D} zoume sto b ma 3

An isq \hat{O} oun phga \hat{D} noume sto b ma 6

Teratismic mejidou kai epistrof exidwn

Kajorismic eisidwn mejidou

g o t \hat{O} poc thc sun \hat{r} thshc

x_0 h pr $_j$ th pros \hat{e} ggish thc r \hat{D} zac

MIT to m \hat{e} gisto pl joc epanal yewn thc mejidou

h akr \hat{D} beia pou j \hat{e} toume

J \hat{e} toume $k = 1$ kai phga \hat{D} noume sto epimeno b ma

Anane $_j$, noume tok ($k = k + 1$) kai phga \hat{D} noume sto epimeno b ma

El \hat{e} ggoume ank $>$ MIT

An den isq \hat{O} ei, j \hat{e} toume $x_{k+1} = g(x_k)$ kai suneq \hat{D} zoume sto epimeno b ma

An isq \hat{O} ei phga \hat{D} noume sto b ma 6

El \hat{e} ggoume ϵ na periss \hat{i} tera krit ria termatismo \hat{O} (ϵ stw $|x_{k+1} - x_k|$)

An den isq \hat{O} oun, suneq \hat{D} zoume sto b ma 3

An isq \hat{O} oun phga \hat{D} noume sto b ma 6

Termatismic mejidou kai epistrof exidwn

x_k h teleuta \hat{D} a pros \hat{e} ggish thc r \hat{D} zac

Kajorismic eisidwn mejidou

g o t \hat{O} poc thc sun \hat{r} thshc

x_0 h pr \hat{t} h pros \hat{e} ggish thc r \hat{D} zac

MIT to m \hat{e} gisto pl joc epanal yewn thc mejidou

h akr \hat{D} beia pou j \hat{e} toume

J \hat{e} toume $k = 1$ kai phga \hat{D} noume sto ep \hat{i} meno b ma

Anane \hat{t} , noume tok ($k = k + 1$) kai phga \hat{D} noume sto ep \hat{i} meno
b ma

El \hat{e} ggoume ank $>$ MIT

An den isq \hat{O} ei, j \hat{e} toume $x_{k+1} = g(x_k)$ kai suneq \hat{D} zoume sto
ep \hat{i} meno b ma

An isq \hat{O} ei phga \hat{D} noume sto b ma 6

El \hat{e} ggoume \hat{e} na periss \hat{i} tera krit ria termatismo \hat{O} (\hat{e} stw
 $|x_{k+1} - x_k|$)

An den isq \hat{O} oun, suneq \hat{D} zoume sto b ma 3

An isq \hat{O} oun phga \hat{D} noume sto b ma 6

Teratismic mejidou kai epistrof exidwn

x_k h teleuta \hat{D} a pros \hat{e} ggish thc r \hat{D} zac

$g(x_k)$ h tim thc sun \hat{r} thshc gia thn pros \hat{e} ggish

Kajorismic eisidwn mejidou

g o t \hat{O} poc thc sun \hat{r} thshc

x_0 h pr $_t$ th pros \hat{e} ggish thc r \hat{D} zac

MIT to m \hat{e} gisto pl joc epanal yewn thc mejidou

h akr \hat{D} beia pou j \hat{e} toume

J \hat{e} toume $k = 1$ kai phga \hat{D} noume sto ep \hat{i} meno b ma

Anane $_j$ noume tok ($k = k + 1$) kai phga \hat{D} noume sto ep \hat{i} meno
b ma

El \hat{e} ggoume ank $>$ MIT

An den isq \hat{O} ei, j \hat{e} toume $x_{k+1} = g(x_k)$ kai suneq \hat{D} zoume sto
ep \hat{i} meno b ma

An isq \hat{O} ei phga \hat{D} noume sto b ma 6

El \hat{e} ggoume \hat{e} na periss \hat{i} tera krit ria termatismo \hat{O} (\hat{e} stw
 $jx_{k+1} \quad x_k$)

An den isq \hat{O} oun, suneq \hat{D} zoume sto b ma 3

An isq \hat{O} oun phga \hat{D} noume sto b ma 6

Teratismic mejidou kai epistrof exidwn

x_k h teleuta \hat{D} a pros \hat{e} ggish thc r \hat{D} zac

$g(x_k)$ h tim thc sun \hat{r} thshc gia thn pros \hat{e} ggish

k to pl joc epanal yewn pou \hat{e} ginan

Anag̃goun to prìblhma thc ep̃lushc enìc ñ dī̃statou
mh-grammikõ̂ sust matoc se aplõ̂sterec mh-grammikèc exis, seic
miac metablht c

Anag̃goun to pr̃blhma thc ep̃lushc enìc n dī̃statou
mh-grammikõ sust matoc se aplõsterc mh-grammikèc exis, seic
miac metablht c

Proèrqontai apì th grammik Gauss-Seidèlh opõDa upolog̃zei thn
ep̃imenh sunist,sa x_i^{k+1} , epil̃ontac wc proc x_i th grammik ex̃dswsh

$$\sum_{j=1}^{X-1} a_{ij} x_j^{k+1} + a_{ii} x_i + \sum_{j=i+1}^X a_{ij} x_j^k = b_i = 0$$

Anagoun to prblhma thc epdlushc enic n di^statou mh-grammiko^ sust matoc se aplo^sterec mh-grammik^c exis, seic miac metablht c

Pro^rqontai ap^ th grammik Gauss-Seidelh opo^a upolog^zei thn epimenh sunist,sa x_i^{k+1} , epil^ontac wc proc x_i th grammik ex^swsh

$$x_i^{k+1} = \frac{1}{a_{ii}} \left(b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{k+1} - \sum_{j=i+1}^n a_{ij} x_j^k \right)$$

H parap^nw diadikas^a mpore^ na efarmoste^ kai sthn per^ptwsh thc arijmhtik c epdlushc susthm^twn mh-grammik, n exis, sewn

Anagoun to prblhma thc epdlushc enic n di^statou mh-grammiko. Sust matoc se aplo^sterec mh-grammikèc exis, seic miac metablht c

Proèrqontai apì th grammik Gauss-Seidelh opoDa upologDzei thn epimenh sunist,sa x_i^{k+1} , epil^ontac wc proc x_i th grammik exDswsh

$$\sum_{j=1}^{i-1} a_{ij} x_j^{k+1} + a_{ii} x_i + \sum_{j=i+1}^n a_{ij} x_j^k = b_i = 0$$

H parap^nw diadikasDa mporeD na efarmosteD kai sthn perDptwsh thc arijmhtik c epdlushc susthm^twn mh-grammik, n exis, sewn

