Επιστημονικοί Υπολογισμοί

Αριθμητική Επίλυση συστημάτων μη-γραμμικών εξισώσεων - Μη γραμμικές μέθοδοι

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Τμήμα Ηλεκτρολόγων Μηχανικών & Μηχανικών Υπολογιστών
Δημοκρίτειο Πανεπιστήμιο Θράκης
Κιμμέρια 67100, Ξάνθη

ΔΗΜΟΚΡΙΤΕΙΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΘΡΑΚΗΣ

30 Μαρτίου 2021

Περιεχόμενα I

epÐlush exÐswsḩc eÔresh stajer¸n shmeÐwn

To prìblhma thc epÐlushc exis¸sewn sqetÐzetai me to prìblhma tou upologismoÔ stajer¸n shmeÐwn miac sunˆrthshc.

epÐlush exÐswshc eÔresh stajer¸n shmeÐwn

To prìblhma thc epÐlushc exis¸sewn sqetÐzetai me to prìblhma tou upologismoÔ stajer¸n shmeÐwn miac sunˆrthshc.

To prìblhma upologismoÔ stajer¸n shmeÐwn

Dedomènhc thc $f : [a; b]$ R ! R
na brejeÐ k˘poio shmeÐo r 2 $(a; b)$ tètoio ¸ste $f(r) = r$

epÐlush exÐswshc eÔresh stajer¸n shmeÐwn

To prìblhma thc epÐlushc exis¸sewn sqetÐzetai me to prìblhma tou upologismoÔ stajer¸n shmeÐwn miac sun˜rthshc.

To prìblhma upologismoÔ stajer¸n shmeÐwn

Dedomènhc thc $f : [a; b] \quad R \to \quad R$

na brejeÐ k˜poio shmeÐo $r \in (a; b)$ tètoio ¸ste $f(r) = r$

epÐlush exÐswshc eÔresh stajer¸n shmeÐwn

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To prìblhma thc epÐlushc exis¸sewn sqetÐzetai me to prìblhma tou upologismoÔ stajer¸n shmeÐwn miac sun¨rthshc.

To prìblhma upologismoÔ stajer¸n shmeÐwn

Dedomènhc thc $f : [a; b] \subset \mathbb{R} \to \mathbb{R}$

na brejeÐ k¨poio shmeÐo $r \in (a; b)$ tètoio ¸ste $f(r) = r$

epÐlush exÐswshc $\Longleftrightarrow$ eÔresh stajer¸n shmeÐwn

me k¨poia arijmhtik mèjodo jèloume na epilÔsoume thn $f(x) = 0$

epÐlush exÐswshc eÔresh stajer¸n shmeÐwn

To prìblhma thc epÐlushc exis¸sewn sqetÐzetai me to prìblhma tou upologismoÔ stajer¸n shmeÐwn miac sun˘rthshc.

To prìblhma upologismoÔ stajer¸n shmeÐwn

Dedomènhc thc $f : [a; b] \quad R ! \quad R$
na brejeÐ k˘poio shmeÐo $r \, 2 \, (a; b)$ tètoio ¸ste $f(r) = r$

epÐlush exÐswshc eÔresh stajer¸n shmeÐwn

- me k˘poia arijmhtik mèjodo jèloume na epilÔsoume thn $f(x) = 0$
- epilÔoume thn $f(x) = g(x) \quad x$ kai brÐskoume th lÔsh r

epÐlush exÐswshc eÔresh stajer¸n shmeÐwn

To prìblhma thc epÐlushc exis¸sewn sqetÐzetai me to prìblhma tou upologismoÔ stajer¸n shmeÐwn miac sun˚rthshc.

To prìblhma upologismoÔ stajer¸n shmeÐwn

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na brejeÐ k˚poio shmeÐo r 2 (a; b) tètoio ¸ste f (r) = r

epÐlush exÐswshc eÔresh stajer¸n shmeÐwn

- me k˚poia arijmhtik mèjodo jèloume na epilÔsoume thn f (x) = 0
- epilÔoume thn f (x) = g(x) x kai brÐskoume th lÔsh r epeid f (r) = 0

epÐlush exÐswshc eÔresh stajer¸n shmeÐwn

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  - epeid f (r) = 0) g(r) r = 0

epÐlush exÐswshc eÔresh stajer¸n shmeÐwn

To prìblhma thc epÐlushc exis¸sewn sqetÐzetai me to prìblhma tou upologismoÔ stajer¸n shmeÐwn miac sun˘rthshc.

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- me k˘poia arijmhtik mèjodo jèloume na epilÔsoume thn f (x) = 0
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- epeid f (r) = 0) g(r) r = 0) g(r) = r

epÐlush exÐswshc eÔresh stajer¸n shmeÐwn

To prìblhma thc epÐlushc exis¸sewn sqetÐzetai me to prìblhma tou upologismoÔ stajer¸n shmeÐwn miac sun˚rthshc.

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- me k˚poia arijmhtik mèjodo jèloume na epilÔsoume thn f (x) = 0
- epilÔoume thn f (x) = g(x) x kai brÐskoume th lÔsh r
- epeid f (r) = 0) g(r) r = 0) g(r) = r
- to r eÐnai stajerì shmeÐo thc g(x)

epÐlush exÐswshc eÔresh stajer¸n shmeÐwn

To prìblhma thc epÐlushc exis¸sewn sqetÐzetai me to prìblhma tou upologismoÔ stajer¸n shmeÐwn miac sunˆrthshc.

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na brejeÐ k˘poio shmeÐo r 2 (a; b) tètoio ¸ste f (r) = r

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- epeid f (r) = 0) g(r) r = 0) g(r) = r
- to r eÐnai stajerì shmeÐo thc g(x)

eÔresh stajer¸n shmeÐwn) epÐlush exÐswshc

- me kˆpoia arijmhtik mèjodo jèloume na broÔme èna stajerì shmeÐo gia thn g(x)

epÐlush exÐswshc eÔresh stajer¸n shmeÐwn

To prìblhma thc epÐlushc exis¸sewn sqetÐzetai me to prìblhma tou upologismoÔ stajer¸n shmeÐwn miac sun˘rthshc.

To prìblhma upologismoÔ stajer¸n shmeÐwn

Dedomènhc thc $f : [a, b] \subset R \to R$
na brejeÐ k˘poio shmeÐo $r \in (a, b)$ tètoio ¸ste $f(r) = r$

- epÐlush exÐswshc $\Rightarrow$ eÔresh stajer¸n shmeÐwn
  - me k˘poia arijmhtik mèjodo jèloume na epilÔsoume thn $f(x) = 0$
  - epilÔoume thn $f(x) = g(x) - x$ kai brÐskoume th lÔsh r
  - epeid $f(r) = 0 \Rightarrow g(r) - r = 0 \Rightarrow g(r) = r$
  - to r eÐnai stajerì shmeÐo thc $g(x)$
- eÔresh stajer¸n shmeÐwn $\Rightarrow$ epÐlush exÐswshc
  - me k˘poia arijmhtik mèjodo jèloume na broÔme èna stajerì shmeÐo gia thn $g(x)$
  - y˘qnoume $g(x) = x$

epÐlush exÐswshc eÔresh stajer¸n shmeÐwn

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  - epilÔoume thn $f(x) = g(x)$ x kai brÐskoume th lÔsh r
  - epeid $f(r) = 0$) $g(r)$ $r = 0$) $g(r) = r$
  - to r eÐnai stajerì shmeÐo thc $g(x)$
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  - me kˆpoia arijmhtik mèjodo jèloume na broÔme èna stajerì shmeÐo gia thn $g(x)$
  - yˆqnoume $g(x) = x$
  - epeid $g(x)$ $x = 0$, ,

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  - me kˆpoia arijmhtik mèjodo jèloume na epilÔsoume thn $f(x) = 0$
  - epilÔoume thn $f(x) = g(x) \quad x$ kai brÐskoume th lÔsh r
  - epeid $f(r) = 0 \;)\; g(r) \quad r = 0 \;)\; g(r) = r$
  - to r eÐnai stajerì shmeÐo thc $g(x)$
- eÔresh stajer¸n shmeÐwn) epÐlush exÐswshc
  - me kˆpoia arijmhtik mèjodo jèloume na broÔme èna stajerì shmeÐo gia thn $g(x)$
  - yˆqnoume $g(x) = x$
  - epeid $g(x) \quad x = 0$, jètoume $f(x) = g(x) \quad x$,

epÐlush exÐswshc eÔresh stajer¸n shmeÐwn

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  - epeid $f(r) = 0 \Rightarrow g(r) - r = 0 \Rightarrow g(r) = r$
  - to r eÐnai stajerì shmeÐo thc $g(x)$
- eÔresh stajer¸n shmeÐwn $\Rightarrow$ epÐlush exÐswshc
  - me kˆpoia arijmhtik mèjodo jèloume na broÔme èna stajerì shmeÐo gia thn $g(x)$
  - yˆqnoume $g(x) = x$
  - epeid $g(x) - x = 0$, jètoume $f(x) = g(x) - x$, katal goume $f(x) = 0$

Je¸rhma ('Uparxh stajer¸n shmeÐwn)

An h sunˆrthsh g : [a; b] R ! [a; b] eÐnai suneq c sto diˆsthma [a; b], tìte h g èqei èna stajerì shmeÐo sto [a; b].

Je¸rhma ('Uparxh stajer¸n shmeÐwn)

An h sunˆrthsh g : [a; b] R ! [a; b] eÐnai suneq c sto diˆsthma [a; b], tìte h g èqei èna stajerì shmeÐo sto [a; b].

Je¸rhma (Monadikìthta stajer¸n shmeÐwn)

An upˆrqei h pr¸th parˆgwgoc $g^{0}(x)$ thc sunˆrthshc g sto (a; b) kai upˆrqei mia stajerˆ p < 1 tètoia ¸ste na isqÔei

$$g^{0}(x) \quad p; \quad \text{gia ìla ta} \quad x\ 2\ (a; b)$$

tìte to stajerì shmeÐo pou an kei sto [a; b] eÐnai monadikì.

Je¸rhma ('Uparxh stajer¸n shmeÐwn)

An h sun˘rthsh g : [a; b] R ! [a; b] eÐnai suneq c sto di˘sthma [a; b], tìte h g èqei èna stajerì shmeÐo sto [a; b].