Se aut thn perDptwsh, to x_i^{k+1} epitugq^netai l^ontac th mh-grammik exDswsh miac metablht c

$$f_i(x_1^{k+1}; x_2^{k+1}; \dots; x_{i-1}^{k+1}; x_i; x_{i+1}^k; \dots; x_n^k) = 0$$

ìpou **metablhtèc** **stajerèc**

Anagoun to prblhma thc epdlushc enic n di^statou mh-grammiko^o sust matoc se aplo^osterec mh-grammik^ec exis, seic miac metablht c

Proerqontai apⁱ th grammik Gauss-Seidelh opo^oDa upolog^oDzei thn epⁱmenh sunist, sa x_i^{k+1} , epil^oontac wc proc x_i th grammik ex^oDswsh

$$\sum_{j=1}^{X^1} a_{ij} x_j^{k+1} + \sum_{j=i+1}^{X^n} a_{ij} x_j^k - b_i = 0$$

H parap^onw diadikas^oDa mpore^oD na efarmoste^oD kai sthn per^oDptwsh thc arijmhtik c epdlushc susthm^otwn mh-grammik, n exis, sewn

Se aut thn per^oDptwsh, to x_i^{k+1} epitug^onetai I^oontac th mh-grammik ex^oDswsh miac metablht c

$$f_i(x_1^{k+1}; x_2^{k+1}; \dots; x_{i-1}^{k+1}; x_i^k; x_{i+1}^k; \dots; x_n^k) = 0$$

ipou **metablht^ec stajer^ec**

An h I^osh thc parap^onw sqeshc e^oDnai h x_i , j^etoume

$$x_i^{k+1} = x_i$$

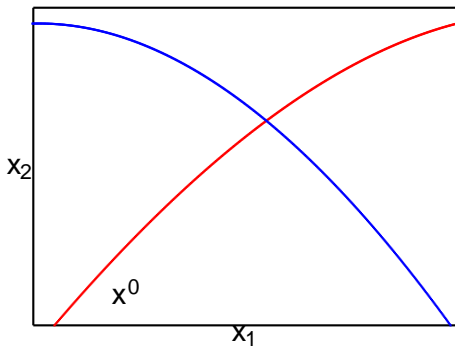
Jewr,ntac kai mia **par^metro qal^rwhc**, to epanalhptikì sq ma gÐnetai

$$x_i^{k+1} = x_i^k + !_k (\mathbb{A}_i \quad x_i^k)$$

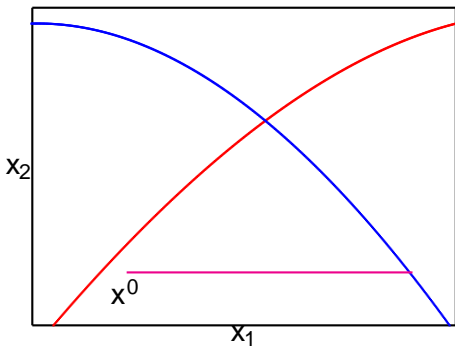
Jewr,ntac kai mia **parâmetro qalârwshc**, to epanalhptikì sq ma gÐnetai

$$x_i^{k+1} = x_i^k + !_k(x_i - x_i^k)$$

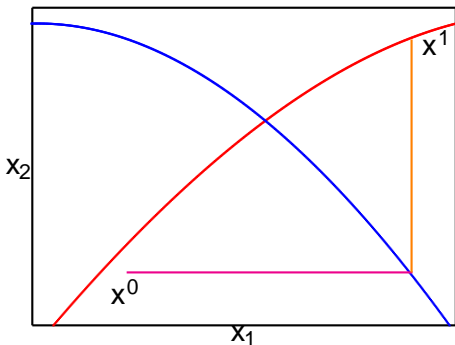
H parapânw mèjodoc èqei nihma mìnò ìtan oi exis,seic f_i èqoun monadikèc IÔseic sth dosmèn h perioq



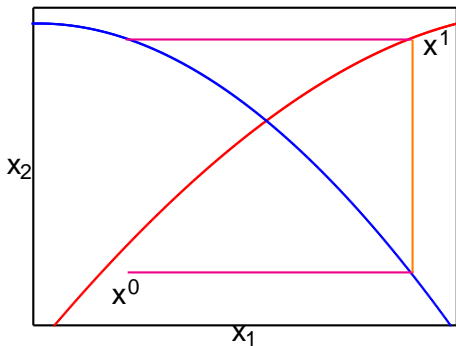
x^0	



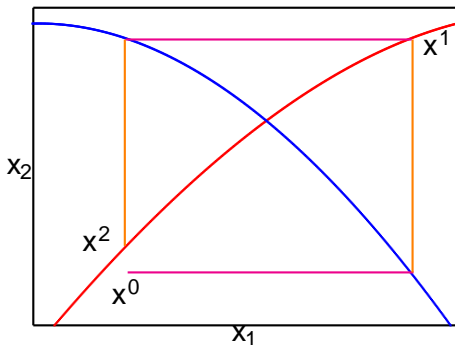
x^0 krat me x_2 stajeri



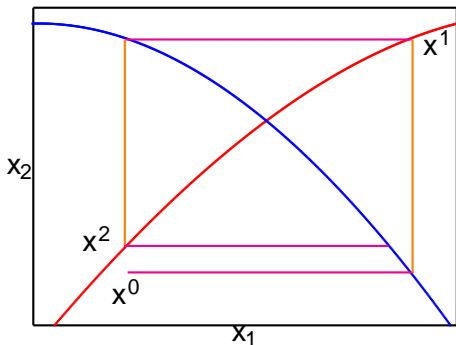
x^0 krat'me x_2 stajeri	krat'me x_1 stajeri	EÔreshx ¹



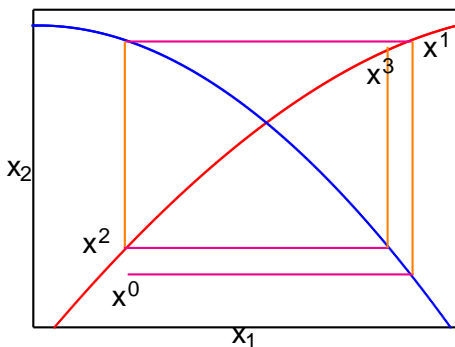
x^0	kratme x_2 stajeri	kratme x_1 stajeri	EÛresh x^1
x^1	kratme x_2 stajeri		