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An up˘rqei h pr¸th par˘gwgoc $g^0(x)$ thc sun˘rthshc g sto (a; b) kai up˘rqei mia stajer˘ p < 1 tètoia ¸ste na isqÔei

$$g^0(x) \quad p; \quad \text{gia ìla ta} \quad x \in (a; b)$$

tìte to stajerì shmeÐo pou an kei sto [a; b] eÐnai monadikì.

Genik epanalhptik mèjodoc

MporoÔme na proseggÐsoume èna stajerì shmeÐo thc sun˘rthshc g(x)

Je¸rhma ('Uparxh stajer¸n shmeÐwn)

An h sun˘rthsh $g : [a;b]$ R ! $[a;b]$ eÐnai suneq c sto di˘sthma $[a;b]$, tìte h g èqei èna stajerì shmeÐo sto $[a;b]$.

Je¸rhma (Monadikìthta stajer¸n shmeÐwn)

An up˘rqei h pr¸th par˘gwgoc $g^0(x)$ thc sun˘rthshc g sto $(a;b)$ kai up˘rqei mia stajer˘ $p < 1$ tètoia ¸ste na isqÔei

$$g^0(x) \quad p; \quad \text{gia ìla ta} \quad x\ 2\ (a;b)$$

tìte to stajerì shmeÐo pou an kei sto $[a;b]$ eÐnai monadikì.

Genik epanalhptik mèjodoc

MporoÔme na proseggÐsoume èna stajerì shmeÐo thc sun˘rthshc $g(x)$ epilègontac mia arqik prosèggish x_0

Je¸rhma ('Uparxh stajer¸n shmeÐwn)

An h sun˘rthsh g : [a; b] R ! [a; b] eÐnai suneq c sto di˘sthma [a; b], tìte h g èqei èna stajerì shmeÐo sto [a; b].

Je¸rhma (Monadikìthta stajer¸n shmeÐwn)

An up˘rqei h pr¸th par˘gwgoc $g^{0}(x)$ thc sun˘rthshc g sto (a; b) kai up˘rqei mia stajer˘ p < 1 tètoia ¸ste na isqÔei

$$g^{0}(x) \quad p; \quad \text{gia ìla ta} \quad x \; 2 \; (a; b)$$

tìte to stajerì shmeÐo pou an kei sto [a; b] eÐnai monadikì.

Genik epanalhptik mèjodoc

MporoÔme na proseggÐsoume èna stajerì shmeÐo thc sun˘rthshc g(x) epilègontac mia arqik prosèggish x_0 dhmiourgeÐtai mia akoloujÐa ìrwn f x_k $g^{1}_{k=0}$ apì th sqèsh

$$x_{k+1} = g(x_k); \quad k = 0; 1; 2; :::$$

Prosoq

O metasqhmatismìc thc exÐswshc $f(x) = 0$ sth morf $x = g(x)$ den eÐnai monadikìc

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O metasqhmatismìc thc exÐswshc $f(x) = 0$ sth morf $x = g(x)$ den eÐnai monadikìc

H genik epanalhptik mèjodoc de sugklÐnei gia opoiad pote sun ˆrthsh $g(x)$

Prosoq

O metasqhmatismìc thc exÐswshc $f(x) = 0$ sth morf $x = g(x)$ den eÐnai monadikìc

H genik epanalhptik mèjodoc de sugklÐnei gia opoiad pote sunˆrthsh $g(x)$

AitÐec apotuqÐac sÔgklishc genik c epanalhptik c mejìdou

1 H sunˆrthsh $g(x)$ apeirÐzetai.

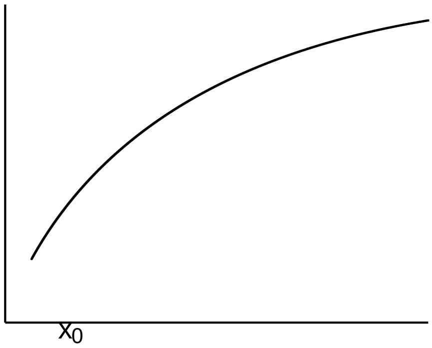
Prosoq

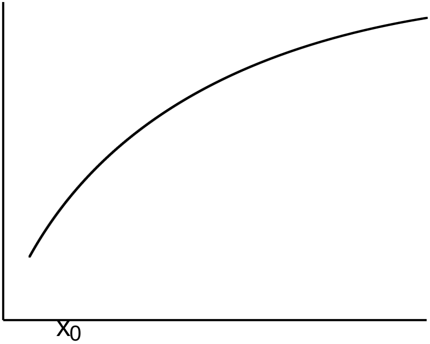
O metasqhmatismìc thc exÐswshc $f(x) = 0$ sth morf $x = g(x)$ den eÐnai monadikìc

H genik epanalhptik mèjodoc de sugklÐnei gia opoiad pote sunˆrthsh $g(x)$

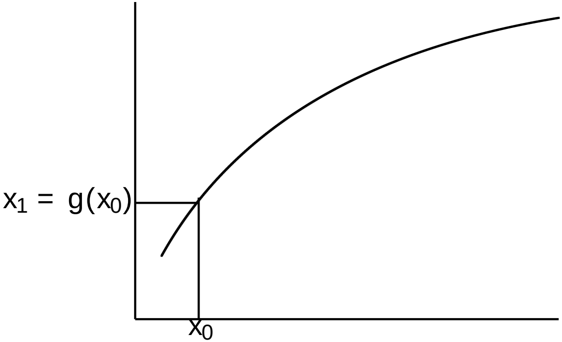
AitÐec apotuqÐac sÔgklishc genik c epanalhptik c mejìdou

1. H sunˆrthsh $g(x)$ apeirÐzetai.
2. H sunˆrthsh $g(x)$ talant¸netai.


x_0

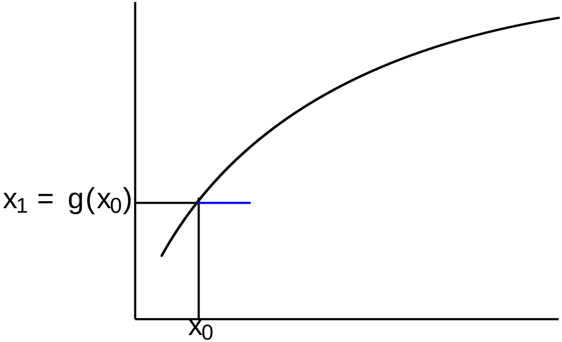


Fèrnoume thn $y = x$



Fèrnoume thn $y = x$

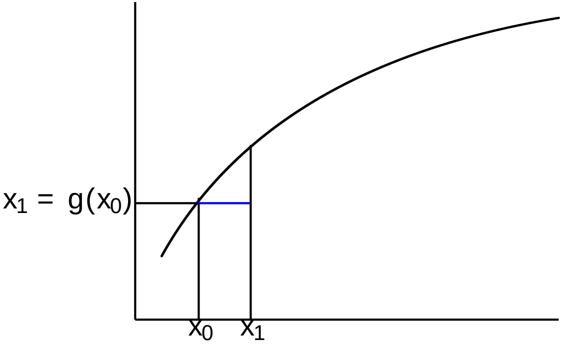
BrÐskoume to $g(x_0)$



Fèrnoume thn $y = x$

BrÐskoume to $g(x_0)$

$y = x_1$ Tom $y = x$

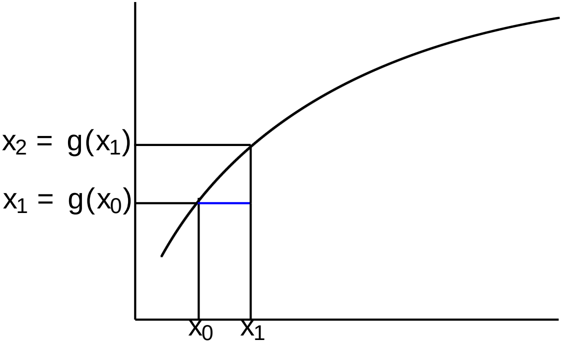


Fèrnoume thn $y = x$

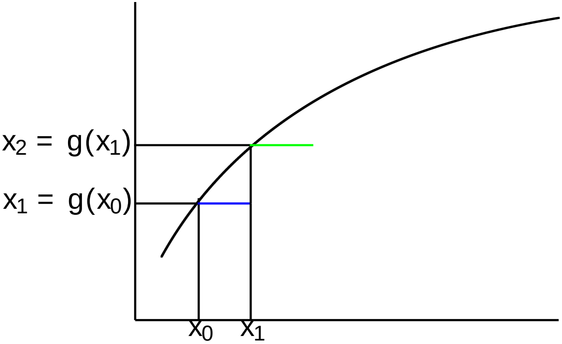
BrÐskoume to $g(x_0)$

$y = x_1$ Tom $y = x$

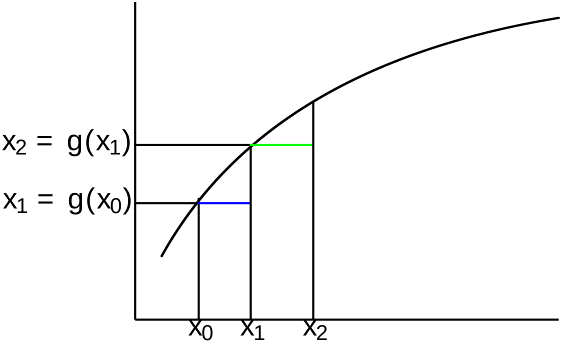
kˆjeth ston $x^{0}x$



Fèrnoume thn $y = x$	
BrÐskoume to $g(x_0)$	BrÐskoume to $g(x_1)$
$y = x_1$ Tom $y = x$	
kˆjeth ston $x^0 x$	



Fèrnoume thn $y = x$	
BrÐskoume to $g(x_0)$	BrÐskoume to $g(x_1)$
$y = x_1$ Tom $y = x$	$y = x_2$ Tom $y = x$
kˆjeth ston $x^0 x$	



Fèrnoume thn $y = x$

BrÐskoume to $g(x_0)$

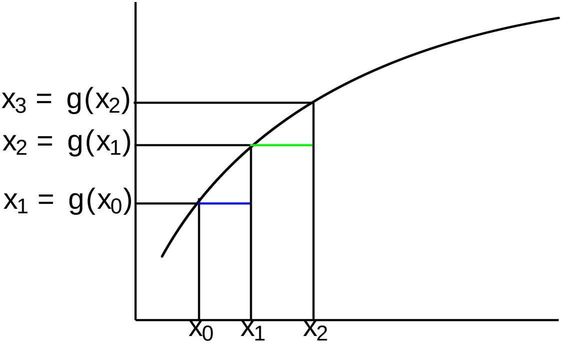
$y = x_1$ Tom $y = x$

kˆjeth ston x^0x

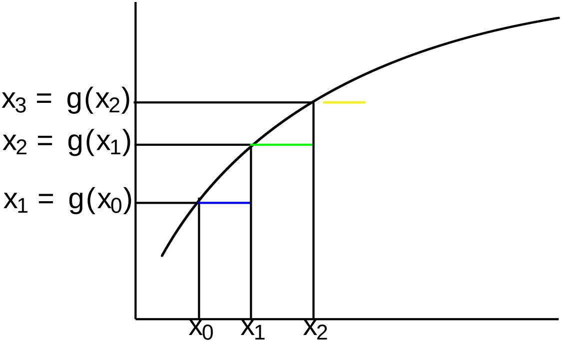
BrÐskoume to $g(x_1)$

$y = x_2$ Tom $y = x$

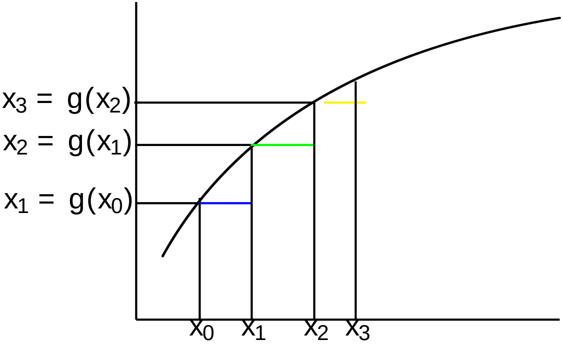
kˆjeth ston x^0x



Fèrnoume thn $y = x$		
BrĐskoume to $g(x_0)$	BrĐskoume to $g(x_1)$	BrĐskoume to $g(x_2)$
$y = x_1$ Tom $y = x$	$y = x_2$ Tom $y = x$	
kˆjeth ston $x^{0}x$	kˆjeth ston $x^{0}x$	



Fèrnoume thn $y = x$		
BrÐskoume to $g(x_0)$	BrÐskoume to $g(x_1)$	BrÐskoume to $g(x_2)$
$y = x_1$ Tom $y = x$	$y = x_2$ Tom $y = x$	$y = x_3$ Tom $y = x$
kˆjeth ston x^0x	kˆjeth ston x^0x	



Fèrnoume thn $y = x$		
BrÐskoume to $g(x_0)$	BrÐskoume to $g(x_1)$	BrÐskoume to $g(x_2)$
$y = x_1$ Tom $y = x$	$y = x_2$ Tom $y = x$	$y = x_3$ Tom $y = x$
kˆjeth ston $x^{0}x$	kˆjeth ston $x^{0}x$	kˆjeth ston $x^{0}x$

Mh sÔgklish thc mejìdou

SÔgklish thc mejìdou

Mh sÔgklish thc mejìdou

H telik tim tou deÐkth epanal yewn, megalÔterh apì to dedomèno mègisto arijmì epanal yewn

SÔgklish thc mejìdou

Mh sÔgklish thc mejìdou

H telik tim tou deÐkth epanal yewn, megalÔterh apì to dedomèno mègisto arijmì epanal yewn

SÔgklish thc mejìdou

DÔo diadoqikèc proseggÐseic, kontˆ (akrÐbeia) h mÐa sthn ˆllh

$$|x_n \quad x_{n \quad 1}| <$$

Mh sÔgklish thc mejìdou

H telik tim tou deÐkth epanal yewn, megalÔterh apì to dedomèno mègisto arijmì epanal yewn

SÔgklish thc mejìdou

DÔo diadoqikèc proseggÐseic, kontˆ (akrÐbeia) h mÐa sthn ˆllh

$|x_n \quad x_{n \quad 1}| <$

H tim thc f (x) kontˆ sto 0

$|f(x_n)| <$

Mh sÔgklish thc mejìdou

H telik tim tou deÐkth epanal yewn, megalÔterh apì to dedomèno mègisto arijmì epanal yewn

SÔgklish thc mejìdou

DÔo diadoqikèc proseggÐseic, kontˆ (akrÐbeia) h mÐa sthn ˆllh

$j x_n \quad x_{n \quad 1} j <$

H tim thc $f(x)$ kontˆ sto 0

$j f(x_n) j <$

Sqetikì sfˆlma mikrìtero apì thn akrÐbeia

$\frac{j x_n \quad x_{n \quad 1} j}{j x_n j} < \quad , \; x_n \, 6 \, 0$

Mh sÔgklish thc mejìdou

H telik tim tou deÐkth epanal yewn, megalÔterh apì to dedomèno mègisto arijmì epanal yewn

SÔgklish thc mejìdou

DÔo diadoqikèc proseggÐseic, kont´ (akrÐbeia) h mÐa sthn ´llh

$$|x_n \quad x_{n \quad 1}| <$$

H tim thc f (x) kont´ sto 0

$$|f(x_n)| <$$

Sqetikì sf´lma mikrìtero apì thn akrÐbeia

$$\frac{|x_n \quad x_{n \quad 1}|}{|x_n|} < \quad , x_n \neq 0$$

Prosoq

Kajorismìc eisìdwn mejìdou

Kajorismìc eisìdwn mejìdou g o tÔpoc thc sunˆrthshc

Kajorismìc eisìdwn mejìdou

g o tÔpoc thc sunˆrthshc

x_0 h pr¸th prosèggish thc rÐzac

Kajorismìc eisìdwn mejìdou

g o tÔpoc thc sunˆrthshc

x_0 h pr¸th prosèggish thc rÐzac

MIT to mègisto pl joc epanal yewn thc mejìdou

Kajorismìc eisìdwn mejìdou

g o tÔpoc thc sunˆrthshc

x_0 h pr¸th prosèggish thc rÐzac

MIT to mègisto pl joc epanal yewn thc mejìdou

h akrÐbeia pou jètoume

Kajorismìc eisìdwn mejìdou

g o tÔpoc thc sunˆrthshc

x_0 h pr¸th prosèggish thc rÐzac

MIT to mègisto pl joc epanal yewn thc mejìdou

h akrÐbeia pou jètoume

Jètoume k = 1 kai phgaÐnoume sto epìmeno b ma

Kajorismìc eisìdwn mejìdou

g o tÔpoc thc sunˆrthshc

x_0 h pr¸th prosèggish thc rÐzac

MIT to mègisto pl joc epanal yewn thc mejìdou

h akrÐbeia pou jètoume

Jètoume $k = 1$ kai phgaÐnoume sto epìmeno b ma

Anane¸noume tok $(k = k + 1)$ kai phgaÐnoume sto epìmeno b ma

Kajorismìc eisìdwn mejìdou

g o tÔpoc thc sunˆrthshc

x_0 h pr¸th prosèggish thc rÐzac

MIT to mègisto pl joc epanal yewn thc mejìdou

h akrÐbeia pou jètoume

Jètoume k = 1 kai phgaÐnoume sto epìmeno b ma

Anane¸noume tok (k = k + 1) kai phgaÐnoume sto epìmeno b ma

Elègqoume ank > MIT

Kajorismìc eisìdwn mejìdou

- g o tÔpoc thc sunˆrthshc
- x_0 h pr¸th prosèggish thc rÐzac
- MIT to mègisto pl joc epanal yewn thc mejìdou
- ϵ h akrÐbeia pou jètoume

Jètoume $k = 1$ kai phgaÐnoume sto epìmeno b ma

Anane¸noume to k $(k = k + 1)$ kai phgaÐnoume sto epìmeno b ma

Elègqoume an $k >$ MIT

- An den isqÔei, jètoume $x_{k+1} = g(x_k)$ kai suneqÐzoume sto epìmeno b ma

Kajorismìc eisìdwn mejìdou

g o tÔpoc thc sunˆrthshc

x_0 h pr¸th prosèggish thc rÐzac

MIT to mègisto pl joc epanal yewn thc mejìdou

h akrÐbeia pou jètoume

Jètoume $k = 1$ kai phgaÐnoume sto epìmeno b ma

Anane¸noume tok $(k = k + 1)$ kai phgaÐnoume sto epìmeno b ma

Elègqoume an $k >$ MIT

An den isqÔei, jètoume $x_{k+1} = g(x_k)$ kai suneqÐzoume sto epìmeno b ma

An isqÔei phgaÐnoume sto b ma 6

Kajorismìc eisìdwn mejìdou

- g o tÔpoc thc sunˆrthshc
- x_0 h pr¸th prosèggish thc rÐzac
- MIT to mègisto pl joc epanal yewn thc mejìdou
- h akrÐbeia pou jètoume

Jètoume $k = 1$ kai phgaÐnoume sto epìmeno b ma

Anane¸noume tok $(k = k + 1)$ kai phgaÐnoume sto epìmeno b ma

Elègqoume an $k > MIT$

- An den isqÔei, jètoume $x_{k+1} = g(x_k)$ kai suneqÐzoume sto epìmeno b ma
- An isqÔei phgaÐnoume sto b ma 6

Elègqoume èna perissìtera krit ria termatismoÔ (èstw $|x_{k+1} \quad x_k| \quad)$

Kajorismìc eisìdwn mejìdou

g o tÔpoc thc sunˆrthshc

x_0 h pr¸th prosèggish thc rÐzac

MIT to mègisto pl joc epanal yewn thc mejìdou

h akrÐbeia pou jètoume

Jètoume k = 1 kai phgaÐnoume sto epìmeno b ma

Anane¸noume tok (k = k + 1) kai phgaÐnoume sto epìmeno b ma

Elègqoume an k > MIT

An den isqÔei, jètoume $x_{k+1} = g(x_k)$ kai suneqÐzoume sto epìmeno b ma

An isqÔei phgaÐnoume sto b ma 6

Elègqoume èna perissìtera krit ria termatismoÔ (èstw $|x_{k+1} \quad x_k|$)