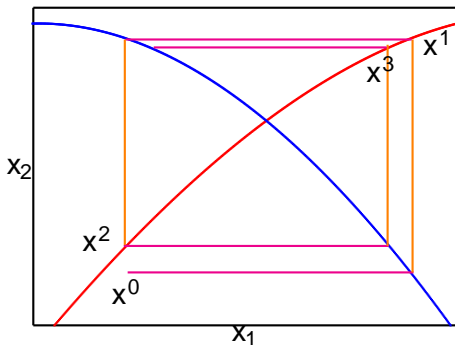
x^0	krat ^{me} x_2 stajeri	krat ^{me} x_1 stajeri	$E\hat{O}reshx^1$
x^1	krat ^{me} x_2 stajeri	krat ^{me} x_1 stajeri	$E\hat{O}reshx^2$



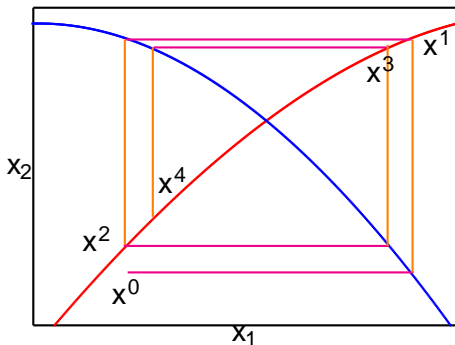
x^0	krat ^{me} x_2 stajeri	krat ^{me} x_1 stajeri	$E\hat{O}reshx^1$
x^1	krat ^{me} x_2 stajeri	krat ^{me} x_1 stajeri	$E\hat{O}reshx^2$
x^2	krat ^{me} x_2 stajeri		



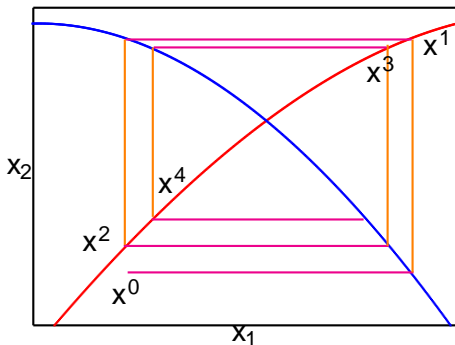
x^0	krat ^{me} x_2 stajeri	krat ^{me} x_1 stajeri	$E\hat{O}reshx^1$
x^1	krat ^{me} x_2 stajeri	krat ^{me} x_1 stajeri	$E\hat{O}reshx^2$
x^2	krat ^{me} x_2 stajeri	krat ^{me} x_1 stajeri	$E\hat{O}reshx^3$



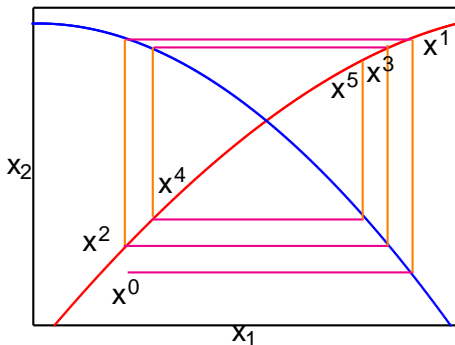
x^0	krat ^{me} x_2 stajeri	krat ^{me} x_1 stajeri	$E\hat{O}reshx^1$
x^1	krat ^{me} x_2 stajeri	krat ^{me} x_1 stajeri	$E\hat{O}reshx^2$
x^2	krat ^{me} x_2 stajeri	krat ^{me} x_1 stajeri	$E\hat{O}reshx^3$
x^3	krat ^{me} x_2 stajeri		



x^0	krat ^{me} x_2 stajeri	krat ^{me} x_1 stajeri	$E\hat{O}reshx^1$
x^1	krat ^{me} x_2 stajeri	krat ^{me} x_1 stajeri	$E\hat{O}reshx^2$
x^2	krat ^{me} x_2 stajeri	krat ^{me} x_1 stajeri	$E\hat{O}reshx^3$
x^3	krat ^{me} x_2 stajeri	krat ^{me} x_1 stajeri	$E\hat{O}reshx^4$



x^0	krat ^{me} x_2 stajeri	krat ^{me} x_1 stajeri	$E\hat{O}reshx^1$
x^1	krat ^{me} x_2 stajeri	krat ^{me} x_1 stajeri	$E\hat{O}reshx^2$
x^2	krat ^{me} x_2 stajeri	krat ^{me} x_1 stajeri	$E\hat{O}reshx^3$
x^3	krat ^{me} x_2 stajeri	krat ^{me} x_1 stajeri	$E\hat{O}reshx^4$
x^4	krat ^{me} x_2 stajeri		



x^0	krat ^{me} x_2 stajeri	krat ^{me} x_1 stajeri	$E\hat{O}reshx^1$
x^1	krat ^{me} x_2 stajeri	krat ^{me} x_1 stajeri	$E\hat{O}reshx^2$
x^2	krat ^{me} x_2 stajeri	krat ^{me} x_1 stajeri	$E\hat{O}reshx^3$
x^3	krat ^{me} x_2 stajeri	krat ^{me} x_1 stajeri	$E\hat{O}reshx^4$
x^4	krat ^{me} x_2 stajeri	krat ^{me} x_1 stajeri	$E\hat{O}reshx^5$

SuneqDzoume thn Ddia diadikasDa mèqri na broÔme èna
shmeDo pou proseggDzei th lÔsh = $(x_1; x_2)$

SuneqDzoume thn Ddia diadikasDa mèqri na broÔme èna
shmeDo pou proseggDzei th IÔsh = ($x_1; x_2$)

Stamat^me ìtan plhreDtai èna perissìtera krit ria
termatismoÔ

Mh sŌgklisch thc mejidou

SŌgklisch thc mejidou

Mh sŌgklish thc mejìdou

H telik tim tou deĐkth epanal yewn, megalŌterh apì to dedomèno mègisto arijmì epanal yewn

SŌgklish thc mejìdou

Mh sÔgklisch thc mejìdou

H telik tim tou deÐkth epanal yewn, megalÔterh apì to dedomèno mègisto arijmì epanal yewn

SÔgklisch thc mejìdou

DÔo diadoqikèc proseggÐseic, kont^ (akrÐbeia) h mÐa sthn
^llh

$$kx^{k+1} \quad x^k k \quad 2$$

Mh sÔgklisch thc mejìdou

H telik tim tou deÐkth epanal yewn, megalÔterh apì to dedomèno mègisto arijmì epanal yewn