An den isqÔoun, suneqÐzoume sto b ma 3

Kajorismìc eisìdwn mejìdou

g o tÔpoc thc sunˆrthshc

x_0 h pr¸th prosèggish thc rÐzac

MIT to mègisto pl joc epanal yewn thc mejìdou

h akrÐbeia pou jètoume

Jètoume $k = 1$ kai phgaÐnoume sto epìmeno b ma

Anane¸noume tok $(k = k + 1)$ kai phgaÐnoume sto epìmeno b ma

Elègqoume an $k > MIT$

An den isqÔei, jètoume $x_{k+1} = g(x_k)$ kai suneqÐzoume sto epìmeno b ma

An isqÔei phgaÐnoume sto b ma 6

Elègqoume èna perissìtera krit ria termatismoÔ (èstw $jx_{k+1} \quad x_k j$)

An den isqÔoun, suneqÐzoume sto b ma 3

An isqÔoun phgaÐnoume sto b ma 6

Kajorismìc eisìdwn mejìdou

g o tÔpoc thc sunˆrthshc

x_0 h pr¸th prosèggish thc rÐzac

MIT to mègisto pl joc epanal yewn thc mejìdou

h akrÐbeia pou jètoume

Jètoume $k = 1$ kai phgaÐnoume sto epìmeno b ma

Anane¸noume tok $(k = k + 1)$ kai phgaÐnoume sto epìmeno b ma

Elègqoume an $k > \text{MIT}$

An den isqÔei, jètoume $x_{k+1} = g(x_k)$ kai suneqÐzoume sto epìmeno b ma

An isqÔei phgaÐnoume sto b ma 6

Elègqoume èna perissìtera krit ria termatismoÔ (èstw $|x_{k+1} \quad x_k| \quad)$

An den isqÔoun, suneqÐzoume sto b ma 3

An isqÔoun phgaÐnoume sto b ma 6

Termatismìc mejìdou kai epistrof exìdwn

Kajorismìc eisìdwn mejìdou

- g o tÔpoc thc sunˆrthshc
- x_0 h pr¸th prosèggish thc rÐzac
- MIT to mègisto pl joc epanal yewn thc mejìdou
  - h akrÐbeia pou jètoume

Jètoume $k = 1$ kai phgaÐnoume sto epìmeno b ma

Anane¸noume tok ($k = k + 1$) kai phgaÐnoume sto epìmeno b ma

Elègqoume an $k >$ MIT

- An den isqÔei, jètoume $x_{k+1} = g(x_k)$ kai suneqÐzoume sto epìmeno b ma
- An isqÔei phgaÐnoume sto b ma 6

Elègqoume èna perissìtera krit ria termatismoÔ (èstw jx_{k+1} x_k j)

- An den isqÔoun, suneqÐzoume sto b ma 3
- An isqÔoun phgaÐnoume sto b ma 6

Termatismìc mejìdou kai epistrof exìdwn

- x_k h teleutaÐa prosèggish thc rÐzac

Kajorismìc eisìdwn mejìdou

- g o tÔpoc thc sunˆrthshc
- x_0 h pr¸th prosèggish thc rÐzac
- MIT to mègisto pl joc epanal yewn thc mejìdou
- h akrÐbeia pou jètoume

Jètoume $k = 1$ kai phgaÐnoume sto epìmeno b ma

Anane¸noume tok $(k = k + 1)$ kai phgaÐnoume sto epìmeno b ma

Elègqoume an $k >$ MIT

- An den isqÔei, jètoume $x_{k+1} = g(x_k)$ kai suneqÐzoume sto epìmeno b ma
- An isqÔei phgaÐnoume sto b ma 6

Elègqoume èna perissìtera krit ria termatismoÔ (èstw $jx_{k+1} \quad x_k j \quad$)

- An den isqÔoun, suneqÐzoume sto b ma 3
- An isqÔoun phgaÐnoume sto b ma 6

Termatismìc mejìdou kai epistrof exìdwn

- x_k h teleutaÐa prosèggish thc rÐzac
- $g(x_k)$ h tim thc sunˆrthshc gia thn prosèggish

Kajorismìc eisìdwn mejìdou

- g o tÔpoc thc sunˆrthshc
- x_0 h pr¸th prosèggish thc rÐzac
- MIT to mègisto pl joc epanal yewn thc mejìdou
  - h akrÐbeia pou jètoume

Jètoume $k = 1$ kai phgaÐnoume sto epìmeno b ma

Anane¸noume tok $(k = k + 1)$ kai phgaÐnoume sto epìmeno b ma

Elègqoume an $k >$ MIT

- An den isqÔei, jètoume $x_{k+1} = g(x_k)$ kai suneqÐzoume sto epìmeno b ma
- An isqÔei phgaÐnoume sto b ma 6

Elègqoume èna perissìtera krit ria termatismoÔ (èstw jx_{k+1} $x_k j$)

- An den isqÔoun, suneqÐzoume sto b ma 3
- An isqÔoun phgaÐnoume sto b ma 6

Termatismìc mejìdou kai epistrof exìdwn

- x_k h teleutaÐa prosèggish thc rÐzac
- $g(x_k)$ h tim thc sunˆrthshc gia thn prosèggish
- k to pl joc epanal yewn pou èginan

Anagˆgoun to prìblhma thc epÐlushc enìc n diˆstatou mh-grammikoÔ sust matoc se aploÔsterec mh-grammikèc exis¸seic miac metablht c

Anagˆgoun to prìblhma thc epÐlushc enìc n diˆstatou mh-grammikoÔ sust matoc se aploÔsterec mh-grammikèc exis¸seic miac metablht c

Proèrqontai apì th grammik Gauss-Seidel, h opoÐa upologÐzei thn epìmenh sunist¸sa x_i^{k+1}, epilÔontac wc proc x_i th grammik exÐswsh

$$\sum_{j=1}^{i\ 1} {}_{ij} x_j^{k+1} + {}_{ii} x_i + \sum_{j=i+1}^{n} {}_{ij} x_j^{k} \quad b_i = 0$$

Anagˆgoun to prìblhma thc epÐlushc enìc n diˆstatou mh-grammikoÔ sust matoc se aploÔsterec mh-grammikèc exis¸seic miac metablht c

Proèrqontai apì th grammik Gauss-Seidel, h opoÐa upologÐzei thn epìmenh sunist¸sa x_i^{k+1}, epilÔontac wc proc x_i th grammik exÐswsh

$$\sum_{j=1}^{i\ 1} {}_{ij}\, x_j^{k+1} + {}_{ii}\, x_i + \sum_{j=i+1}^{n} {}_{ij}\, x_j^{k} \quad b_i = 0$$

H parapˆnw diadikasÐa mporeÐ na efarmosteÐ kai sthn perÐptwsh thc arijmhtik c epÐlushc susthmˆtwn mh-grammik¸n exis¸sewn

Anagˆgoun to prìblhma thc epÐlushc enìc n diˆstatou mh-grammikoÔ sust matoc se aploÔsterec mh-grammikèc exis¸seic miac metablht c

Proèrqontai apì th grammik Gauss-Seidelh opoÐa upologÐzei thn epìmenh sunist¸sa x_i^{k+1}, epilÔontac wc proc x_i th grammik exÐswsh

$$\sum_{j=1}^{i-1} {}_{ij}\, x_j^{k+1} + {}_{ii}\, x_i + \sum_{j=i+1}^{n} {}_{ij}\, x_j^{k} \quad b_i = 0$$

H parapˆnw diadikasÐa mporeÐ na efarmosteÐ kai sthn perÐptwsh thc arijmhtik c epÐlushc susthmˆtwn mh-grammik¸n exis¸sewn

Se aut thn perÐptwsh, to x_i^{k+1} epitugqˆnetai lÔnontac th mh-grammik exÐswsh miac metablht c

$$f_i(x_1^{k+1}; x_2^{k+1}; :::; x_{i-1}^{k+1}; x; x_{i+1}^{k}; :::; x_n^{k}) = 0$$

ìpou metablhtèc stajerèc

Anagˆgoun to prìblhma thc epÐlushc enìc n diˆstatou mh-grammikoÔ sust matoc se aploÔsterec mh-grammikèc exis¸seic miac metablht c

Proèrqontai apì th grammik Gauss-Seidel h opoÐa upologÐzei thn epìmenh sunist¸sa x_i^{k+1}, epilÔontac wc proc x_i th grammik exÐswsh

$$\sum_{j=1}^{i\ 1} {}_{ij}\, x_j^{k+1} + {}_{ii}\, x_i + \sum_{j=i+1}^{n} {}_{ij}\, x_j^{k} \quad b_i = 0$$

H parapˆnw diadikasÐa mporeÐ na efarmosteÐ kai sthn perÐptwsh thc arijmhtik c epÐlushc susthmˆtwn mh-grammik¸n exis¸sewn

Se aut thn perÐptwsh, to x_i^{k+1} epitugqˆnetai lÔnontac th mh-grammik exÐswsh miac metablht c

$$f_i(x_1^{k+1}; x_2^{k+1}; :::; x_{i\ 1}^{k+1}; x; x_{i+1}^{k}; :::; x_n^{k}) = 0$$

ìpou metablhtèc stajerèc

An h lÔsh thc parapˆnw sqeshc eÐnai h $\hat{x}_i$, jètoume

$$x_i^{k+1} = \hat{x}_i$$

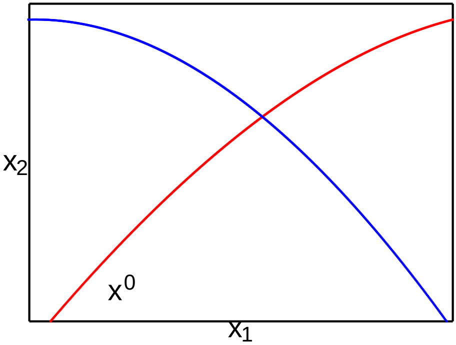
Jewr¸ntac kai mia parˆmetro qalˆrwshc, to epanalhptikì sq ma gÐnetai

$$x_i^{k+1} = x_i^k + !_k(\hat{x}_i \quad x_i^k)$$

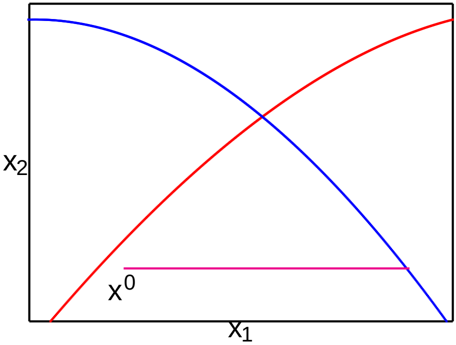
Jewr¸ntac kai mia par˜metro qal˜rwshc, to epanalhptikì sq ma gÐnetai

$$x_i^{k+1} = x_i^k + !_k(\hat{x}_i \quad x_i^k)$$

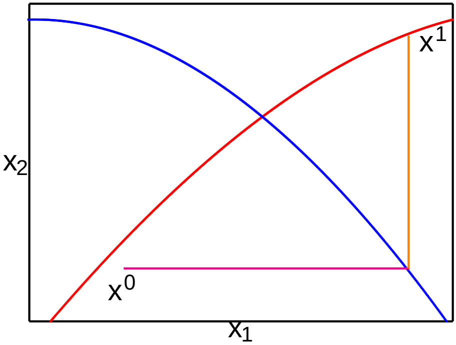
H parap˜nw mèjodoc èqei nìhma mìno ìtan oi exis¸seic f_i èqoun monadikèc lÔseic sth dosmènh perioq



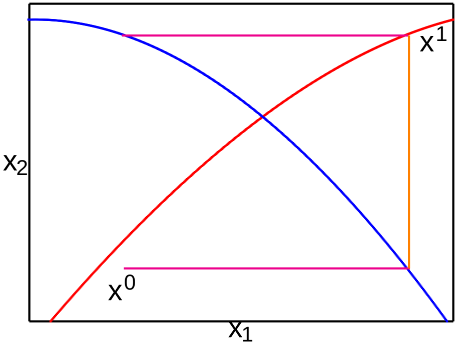
x^0	



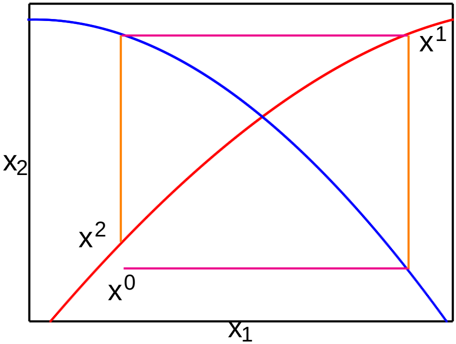
x^0 kratˆme x_2 stajerì



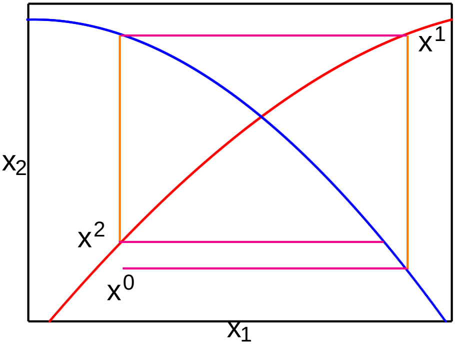
x^0 kratˆme x_2 stajerì kratˆme x_1 stajerì	EÔresh x^1



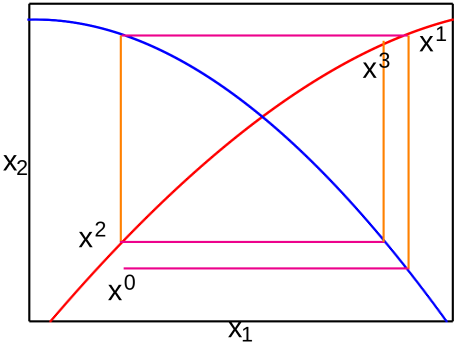
x^0 kratˆme x_2 stajerì kratˆme x_1 stajerì	EÔresh x^1
x^1 kratˆme x_2 stajerì	



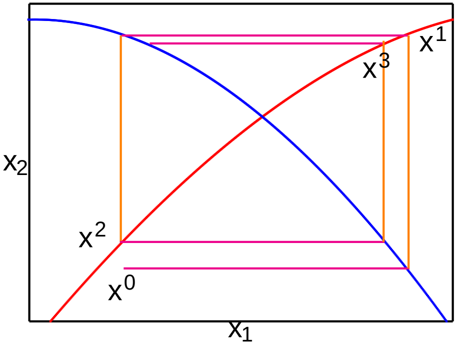
x^0	kratˆme x_2 stajerì	kratˆme x_1 stajerì	EÔreshx^1
x^1	kratˆme x_2 stajerì	kratˆme x_1 stajerì	EÔreshx^2



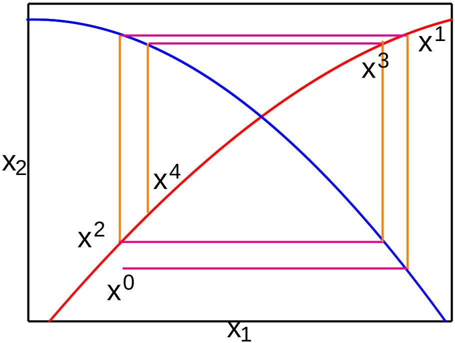
x^0	kratˆme x_2 stajerì	kratˆme x_1 stajerì	EÔreshx x^1
x^1	kratˆme x_2 stajerì	kratˆme x_1 stajerì	EÔreshx x^2
x^2	kratˆme x_2 stajerì		



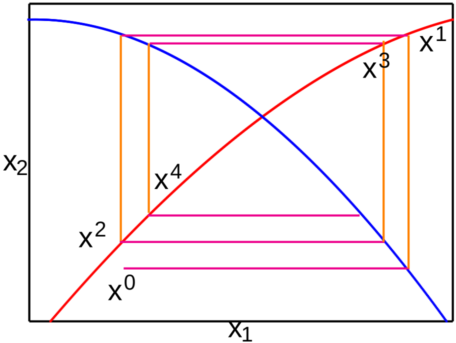
x^0	kratˆme x_2 stajerì	kratˆme x_1 stajerì	EÔreshx^1
x^1	kratˆme x_2 stajerì	kratˆme x_1 stajerì	EÔreshx^2
x^2	kratˆme x_2 stajerì	kratˆme x_1 stajerì	EÔreshx^3



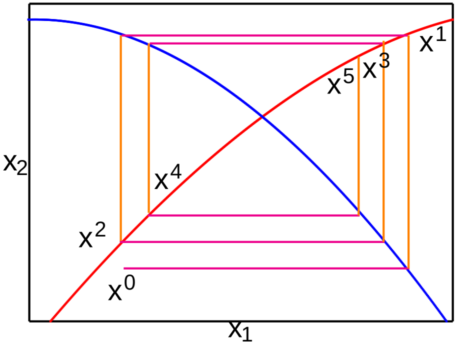
x^0	krat^me x_2 stajerì	krat^me x_1 stajerì	EÔreshx^1
x^1	krat^me x_2 stajerì	krat^me x_1 stajerì	EÔreshx^2
x^2	krat^me x_2 stajerì	krat^me x_1 stajerì	EÔreshx^3
x^3	krat^me x_2 stajerì		



x^0	kratˆme x_2 stajerì	kratˆme x_1 stajerì	EÔreshx^1
x^1	kratˆme x_2 stajerì	kratˆme x_1 stajerì	EÔreshx^2
x^2	kratˆme x_2 stajerì	kratˆme x_1 stajerì	EÔreshx^3
x^3	kratˆme x_2 stajerì	kratˆme x_1 stajerì	EÔreshx^4



x^0	kratˆme x_2 stajerì	kratˆme x_1 stajerì	EÔreshx^1
x^1	kratˆme x_2 stajerì	kratˆme x_1 stajerì	EÔreshx^2
x^2	kratˆme x_2 stajerì	kratˆme x_1 stajerì	EÔreshx^3
x^3	kratˆme x_2 stajerì	kratˆme x_1 stajerì	EÔreshx^4
x^4	kratˆme x_2 stajerì		



x^0	kratˆme x_2 stajerì	kratˆme x_1 stajerì	EÔreshx^1
x^1	kratˆme x_2 stajerì	kratˆme x_1 stajerì	EÔreshx^2
x^2	kratˆme x_2 stajerì	kratˆme x_1 stajerì	EÔreshx^3
x^3	kratˆme x_2 stajerì	kratˆme x_1 stajerì	EÔreshx^4
x^4	kratˆme x_2 stajerì	kratˆme x_1 stajerì	EÔreshx^5

SuneqÐzoume thn Ðdia diadikasÐa mèqri na broÔme èna shmeÐo pou proseggÐzei th lÔsh $x = (x_1; x_2)$

SuneqÐzoume thn Ðdia diadikasÐa mèqri na broÔme èna shmeÐo pou proseggÐzei th lÔsh $x = (x_1; x_2)$

Stamatˆme ìtan plhreÐtai èna perissìtera krit ria termatismoÔ

Mh sÔgklish thc mejìdou

SÔgklish thc mejìdou

Mh sÔgklish thc mejìdou

H telik tim tou deÐkth epanal yewn, megalÔterh apì to dedomèno mègisto arijmì epanal yewn

SÔgklish thc mejìdou

Mh sÔgklish thc mejìdou

H telik tim tou deÐkth epanal yewn, megalÔterh apì to dedomèno mègisto arijmì epanal yewn

SÔgklish thc mejìdou

DÔo diadoqikèc proseggÐseic, kontˆ (akrÐbeia) h mÐa sthn ˆllh

$\|x^{k+1} - x^k\|_2$

Mh sÔgklish thc mejìdou

H telik tim tou deÐkth epanal yewn, megalÔterh apì to dedomèno mègisto arijmì epanal yewn

SÔgklish thc mejìdou

DÔo diadoqikèc proseggÐseic, kont̂ (akrÐbeia) h mÐa sthn ̂llh

$\|x^{k+1} \quad x^k\| \quad _2$

H nìrma thc F_n kont̂ sto 0

$\|F_n(x^{k+1})\| \quad _1$

Mh sÔgklish thc mejìdou

H telik tim tou deÐkth epanal yewn, megalÔterh apì to dedomèno mègisto arijmì epanal yewn

SÔgklish thc mejìdou

DÔo diadoqikèc proseggÐseic, kont˘ (akrÐbeia) h mÐa sthn ˘llh

$\|x^{k+1} \quad x^k\| \quad _2$

H nìrma thc F_n kont˘ sto 0

$\|F_n(x^{k+1})\| \quad _1$

Prosoq

An˘loga me th fÔsh thc sun˘rthshc, mporeÐ na isqÔei k˘poio krit rio termatismoÔ kai h mèjodoc na apoklÐnei