SÔgklisch thc mejìdou

DÔo diadoqikèc proseggÐseic, kont^ (akrÐbeia) h mÐa sthn
^llh

$$kx^{k+1} \quad x^k k \quad 2$$

H nirma thc F_n kont^ sto 0

$$kF_n(x^{k+1})k \quad 1$$

Mh sOgklish thc mejidou

H telik tim tou deDkth epanal yewn, megalOterh apì to dedomèno mègisto arijmì epanal yewn

SOgklish thc mejidou

DÔo diadoqikèc proseggDseic, kont^ (akrDbeia) h mDa sthn
^llh

$$kx^{k+1} \quad x^k k \quad 2$$

H nirma thc F_n kont^ sto 0

$$kF_n(x^{k+1})k \quad 1$$

Prosoq

An^loga me th fOsh thc sun^rthshc, mporeD na isqOei k^poio krit rio termatismoO kai h mèjodoc na apoklDnei

Eḏsodoi-'Exodoi

Eḏsodoi

EĐsodoi-'Exodoi

EĐsodoi

n to pl joc tw_n exis₃sewn kai tw_n agn₃stwn

Eðsodoi-'Exodoi

Eðsodoi

n to pl joc tw n exis₃sewn kai tw n agn₃stwn
 $F_n(x)$ o tÔpoc thc sun[^]rthshc

EĐsodoi-'Exodoi

EĐsodoi

n to pl joc twñ exis₃sewn kai twñ agn₃stwn

$F_n(x)$ o tÔpoc thc sun^rthshc

x_0 h arqik prosèggish thc rĐzac

Eðsodoi-'Exodoi

Eðsodoi

n to pl joc tw n exis₃sewn kai tw n agn₃stwn

$F_n(x)$ o tÔpoc thc sun^rthshc

x_0 h arqik prosèggish thc rÐzac

! _k oi par^metroi qal^rwshe

EĐsodoi-'Exodoi

EĐsodoi

n to pl joc tw_n exis₃sewn kai tw_n agn₃stwn

$F_n(x)$ o tÔpoc thc sun^rthshc

x_0 h arqik prosèggish thc rĐzac

! _k oi par^metroi qal^rwshe

MIT to mègisto pl joc epanal yewn thc mejìdou

EĐsodoi-'Exodoi

EĐsodoi

n to pl joc tw_n exis₃sewn kai tw_n agn₃stwn

$F_n(x)$ o tÔpoc thc sun^rthshc

x_0 h arqik prosèggish thc rĐzac

! _k oi par^metroi qal^rwshe

MIT to mègisto pl joc epanal yewn thc mejìdou

₁ h akrĐbeia pou jètoume gia tox

EḐsodoi-'Exodoi

EḐsodoi

n to pl joc tw_n exis₃sewn kai tw_n agn₃stwn

$F_n(x)$ o tÔpoc thc sun^rthshc

x_0 h arqik prosèggish thc rḐzac

! _k oi par^metroi qal^rwshe

MIT to mègisto pl joc epanal yewn thc mejìdou

1 h akrḐbeia pou jètoume gia tox

2 h akrḐbeia pou jètoume gia thn $F_n(x)$

Eðsodoi-'Exodoi

Eðsodoi

n to pl joc tw_n exis₃sewn kai tw_n agn₃stwn

$F_n(x)$ o tÔpoc thc sun[^]rthshc

x_0 h arqik prosèggish thc rÐzac

! _k oi par[^]metroi qal[^]rwshe

MIT to mègisto pl joc epanal yewn thc mejìdou

₁ h akrÐbeia pou jètoume gia tox

₂ h akrÐbeia pou jètoume gia thn $F_n(x)$

'Exodoi

Eðsodoi-'Exodoi

Eðsodoi

n to pl joc tw_n exis₃sewn kai tw_n agn₃stwn

$F_n(x)$ o tÔpoc thc sun[^]rthshc

x_0 h arqik prosèggish thc rÐzac

! _k oi par[^]metroi qal[^]rwhc

MIT to mègisto pl joc epanal yewn thc mejìdou

₁ h akrÐbeia pou jètoume gia tox

₂ h akrÐbeia pou jètoume gia thn $F_n(x)$

'Exodoi

x^k h teleutaÐa prosèggish thc rÐzac

EĐsodoi-'Exodoi

EĐsodoi

n to pl joc tw_n exis₃sewn kai tw_n agn₃stwn

$F_n(x)$ o tÔpoc thc sun^rthshc

x_0 h arqik prosèggish thc rĐzac

! _k oi par^metroi qal^rwshe

MIT to mègisto pl joc epanal yewn thc mejidou

₁ h akrĐbeia pou jètoume gia tox

₂ h akrĐbeia pou jètoume gia thn $F_n(x)$

'Exodoi

x^k h teleutaĐa prosèggish thc rĐzac

$F_n(x^k)$ h tim thc sun^rthshc sthn _k epan^lhyh

EĐsodoi-'Exodoi

EĐsodoi

n to pl joc tw_n exis₃sewn kai tw_n agn₃stwn

$F_n(x)$ o tÔpoc thc sun^rthshc

x_0 h arqik prosèggish thc rĐzac

! _k oi par^metroi qal^rwshe

MIT to mègisto pl joc epanal yewn thc mejidou

₁ h akrĐbeia pou jètoume gia tox

₂ h akrĐbeia pou jètoume gia thn $F_n(x)$

'Exodoi

x^k h teleutaĐa prosèggish thc rĐzac

$F_n(x^k)$ h tim thc sun^rthshc sthn _k epan^lhyh

_k to pl joc epanal yewn pou èginan

Kajorismic eisidwn mejidou

Kajorismic eisidwn mejidou

Jètoume $k = 1$

Kajorismic eisidwn mejidou

Jètoume $k = 1$

Elègqoume $ank < MIT$

An den isqÔei, phgaÐnoume sto b ma 9. Diaforetik[^], aux[^]noume to k kai suneqÐzoume sto eplomeno b ma

Kajorismic eisidwn mejidou

Jètoume $k = 1$

Elègqoume $ank < MIT$

An den isqÔei, phgaÐnoume sto b ma 9. Diaforetik[^], aux[^]noume to k kai suneqÐzoume sto eplomeno b ma

Elègqoume $ankF_n(x^k)k \quad 2$

An den isqÔei, suneqÐzoume sto epimeno b ma. Diaforetik[^], phgaÐnoume sto b ma 9

Kajorismic eisidwn mejidou

Jètoume $k = 1$

Elèqqoume $\text{ank} < \text{MIT}$

An den isqÔei, phgaÐnoume sto b ma 9. Diaforetik[^], aux[^]noume to k kai suneqÐzoume sto eplomeno b ma