EĐsodoi-'Exodoi

EĐsodoi

EĐsodoi-'Exodoi

EĐsodoi

n to pl joc twn exis¸sewn kai twn agn¸stwn

EÐsodoi-'Exodoi

EÐsodoi

- n to pl joc twn exis¸sewn kai twn agn¸stwn
- $F_n(x)$ o tÔpoc thc sun˘rthshc

EÐsodoi-'Exodoi

EÐsodoi

- n to pl joc twn exis¸sewn kai twn agn¸stwn
- $F_n(x)$ o tÔpoc thc sunˆrthshc
- x_0 h arqik prosèggish thc rÐzac

EÐsodoi-'Exodoi

EÐsodoi

- n to pl joc twn exis¸sewn kai twn agn¸stwn
- $F_n(x)$ o tÔpoc thc sunˆrthshc
- x_0 h arqik prosèggish thc rÐzac
- $!_k$ oi parˆmetroi qalˆrwshc

EÐsodoi-'Exodoi

EÐsodoi

- n to pl joc twn exis¸sewn kai twn agn¸stwn
- $F_n(x)$ o tÔpoc thc sunˆrthshc
- x_0 h arqik prosèggish thc rÐzac
- !$_k$ oi parˆmetroi qalˆrwshc
- MIT to mègisto pl joc epanal yewn thc mejìdou

EÐsodoi-'Exodoi

EÐsodoi

- n to pl joc twn exis¸sewn kai twn agn¸stwn
- $F_n(x)$ o tÔpoc thc sunˆrthshc
- x_0 h arqik prosèggish thc rÐzac
- ! $_k$ oi parˆmetroi qalˆrwshc
- MIT to mègisto pl joc epanal yewn thc mejìdou
- $_1$ h akrÐbeia pou jètoume gia tox

EÐsodoi-'Exodoi

EÐsodoi

- n to pl joc twn exis¸sewn kai twn agn¸stwn
- $F_n(x)$ o tÔpoc thc sunˆrthshc
- x_0 h arqik prosèggish thc rÐzac
- ! $_k$ oi parˆmetroi qalˆrwshc
- MIT to mègisto pl joc epanal yewn thc mejìdou
- $_1$ h akrÐbeia pou jètoume gia tox
- $_2$ h akrÐbeia pou jètoume gia thn$F_n(x)$

EÐsodoi-'Exodoi

EÐsodoi

- n to pl joc twn exis¸sewn kai twn agn¸stwn
- $F_n(x)$ o tÔpoc thc sunˆrthshc
- x_0 h arqik prosèggish thc rÐzac
- ! k oi parˆmetroi qalˆrwshc
- MIT to mègisto pl joc epanal yewn thc mejìdou
- 1 h akrÐbeia pou jètoume gia tox
- 2 h akrÐbeia pou jètoume gia thn$F_n(x)$

'Exodoi

EÐsodoi-'Exodoi

EÐsodoi

- n to pl joc twn exis¸sewn kai twn agn¸stwn
- $F_n(x)$ o tÔpoc thc sunˆrthshc
- x_0 h arqik prosèggish thc rÐzac
- $!_k$ oi parˆmetroi qalˆrwshc
- MIT to mègisto pl joc epanal yewn thc mejìdou
- $_1$ h akrÐbeia pou jètoume gia tox
- $_2$ h akrÐbeia pou jètoume gia thn$F_n(x)$

'Exodoi

- x^k h teleutaÐa prosèggish thc rÐzac

EÐsodoi-'Exodoi

EÐsodoi

- n to pl joc twn exis¸sewn kai twn agn¸stwn
- $F_n(x)$ o tÔpoc thc sunˆrthshc
- x_0 h arqik prosèggish thc rÐzac
- $!_k$ oi parˆmetroi qalˆrwshc
- MIT to mègisto pl joc epanal yewn thc mejìdou
- $_1$ h akrÐbeia pou jètoume gia tox
- $_2$ h akrÐbeia pou jètoume gia thn$F_n(x)$

'Exodoi

- x^k h teleutaÐa prosèggish thc rÐzac
- $F_n(x^k)$ h tim thc sunˆrthshc sthn k epanˆlhyh

EÐsodoi-'Exodoi

EÐsodoi

- n to pl joc twn exis¸sewn kai twn agn¸stwn
- $F_n(x)$ o tÔpoc thc sun˚rthshc
- x_0 h arqik prosèggish thc rÐzac
- ! $_k$ oi par˚metroi qal˚rwshc
- MIT to mègisto pl joc epanal yewn thc mejìdou
- $_1$ h akrÐbeia pou jètoume gia tox
- $_2$ h akrÐbeia pou jètoume gia thn$F_n(x)$

'Exodoi

- x^k h teleutaÐa prosèggish thc rÐzac
- $F_n(x^k)$ h tim thc sun˚rthshc sthn k epan˚lhyh
- k to pl joc epanal yewn pou èginan

Kajorismìc eisìdwn mejìdou

Kajorismìc eisìdwn mejìdou

Jètoume $k = 1$

Kajorismìc eisìdwn mejìdou

Jètoume k = 1

Elègqoume an k < MIT

An den isqÔei, phgaÐnoume sto b ma 9. Diaforetikˆ, auxˆnoume to k kai suneqÐzoume sto eplomeno b ma

Kajorismìc eisìdwn mejìdou

Jètoume $k = 1$

Elègqoume an $k <$ MIT

An den isqÔei, phgaÐnoume sto b ma 9. Diaforetikˆ, auxˆnoume to k kai suneqÐzoume sto eplomeno b ma

Elègqoume an $\|F_n(x^k)\|_2$

An den isqÔei, suneqÐzoume sto epìmeno b ma. Diaforetikˆ, phgaÐnoume sto b ma 9

Kajorismìc eisìdwn mejìdou

Jètoume k = 1

Elègqoume ank < MIT

An den isqÔei, phgaÐnoume sto b ma 9. Diaforetikˆ, auxˆnoume to k kai suneqÐzoume sto eplomeno b ma

Elègqoume an$kF_n(x^k)k_2$

An den isqÔei, suneqÐzoume sto epìmeno b ma. Diaforetikˆ, phgaÐnoume sto b ma 9

Jètoume i = 0

Kajorismìc eisìdwn mejìdou

Jètoume k = 1

Elègqoume an k < MIT

An den isqÔei, phgaÐnoume sto b ma 9. Diaforetik̂, aux̂noume to k kai suneqÐzoume sto eplomeno b ma

Elègqoume an $\|F_n(x^k)\|_2$

An den isqÔei, suneqÐzoume sto epìmeno b ma. Diaforetik̂, phgaÐnoume sto b ma 9

Jètoume i = 0

An isqÔei i < n, aux̂noume to i kai phgaÐnoume sto epìmeno b ma. Diaforetik̂, phgaÐnoume sto b ma 8

Kajorismìc eisìdwn mejìdou

Jètoume $k = 1$

Elègqoume an $k <$ MIT
An den isqÔei, phgaÐnoume sto b ma 9. Diaforetik^, aux^noume to k kai suneqÐzoume sto eplomeno b ma

Elègqoume an $kF_n(x^k)k_2$
An den isqÔei, suneqÐzoume sto epìmeno b ma. Diaforetik^, phgaÐnoume sto b ma 9

Jètoume $i = 0$

An isqÔei $i < n$, aux^noume to i kai phgaÐnoume sto epìmeno b ma. Diaforetik^, phgaÐnoume sto b ma 8

BrÐskoume th lÔsh $\hat{x}_i$ thc monodi^stathc mh-grammik c exÐswshc $f_i(x_1^{k+1}; x_2^{k+1}; \ldots; x_{i-1}^{k+1}; x; x_{i+1}^{k+1}; \ldots; x_n^{k+1}) = 0$ wc proc x, jètoume $x_i^{k+1} = x_i^k + !_k(\hat{x}_i - x_i^k)$ kai p gaine sto b ma 6

Kajorismìc eisìdwn mejìdou

Jètoume $k = 1$

Elègqoume an $k <$ MIT
An den isqÔei, phgaÐnoume sto b ma 9. Diaforetikˆ, auxˆnoume to k kai suneqÐzoume sto eplomeno b ma

Elègqoume an $kF_n(x^k)k \quad 2$
An den isqÔei, suneqÐzoume sto epìmeno b ma. Diaforetikˆ, phgaÐnoume sto b ma 9

Jètoume $i = 0$

An isqÔei $i < n$, auxˆnoume to i kai phgaÐnoume sto epìmeno b ma. Diaforetikˆ, phgaÐnoume sto b ma 8

BrÐskoume th lÔsh $\hat{x}_i$ thc monodiˆstathc mh-grammik c exÐswshc $f_i(x_1^{k+1}; x_2^{k+1}; :::; x_{i-1}^{k+1}; x; x_{i+1}^{k+1}; :::; x_n^{k+1}) = 0$ wc proc x, jètoume $x_i^{k+1} = x_i^k + !_k(\hat{x}_i \quad x_i^k)$ kai p gaine sto b ma 6

Elègqoume an $kx_i^{k+1} \quad x_i^k k \quad 1$
An den isqÔei, suneqÐzoume sto b ma 3. Diaforetikˆ, phgaÐnoume sto b ma 9

Kajorismìc eisìdwn mejìdou

Jètoume $k = 1$

Elègqoume an $k <$ MIT
An den isqÔei, phgaÐnoume sto b ma 9. Diaforetik˘, aux˘noume to k kai suneqÐzoume sto eplomeno b ma

Elègqoume an $\|F_n(x^k)\|$ 2
An den isqÔei, suneqÐzoume sto epìmeno b ma. Diaforetik˘, phgaÐnoume sto b ma 9

Jètoume $i = 0$

An isqÔei $i < n$, aux˘noume to i kai phgaÐnoume sto epìmeno b ma. Diaforetik˘, phgaÐnoume sto b ma 8

BrÐskoume th lÔsh $\hat{x}_i$ thc monodi˘stathc mh-grammik c exÐswshc $f_i(x_1^{k+1}; x_2^{k+1}; \ldots; x_{i-1}^{k+1}; x; x_{i+1}^{k}; \ldots; x_n^{k+1}) = 0$ wc proc x, jètoume $x_i^{k+1} = x_i^k + \omega_k(\hat{x}_i - x_i^k)$ kai p gaine sto b ma 6

Elègqoume an $\|x_i^{k+1} - x_i^k\|$ 1
An den isqÔei, suneqÐzoume sto b ma 3. Diaforetik˘, phgaÐnoume sto b ma 9

Termatismìc mejìdou kai epistrof exìdwn

Anagˆgoun to prìblhma thc epÐlushc enìc n diˆstatou mh-grammikoÔ sust matoc se aploÔsterec mh-grammikèc exis¸seic miac metablht c

Anagˆgoun to prìblhma thc epÐlushc enìc n diˆstatou mh-grammikoÔ sust matoc se aploÔsterec mh-grammikèc exis¸seic miac metablht c

Proèrqontai apì th grammik Jacobi, h opoÐa upologÐzei thn epìmenh sunist¸sa x_i^{k+1}, epilÔontac wc proc x_i th grammik exÐswsh

$$\sum_{j=1}^{i-1} {}_{ij} x_j^{k} + {}_{ii} x_i + \sum_{j=i+1}^{n} {}_{ij} x_j^{k} \quad b_i = 0$$

Anagˆgoun to prìblhma thc epÐlushc enìc n diˆstatou mh-grammikoÔ sust matoc se aploÔsterec mh-grammikèc exis¸seic miac metablht c

Proèrqontai apì th grammik Jacobi, h opoÐa upologÐzei thn epìmenh sunist¸sa x_i^{k+1}, epilÔontac wc proc x_i th grammik exÐswsh

$$\sum_{j=1}^{i\ 1} {}_{ij}\, x_j^k + {}_{ii}\, x_i + \sum_{j=i+1}^{n} {}_{ij}\, x_j^k \quad b_i = 0$$

H parapˆnw diadikasÐa mporeÐ na efarmosteÐ kai sthn perÐptwsh thc arijmhtik c epÐlushc susthmˆtwn mh-grammik¸n exis¸sewn

Anagˆgoun to prìblhma thc epÐlushc enìc n diˆstatou mh-grammikoÔ sust matoc se aploÔsterec mh-grammikèc exis¸seic miac metablht c

Proèrqontai apì th grammik Jacobi, h opoÐa upologÐzei thn epìmenh sunist¸sa x_i^{k+1}, epilÔontac wc proc x_i th grammik exÐswsh

$$\sum_{j=1}^{i-1} {}_{ij} x_j^k + {}_{ii} x_i + \sum_{j=i+1}^{n} {}_{ij} x_j^k \quad b_i = 0$$

H parapˆnw diadikasÐa mporeÐ na efarmosteÐ kai sthn perÐptwsh thc arijmhtik c epÐlushc susthmˆtwn mh-grammik¸n exis¸sewn

Se aut thn perÐptwsh, to x_i^{k+1} epitugqˆnetai lÔnontac th mh-grammik exÐswsh miac metablht c

$$f_i(x_1^k; x_2^k; \dots; x_{i-1}^k; x; x_{i+1}^k; \dots; x_n^k) = 0$$

ìpou metablhtèc stajerèc

Anag˘goun to prìblhma thc epÐlushc enìc n di˘statou mh-grammikoÔ sust matoc se aploÔsterec mh-grammikèc exis¸seic miac metablht c

Proèrqontai apì th grammik Jacobi, h opoÐa upologÐzei thn epìmenh sunist¸sa x_i^{k+1}, epilÔontac wc proc x_i th grammik exÐswsh

$$\sum_{j=1}^{i\;1} {}_{ij}\, x_j^{k} + {}_{ii}\, x_i + \sum_{j=i+1}^{n} {}_{ij}\, x_j^{k} \quad b_i = 0$$

H parap˘nw diadikasÐa mporeÐ na efarmosteÐ kai sthn perÐptwsh thc arijmhtik c epÐlushc susthm˘twn mh-grammik¸n exis¸sewn

Se aut thn perÐptwsh, to x_i^{k+1} epitugq˘netai lÔnontac th mh-grammik exÐswsh miac metablht c

$$f_i(x_1^k; x_2^k; :::; x_{i\;1}^k; x; x_{i+1}^k; :::; x_n^k) = 0$$

ìpou metablhtèc stajerèc

An h lÔsh thc parap˘nw sqeshc eÐnai h $\hat{x}_i$, jètoume

$$x_i^{k+1} = \hat{x}_i$$

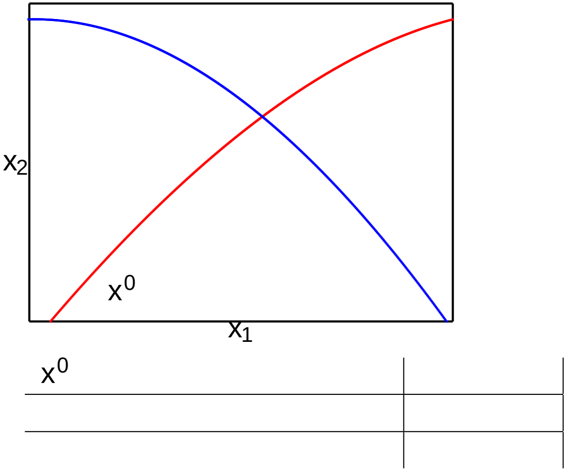
Jewr¸ntac kai mia parˆmetro qalˆrwshc, to epanalhptikì sq ma gÐnetai

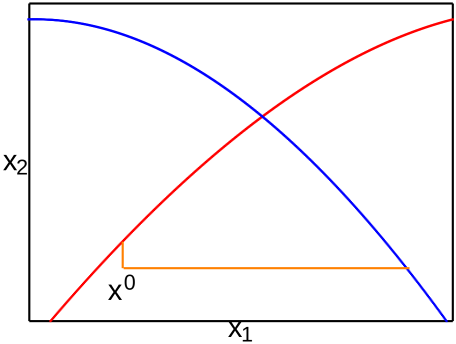
$$x_i^{k+1} = x_i^k + !_k(\hat{x}_i \quad x_i^k)$$

Jewr¸ntac kai mia parˆmetro qalˆrwshc, to epanalhptikì sq ma gĐnetai

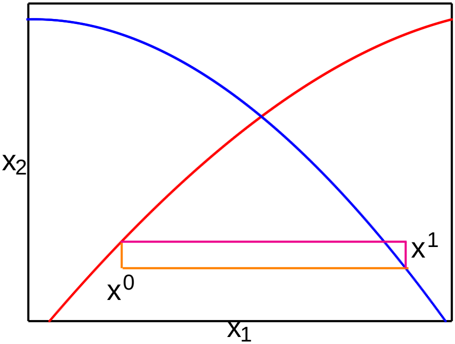
$$x_i^{k+1} = x_i^k + !_k(\hat{x}_i \quad x_i^k)$$

H parapˆnw mèjodoc èqei nìhma mìno ìtan oi exis¸seic f_i èqoun monadikèc lÔseic sth dosmènh perioq

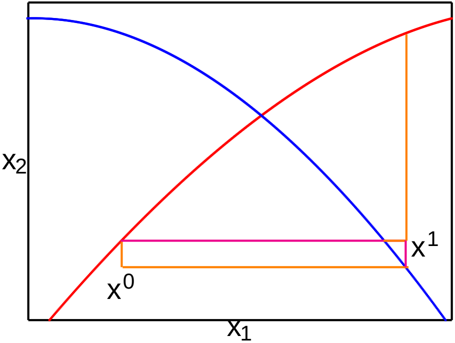

x_2
x^0
x_1
x^0



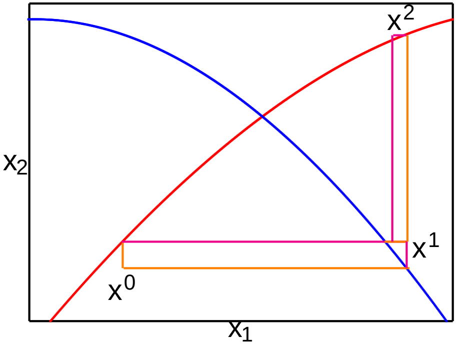
x^0 epilÔoume wc x_2 kai x_1



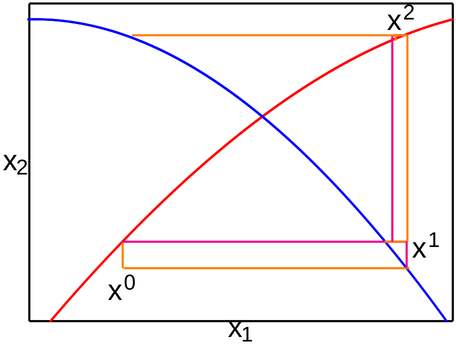
x^0 epilÔoume wc x_2 kai x_1	EÔresh x^1



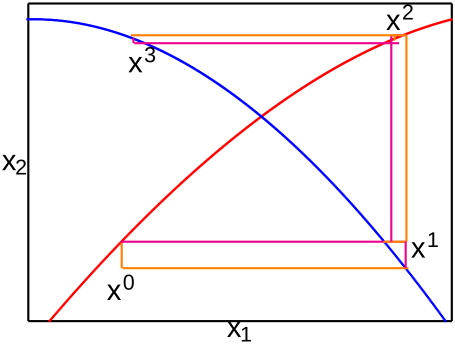
x^0	epilÔoume wc x_2 kai x_1	EÔresh x^1
x^1	epilÔoume wc x_2 kai x_1	



x^0	epilÔoume wc x_2 kai x_1	EÔresh x^1
x^1	epilÔoume wc x_2 kai x_1	EÔresh x^2



x^0	epilÔoume wc x_2 kai x_1	EÔresh x^1
x^1	epilÔoume wc x_2 kai x_1	EÔresh x^2
x^2	epilÔoume wc x_2 kai x_1	



x^0	epilÔoume wc x_2 kai x_1	EÔresh x^1
x^1	epilÔoume wc x_2 kai x_1	EÔresh x^2
x^2	epilÔoume wc x_2 kai x_1	EÔresh x^3

SuneqÐzoume thn Ðdia diadikasÐa mèqri na broÔme èna shmeÐo pou proseggÐzei th lÔsh $x = (x_1; x_2)$

SuneqÐzoume thn Ðdia diadikasÐa mèqri na broÔme èna shmeÐo pou proseggÐzei th lÔsh $x = (x_1; x_2)$