Elèqqoume $\text{ank} F_n(x^k)k \quad 2$

An den isqÔei, suneqÐzoume sto epimeno b ma. Diaforetik[^], phgaÐnoume sto b ma 9

Jètoume $i = 0$

Kajorismic eisidwn mejidou

Jètoume $k = 1$

Elègqoume $ank < MIT$

An den isqÔei, phgaÐnoume sto b ma 9. Diaforetik[^], aux[^]noume to k kai suneqÐzoume sto eplomeno b ma

Elègqoume $ankF_n(x^k)k \quad 2$

An den isqÔei, suneqÐzoume sto epimeno b ma. Diaforetik[^], phgaÐnoume sto b ma 9

Jètoume $i = 0$

An isqÔeii $< n$, aux[^]noume to i kai phgaÐnoume sto epimeno b ma. Diaforetik[^], phgaÐnoume sto b ma 8

Kajorismic eisidwn mejidou

Jètoume $k = 1$

Elèqqoume $ank < MIT$

An den isqÔei, phgaÐnoume sto b ma 9. Diaforetik[^], aux[^] noume to k kai suneqÐzoume sto eplomeno b ma

Elèqqoume $ankF_n(x^k)k \quad 2$

An den isqÔei, suneqÐzoume sto epimeno b ma. Diaforetik[^], phgaÐnoume sto b ma 9

Jètoume $i = 0$

An isqÔeii $< n$, aux[^] noume to i kai phgaÐnoume sto epimeno b ma. Diaforetik[^], phgaÐnoume sto b ma 8

BrÐskoume th IÔsh[^] thc monodi[^] stathc mh-grammik c exÐswshc
 $f_i(x_1^{k+1}; x_2^{k+1}; \dots; x_{i-1}^{k+1}; x; x_{i+1}^{k+1}; \dots; x_n^{k+1}) = 0$ wc proc x , jètoume
 $x_i^{k+1} = x_i^k + !_k(x_i \quad x_i^k)$ kai p gaine sto b ma 6

Kajorismic eisidwn mejidou

Jètoume $k = 1$

Elèqqoume ank $< MIT$

An den isqÔei, phgaÐnoume sto b ma 9. Diaforetik[^], aux[^] noume to k kai suneqÐzoume sto eplomeno b ma

Elèqqoume ank $F_n(x^k)k \quad 2$

An den isqÔei, suneqÐzoume sto epimeno b ma. Diaforetik[^], phgaÐnoume sto b ma 9

Jètoume $i = 0$

An isqÔeii $< n$, aux[^] noume to i kai phgaÐnoume sto epimeno b ma. Diaforetik[^], phgaÐnoume sto b ma 8

BrÐskoume th IÔsh[^] thc monodi[^] stathc mh-grammik c exÐswshc $f_i(x_1^{k+1}; x_2^{k+1}; \dots; x_{i-1}^{k+1}; x; x_{i+1}^{k+1}; \dots; x_n^{k+1}) = 0$ wc proc x , jètoume $x_i^{k+1} = x_i^k + !_k(x_i^k)$ kai p gaine sto b ma 6

Elèqqoume ank $x_i^{k+1} \quad x_i^k \quad 1$

An den isqÔei, suneqÐzoume sto b ma 3. Diaforetik[^], phgaÐnoume sto b ma 9

Kajorismic eisidwn mejidou

Jètoume $k = 1$

Elèqqoume $ank < MIT$

An den isqÔei, phgaÐnoume sto b ma 9. Diaforetik[^], aux[^] noume to k kai suneqÐzoume sto eplomeno b ma

Elèqqoume $ank F_n(x^k)k \quad 2$

An den isqÔei, suneqÐzoume sto epimeno b ma. Diaforetik[^], phgaÐnoume sto b ma 9

Jètoume $i = 0$

An isqÔeii $< n$, aux[^] noume to i kai phgaÐnoume sto epimeno b ma. Diaforetik[^], phgaÐnoume sto b ma 8

BrÐskoume th lÔsh[^] thc monodi[^] stathc mh-grammik c exÐswshc $f_i(x_1^{k+1}; x_2^{k+1}; \dots; x_{i-1}^{k+1}; x; x_{i+1}^{k+1}; \dots; x_n^{k+1}) = 0$ wc proc x , jètoume $x_i^{k+1} = x_i^k + !_k(x_i^k)$ kai p gaine sto b ma 6

Elèqqoume $ank x_i^{k+1} \quad x_i^k k \quad 1$

An den isqÔei, suneqÐzoume sto b ma 3. Diaforetik[^], phgaÐnoume sto b ma 9

Teratismic mejidou kai epistrof exidwn

Anag^goun to pìblhma thc epÐlushc enìc n di^statou
mh-grammikoÔ sust matoc se aploÔsterc mh-grammikèc
exis,seic miac metablht c

Anag^goun to prìblhma thc epÐlushc enìc n di^statou
mh-grammikoÔ sust matoc se aploÔsterc mh-grammikèc
exis,seic miac metablht c

Proèrqontai apì th grammik Jacobi h opoÐa upologÐzei thn
epìmenh sunist,sa x_i^{k+1} , epilÔontac wc proc x_i th grammik
exÐswsh

$$x_i^{k+1} = \frac{1}{a_{ii}} \left(b_i - \sum_{j=1}^{i-1} a_{ij} x_j^k - \sum_{j=i+1}^n a_{ij} x_j^k \right)$$

Anagoun to prblhma thc epdlushc enic n di^statou mh-grammiko^ sust matoc se aplo^sterec mh-grammikèc exis, seic miac metablht c

Proèrqontai apì th grammik Jacobij h opoDa upologDzei thn epimenh sunist,sa x_i^{k+1} , epil^ontac wc proc x_i th grammik exDswsh

$$\sum_{j=1}^{i-1} a_{ij} x_j^k + x_i + \sum_{j=i+1}^n a_{ij} x_j^k = b_i = 0$$

H parap^nw diadikasDa mporeD na efarmosteD kai sthn perDptwsh thc arijmhtik c epdlushc susthm^twn mh-grammik, n exis, sewn

Anagoun to prblhma thc epdlushc enic n di^statou mh-grammiko^ sust matoc se aplo^sterec mh-grammik^c exis,seic miac metablht c

Pro^rcontai ap^ th grammik Jacobi h opo^Da upolog^zei thn ep^menh sunist,sa x_i^{k+1} , epil^ontac wc proc x_i th grammik ex^swsh

$$\sum_{j=1}^{i-1} a_{ij} x_j^k + a_{ii} x_i + \sum_{j=i+1}^n a_{ij} x_j^k - b_i = 0$$

H parap^nw diadikas^Da mpore^ na efarmoste^ kai sthn per^ptwsh thc arijmhtik c epdlushc susthm^tw n mh-grammik, n exis,sewn

Se aut thn per^ptwsh, to x_i^{k+1} epitugq^netai l^ontac th mh-grammik ex^swsh miac metablht c

$$f_i(x_1^k, x_2^k, \dots, x_{i-1}^k; x_i, x_{i+1}^k, \dots, x_n^k) = 0$$

ipou metablht^c stajer^c

Anagoun to pribhma thc epdlushc enic n di^statou mh-grammiko^ sust matoc se aplo^sterec mh-grammik^c exis, seic miac metablht c

Pro^rcontai apì th grammik Jacobi h opo^Da upolog^zei thn epimenh sunist,sa x_i^{k+1} , epil^ontac wc proc x_i th grammik ex^swsh

$$\sum_{j=1}^{i-1} a_{ij} x_j^k + \sum_{j=i+1}^n a_{ij} x_j^k - b_i = 0$$

H parap^nw diadikas^Da mpore^D na efarmoste^D kai sthn per^ptwsh thc arijmhtik c epdlushc susthm^tw n mh-grammik, n exis, sewn

Se aut thn per^ptwsh, to x_i^{k+1} epitugq^netai l^onontac th mh-grammik ex^swsh miac metablht c

$$f_i(x_1^k, x_2^k, \dots, x_{i-1}^k; x_i, x_{i+1}^k, \dots, x_n^k) = 0$$

ì pou **metablht^c stajer^c**

An h l^osh thc parap^nw sqeshc e^Dnai h \mathbb{A}_i , j^tουμε

$$x_i^{k+1} = \mathbb{A}_i$$

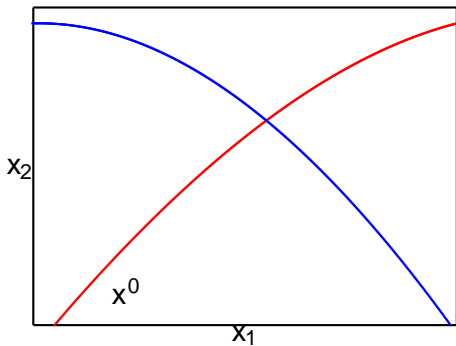
Jewr,ntac kai mia **par^metro qal^rwhc**, to epanalhptikì sq ma gDnetai

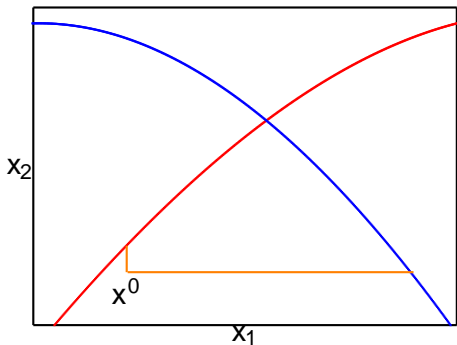
$$x_i^{k+1} = x_i^k + !_k (\mathbb{A}_i \quad x_i^k)$$

Jewr,ntac kai mia **par^metro qal^rwshe**, to epanalhptikì sq ma gÐnetai

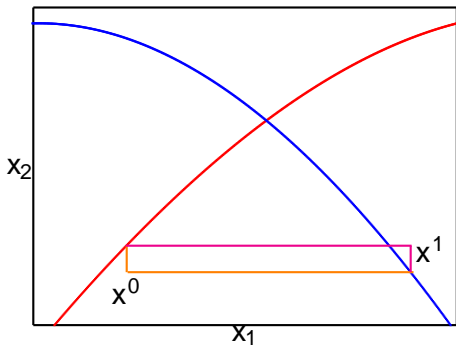
$$x_i^{k+1} = x_i^k + !_k (x_i - x_i^k)$$

H parap^nw mèjodoc èqei nihma mìno ìtan oi exis,seic f_i èqoun monadikèc IÔseic sth dosmèn h perioq

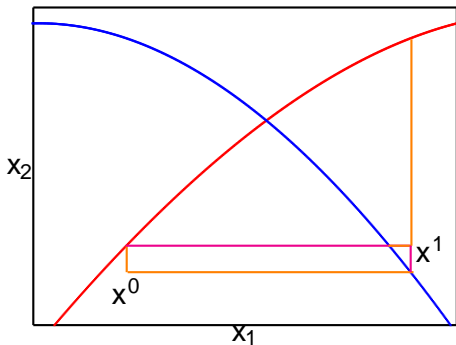




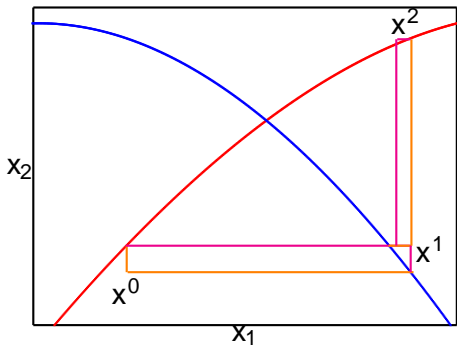
x^0 epilôoume wo x_2 kai x_1	



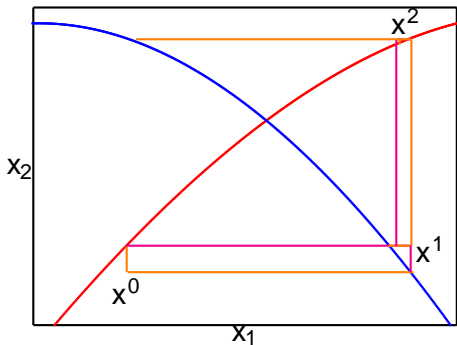
x^0	epilÔoume wo x_2 kai x_1	EÔresh x^1



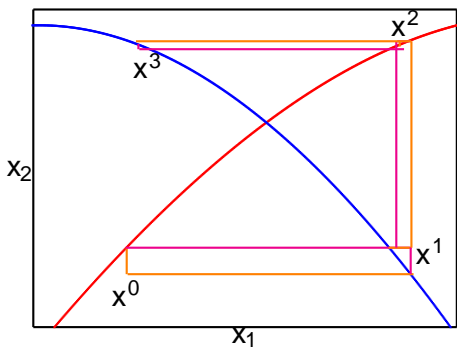
x^0	epilÔoume wo x_2 kai x_1	EÔresh x^1
x^1	epilÔoume wo x_2 kai x_1	



x^0	epilÔoume wo x_2 kai x_1	EÔresh x^1
x^1	epilÔoume wo x_2 kai x_1	EÔresh x^2



x^0	epilÔoume wo x_2 kai x_1	EÔresh x^1
x^1	epilÔoume wo x_2 kai x_1	EÔresh x^2
x^2	epilÔoume wo x_2 kai x_1	



x^0	epilÔoume wo x_2 kai x_1	EÔresh x^1
x^1	epilÔoume wo x_2 kai x_1	EÔresh x^2
x^2	epilÔoume wo x_2 kai x_1	EÔresh x^3

SuneqDzoume thn Ddia diadikasDa mèqri na broÔme èna
shmeDo pou proseggDzei th IÔsh = (x₁ ; x₂)

SuneqDzoume thn Ddia diadikasDa mèqri na broÔme èna
shmeDo pou proseggDzei th IÔsh = (x₁ ; x₂)
Stamat^me ìtan plhreDtai èna perissìtera krit ria
termatismoÔ

Mh sŌgklish thc mejidou

SŌgklish thc mejidou

Mh sÔgklish thc mejìdou

H telik tim tou deĐkth epanal yewn, megalÔterh apì to dedomèno mègisto arijmì epanal yewn

SÔgklish thc mejìdou

Mh sÔgklisch thc mejìdou

H telik tim tou deÐkth epanal yewn, megalÔterh apì to dedomèno mègisto arijmì epanal yewn

SÔgklisch thc mejìdou

DÔo diadoqikèc proseggÐseic, kont^ (akrÐbeia) h mÐa sthn
^llh

$$kx^{k+1} \quad x^k k \quad 2$$

Mh sÔgklish thc mejìdou

H telik tim tou deÐkth epanal yewn, megalÔterh apì to dedomèno mègisto arijmì epanal yewn

SÔgklish thc mejìdou

DÔo diadoqikèc proseggÐseic, kont^ (akrÐbeia) h mÐa sthn
^llh

$$kx^{k+1} \quad x^k k \quad 2$$

H nìrma thc F_n kont^ sto 0

$$kF_n(x^{k+1})k \quad 1$$

Mh sOgklish thc mejidou

H telik tim tou deDkth epanal yewn, megalOterh apì to dedomèno mègisto arijmì epanal yewn

SOgklish thc mejidou

DÔo diadoqikèc proseggDseic, kont^ (akrDbeia) h mDa sthn
^llh

$$kx^{k+1} \quad x^k k \quad 2$$

H nirma thc F_n kont^ sto 0

$$kF_n(x^{k+1})k \quad 1$$

Prosoq

An^loga me th fOsh thc sun^rthshc, mporeD na isqOei k^poio krit rio termatismoO kai h mèjodoc na apoklDnei

Eðsodoi-'Exodoi

Eðsodoi

EḐsodoi-'Exodoi

EḐsodoi

n to pl joc tw n exis₃sewn kai tw n agn₃stwn

EĐsodoi-'Exodoi

EĐsodoi

n to pl joc tw n exis₃sewn kai tw n agn₃stwn
 $F_n(x)$ o tÔpoc thc sun[^]rthshc

EĐsodoi-'Exodoi

EĐsodoi

n to pl joc tw n exis₃sewn kai tw n agn₃stwn

$F_n(x)$ o tÔpoc thc sun^rthshc

x_0 h arqik prosèggish thc rĐzac

EĐsodoi-'Exodoi

EĐsodoi

n to pl joc tw n exis₃sewn kai tw n agn₃stwn

$F_n(x)$ o tÔpoc thc sun^rthshc

x_0 h arqik prosèggish thc rĐzac

! k oi par^metroi qal^rwshe

EĐsodoi-'Exodoi

EĐsodoi

n to pl joc tw_n exis₃sewn kai tw_n agn₃stwn

$F_n(x)$ o tÔpoc thc sun[^]rthshc

x_0 h arqik prosèggish thc rĐzac

! _k oi par[^]metroi qal[^]rwshe

MIT to mègisto pl joc epanal yewn thc mejìdou

EĐsodoi-'Exodoi

EĐsodoi

n to pl joc tw_n exis₃sewn kai tw_n agn₃stwn

$F_n(x)$ o tÔpoc thc sun[^]rthshc

x_0 h arqik prosèggish thc rĐzac

! _k oi par[^]metroi qal[^]rwhc

MIT to mègisto pl joc epanal yewn thc mejìdou

₁ h akrĐbeia pou jètoume gia tox

EḌsodoi-'Exodoi

EḌsodoi

n to pl joc tw_n exis₃sewn kai tw_n agn₃stwn

$F_n(x)$ o tÔpoc thc sun[^]rthshc

x_0 h arqik prosèggish thc rḌzac

! _k oi par[^]metroi qal[^]rwshe

MIT to mègisto pl joc epanal yewn thc mejìdou

1 h akrḌbeia pou jètoume gia tox

2 h akrḌbeia pou jètoume gia thn $F_n(x)$

Eðsodoi-'Exodoi

Eðsodoi

n to pl joc tw_n exis₃sewn kai tw_n agn₃stwn

$F_n(x)$ o tÔpoc thc sun[^]rthshc

x_0 h arqik prosèggish thc rÐzac

! _k oi par[^]metroi qal[^]rwshe

MIT to mègisto pl joc epanal yewn thc mejìdou

₁ h akrÐbeia pou jètoume gia tox

₂ h akrÐbeia pou jètoume gia thn $F_n(x)$

'Exodoi

EḐsodoi-'Exodoi

EḐsodoi

n to pl joc tw_n exis₃sewn kai tw_n agn₃stwn

$F_n(x)$ o tÔpoc thc sun[^]rthshc

x_0 h arqik prosèggish thc rḐzac

! _k oi par[^]metroi qal[^]rwhc

MIT to mègisto pl joc epanal yewn thc mejìdou

₁ h akrḐbeia pou jètoume gia tox

₂ h akrḐbeia pou jètoume gia thn $F_n(x)$

'Exodoi

x^k h teleutaḐa prosèggish thc rḐzac

EĐsodoi-'Exodoi

EĐsodoi

n to pl joc tw_n exis₃sewn kai tw_n agn₃stwn

$F_n(x)$ o tÔpoc thc sun[^]rthshc

x_0 h arqik prosèggish thc rĐzac

! _k oi par[^]metroi qal[^]rwhc

MIT to mègisto pl joc epanal yewn thc mejìdou

₁ h akrĐbeia pou jètoume gia tox

₂ h akrĐbeia pou jètoume gia thn $F_n(x)$

'Exodoi

x^k h teleutaĐa prosèggish thc rĐzac

$F_n(x^k)$ h tim thc sun[^]rthshc sthn _k epan[^]lhyh

EĐsodoi-'Exodoi

EĐsodoi

n to pl joc tw_n exis₃sewn kai tw_n agn₃stwn

$F_n(x)$ o tÔpoc thc sun[^]rthshc

x_0 h arqik prosèggish thc rĐzac

! _k oi par[^]metroi qal[^]rwhc

MIT to mègisto pl joc epanal yewn thc mejìdou

₁ h akrĐbeia pou jètoume gia tox

₂ h akrĐbeia pou jètoume gia thn $F_n(x)$

'Exodoi

x^k h teleutaĐa prosèggish thc rĐzac

$F_n(x^k)$ h tim thc sun[^]rthshc sthn _k epan[^]lhyh

_k to pl joc epanal yewn pou èginan

Kajorismic eisidwn mejidou

Kajorismic eisidwn mejidou

Jètoume $k = 1$

Kajorismic eisidwn mejidou

Jètoume $k = 1$

Elègqoume $ank < MIT$

An den isqÔei, phgaÐnoume sto b ma 9. Diaforetik[^], aux[^]noume to k kai suneqÐzoume sto eplomeno b ma

Kajorismic eisidwn mejidou

Jètoume $k = 1$

Elègqoume $ank < MIT$

An den isqÔei, phgaÐnoume sto b ma 9. Diaforetik[^], aux[^]noume to k kai suneqÐzoume sto eplomeno b ma

Elègqoume $ankF_n(x^k)k \quad 2$

An den isqÔei, suneqÐzoume sto epimeno b ma. Diaforetik[^], phgaÐnoume sto b ma 9

Kajorismic eisidwn mejidou

Jètoume $k = 1$

Elègqoume $ank < MIT$

An den isqÔei, phgaÐnoume sto b ma 9. Diaforetik[^], aux[^]noume to k kai suneqÐzoume sto eplomeno b ma

Elègqoume $ankF_n(x^k)k \quad 2$

An den isqÔei, suneqÐzoume sto epimeno b ma. Diaforetik[^], phgaÐnoume sto b ma 9

Jètoume $i = 0$

Kajorismic eisidwn mejidou

Jètoume $k = 1$

Elègqoume $ank < MIT$

An den isqÔei, phgaÐnoume sto b ma 9. Diaforetik[^], aux[^]noume to k kai suneqÐzoume sto eplomeno b ma

Elègqoume $ankF_n(x^k)k \quad 2$

An den isqÔei, suneqÐzoume sto epimeno b ma. Diaforetik[^], phgaÐnoume sto b ma 9

Jètoume $i = 0$

An isqÔeii $< n$, aux[^]noume to i kai phgaÐnoume sto epimeno b ma. Diaforetik[^], phgaÐnoume sto b ma 8

Kajorismic eisidwn mejidou

Jètoume $k = 1$

Elèqqoume $ank < MIT$

An den isqÔei, phgaÐnoume sto b ma 9. Diaforetik[^], aux[^] noume to k kai suneqÐzoume sto eplomeno b ma

Elèqqoume $ank F_n(x^k)k \quad 2$

An den isqÔei, suneqÐzoume sto epimeno b ma. Diaforetik[^], phgaÐnoume sto b ma 9

Jètoume $i = 0$

An isqÔeii $< n$, aux[^] noume to i kai phgaÐnoume sto epimeno b ma. Diaforetik[^], phgaÐnoume sto b ma 8

BrÐskoume th lÔsh[^] thc monodi[^]stathc mh-grammik c exÐswshc
 $f_i(x_1^k; x_2^k; \dots; x_{i-1}^k; x; x_{i+1}^k; \dots; x_n^k) = 0$ wc proc x , jètoume
 $x_i^{k+1} = x_i^k + !_k(x_i^k)$ kai p gaine sto b ma 6

Kajorismic eisidwn mejidou

Jètoume $k = 1$

Elèqqoume $\text{ank} < \text{MIT}$

An den isqÔei, phgaÐnoume sto b ma 9. Diaforetik[^], aux[^] noume to k kai suneqÐzoume sto eplomeno b ma

Elèqqoume $\text{ank} F_n(x^k) k \quad 2$

An den isqÔei, suneqÐzoume sto epimeno b ma. Diaforetik[^], phgaÐnoume sto b ma 9

Jètoume $i = 0$

An isqÔeii $< n$, aux[^] noume to i kai phgaÐnoume sto epimeno b ma. Diaforetik[^], phgaÐnoume sto b ma 8

BrÐskoume th lÔsh[^] thc monodi[^]stathc mh-grammik c exÐswshc $f_i(x_1^k; x_2^k; \dots; x_{i-1}^k; x; x_{i+1}^k; \dots; x_n^k) = 0$ wc proc x , jètoume $x_i^{k+1} = x_i^k + !_k(x_i - x_i^k)$ kai p gaine sto b ma 6

Elèqqoume $\text{ank} x_i^{k+1} \quad x_i^k \quad 1$

An den isqÔei, suneqÐzoume sto b ma 3. Diaforetik[^], phgaÐnoume sto b ma 9

Kajorismic eisidwn mejidou

Jètoume $k = 1$

Elèqqoume $ank < MIT$

An den isqÔei, phgaÐnoume sto b ma 9. Diaforetik[^], aux[^] noume to k kai suneqÐzoume sto eplomeno b ma

Elèqqoume $ank F_n(x^k)k \quad 2$

An den isqÔei, suneqÐzoume sto epimeno b ma. Diaforetik[^], phgaÐnoume sto b ma 9

Jètoume $i = 0$

An isqÔeii $< n$, aux[^] noume to i kai phgaÐnoume sto epimeno b ma. Diaforetik[^], phgaÐnoume sto b ma 8

BrÐskoume th lÔsh[^] thc monodi[^]stathc mh-grammik c exÐswshc $f_i(x_1^k; x_2^k; \dots; x_{i-1}^k; x; x_{i+1}^k; \dots; x_n^k) = 0$ wc proc x , jètoume $x_i^{k+1} = x_i^k + !_k(x_i - x_i^k)$ kai p gaine sto b ma 6

Elèqqoume $ank x_i^{k+1} \quad x_i^k \quad 1$

An den isqÔei, suneqÐzoume sto b ma 3. Diaforetik[^], phgaÐnoume sto b ma 9

Teratismic mejidou kai epistrof exidwn

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