Stamat^me ìtan plhreÐtai èna perissìtera krit ria termatismoÔ

Mh sÔgklish thc mejìdou

SÔgklish thc mejìdou

Mh sÔgklish thc mejìdou

H telik tim tou deÐkth epanal yewn, megalÔterh apì to dedomèno mègisto arijmì epanal yewn

SÔgklish thc mejìdou

Mh sÔgklish thc mejìdou

H telik tim tou deÐkth epanal yewn, megalÔterh apì to dedomèno mègisto arijmì epanal yewn

SÔgklish thc mejìdou

DÔo diadoqikèc proseggÐseic, kontˆ (akrÐbeia) h mÐa sthn ˆllh

$\| x^{k+1} \quad x^k \|_2$

Mh sÔgklish thc mejìdou

H telik tim tou deÐkth epanal yewn, megalÔterh apì to dedomèno mègisto arijmì epanal yewn

SÔgklish thc mejìdou

DÔo diadoqikèc proseggÐseic, kont̂ (akrÐbeia) h mÐa sthn ̂llh

$kx^{k+1} \quad x^k k \quad {}_2$

H nìrma thc F_n kont̂ sto 0

$kF_n(x^{k+1})k \quad {}_1$

Mh sÔgklish thc mejìdou

H telik tim tou deĐkth epanal yewn, megalÔterh apì to dedomèno mègisto arijmì epanal yewn

SÔgklish thc mejìdou

DÔo diadoqikèc proseggĐseic, kontˆ (akrĐbeia) h mĐa sthn ˆllh

$$\|x^{k+1} - x^k\|_2$$

H nìrma thc F_n kontˆ sto 0

$$\|F_n(x^{k+1})\|_1$$

Prosoq

Anˆloga me th fÔsh thc sunˆrthshc, mporeĐ na isqÔei kˆpoio krit rio termatismoÔ kai h mèjodoc na apoklĐnei

EĐsodoi-'Exodoi

EĐsodoi

EÐsodoi-'Exodoi

EÐsodoi

n to pl joc twn exis¸sewn kai twn agn¸stwn

EÐsodoi-'Exodoi

EÐsodoi

n to pl joc twn exis¸sewn kai twn agn¸stwn
$F_n(x)$ o tÔpoc thc sun˚rthshc

EÐsodoi-'Exodoi

EÐsodoi

n to pl joc twn exis¸sewn kai twn agn¸stwn
$F_n(x)$ o tÔpoc thc sun˘rthshc
x_0 h arqik prosèggish thc rÐzac

EÐsodoi-'Exodoi

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- n to pl joc twn exis¸sewn kai twn agn¸stwn
- $F_n(x)$ o tÔpoc thc sunˆrthshc
- x_0 h arqik prosèggish thc rÐzac
- !$_k$ oi parˆmetroi qalˆrwshc

EÐsodoi-'Exodoi

EÐsodoi

- n to pl joc twn exis¸sewn kai twn agn¸stwn
- $F_n(x)$ o tÔpoc thc sunˆrthshc
- x_0 h arqik prosèggish thc rÐzac
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- MIT to mègisto pl joc epanal yewn thc mejìdou

EÐsodoi-'Exodoi

EÐsodoi

- n to pl joc twn exis¸sewn kai twn agn¸stwn
- $F_n(x)$ o tÔpoc thc sunˆrthshc
- x_0 h arqik prosèggish thc rÐzac
- $!_k$ oi parˆmetroi qalˆrwshc
- MIT to mègisto pl joc epanal yewn thc mejìdou
- $_1$ h akrÐbeia pou jètoume gia tox

EÐsodoi-'Exodoi

EÐsodoi

n to pl joc twn exis¸sewn kai twn agn¸stwn
$F_n(x)$ o tÔpoc thc sunˆrthshc
x_0 h arqik prosèggish thc rÐzac
$!_k$ oi parˆmetroi qalˆrwshc
MIT to mègisto pl joc epanal yewn thc mejìdou
$_1$ h akrÐbeia pou jètoume gia tox
$_2$ h akrÐbeia pou jètoume gia thn$F_n(x)$

EÐsodoi-'Exodoi

EÐsodoi

- n to pl joc twn exis¸sewn kai twn agn¸stwn
- $F_n(x)$ o tÔpoc thc sunˆrthshc
- x_0 h arqik prosèggish thc rÐzac
- ! $_k$ oi parˆmetroi qalˆrwshc
- MIT to mègisto pl joc epanal yewn thc mejìdou
- $_1$ h akrÐbeia pou jètoume gia tox
- $_2$ h akrÐbeia pou jètoume gia thn$F_n(x)$

'Exodoi

EÐsodoi-'Exodoi

EÐsodoi

- n to pl joc twn exis¸sewn kai twn agn¸stwn
- $F_n(x)$ o tÔpoc thc sun˜rthshc
- x_0 h arqik prosèggish thc rÐzac
- $!_k$ oi par˜metroi qal˜rwshc
- MIT to mègisto pl joc epanal yewn thc mejìdou
- $_1$ h akrÐbeia pou jètoume gia tox
- $_2$ h akrÐbeia pou jètoume gia thn$F_n(x)$

'Exodoi

- x^k h teleutaÐa prosèggish thc rÐzac

EÐsodoi-'Exodoi

EÐsodoi

- n to pl joc twn exis¸sewn kai twn agn¸stwn
- $F_n(x)$ o tÔpoc thc sunˆrthshc
- x_0 h arqik prosèggish thc rÐzac
- ! $_k$ oi parˆmetroi qalˆrwshc
- MIT to mègisto pl joc epanal yewn thc mejìdou
- $_1$ h akrÐbeia pou jètoume gia tox
- $_2$ h akrÐbeia pou jètoume gia thn$F_n(x)$

'Exodoi

- x^k h teleutaÐa prosèggish thc rÐzac
- $F_n(x^k)$ h tim thc sunˆrthshc sthn k epanˆlhyh

EÐsodoi-'Exodoi

EÐsodoi

- n to pl joc twn exis¸sewn kai twn agn¸stwn
- $F_n(x)$ o tÔpoc thc sun˘rthshc
- x_0 h arqik prosèggish thc rÐzac
- $!_k$ oi par˘metroi qal˘rwshc
- MIT to mègisto pl joc epanal yewn thc mejìdou
- $_1$ h akrÐbeia pou jètoume gia to x
- $_2$ h akrÐbeia pou jètoume gia thn $F_n(x)$

'Exodoi

- x^k h teleutaÐa prosèggish thc rÐzac
- $F_n(x^k)$ h tim thc sun˘rthshc sthn k epan˘lhyh
- k to pl joc epanal yewn pou èginan

Kajorismìc eisìdwn mejìdou

Kajorismìc eisìdwn mejìdou

Jètoume $k = 1$

Kajorismìc eisìdwn mejìdou

Jètoume k = 1

Elègqoume an k < MIT

An den isqÔei, phgaÐnoume sto b ma 9. Diaforetikˆ, auxˆnoume to k kai suneqÐzoume sto eplomeno b ma

Kajorismìc eisìdwn mejìdou

Jètoume $k = 1$

Elègqoume an $k <$ MIT

An den isqÔei, phgaÐnoume sto b ma 9. Diaforetikˆ, auxˆnoume to k kai suneqÐzoume sto eplomeno b ma

Elègqoume an $\|F_n(x^k)\|_2$

An den isqÔei, suneqÐzoume sto epìmeno b ma. Diaforetikˆ, phgaÐnoume sto b ma 9

Kajorismìc eisìdwn mejìdou

Jètoume $k = 1$

Elègqoume an $k < MIT$
An den isqÔei, phgaÐnoume sto b ma 9. Diaforetikˆ, auxˆnoume to k kai suneqÐzoume sto eplomeno b ma

Elègqoume an $kF_n(x^k)k_2$
An den isqÔei, suneqÐzoume sto epìmeno b ma. Diaforetikˆ, phgaÐnoume sto b ma 9

Jètoume $i = 0$

Kajorismìc eisìdwn mejìdou

Jètoume k = 1

Elègqoume an k < MIT
An den isqÔei, phgaÐnoume sto b ma 9. Diaforetikˆ, auxˆnoume to k kai suneqÐzoume sto eplomeno b ma

Elègqoume an $kF_n(x^k)k_2$
An den isqÔei, suneqÐzoume sto epìmeno b ma. Diaforetikˆ, phgaÐnoume sto b ma 9

Jètoume i = 0

An isqÔei i < n, auxˆnoume to i kai phgaÐnoume sto epìmeno b ma. Diaforetikˆ, phgaÐnoume sto b ma 8

Kajorismìc eisìdwn mejìdou

Jètoume $k = 1$

Elègqoume an $k <$ MIT
An den isqÔei, phgaÐnoume sto b ma 9. Diaforetikˆ, auxˆnoume to k kai suneqÐzoume sto eplomeno b ma

Elègqoume an $\|F_n(x^k)\|_2$
An den isqÔei, suneqÐzoume sto epìmeno b ma. Diaforetikˆ, phgaÐnoume sto b ma 9

Jètoume $i = 0$

An isqÔei $i < n$, auxˆnoume to i kai phgaÐnoume sto epìmeno b ma. Diaforetikˆ, phgaÐnoume sto b ma 8

BrÐskoume th lÔsh $\hat{x}_i$ thc monodiˆstathc mh-grammik c exÐswshc $f_i(x_1^k; x_2^k; \ldots; x_{i-1}^k; x; x_{i+1}^k; \ldots; x_n^k) = 0$ wc proc x, jètoume $x_i^{k+1} = x_i^k + \omega_k(\hat{x}_i - x_i^k)$ kai p gaine sto b ma 6

Kajorismìc eisìdwn mejìdou

Jètoume $k = \quad 1$

Elègqoume an $k < $ MIT
An den isqÔei, phgaÐnoume sto b ma 9. Diaforetikˆ, auxˆnoume to k kai suneqÐzoume sto eplomeno b ma

Elègqoume an $kF_n(x^k)k \quad _2$
An den isqÔei, suneqÐzoume sto epìmeno b ma. Diaforetikˆ, phgaÐnoume sto b ma 9

Jètoume $i = 0$

An isqÔei $i < n$, auxˆnoume to i kai phgaÐnoume sto epìmeno b ma. Diaforetikˆ, phgaÐnoume sto b ma 8

BrÐskoume th lÔsh $\hat{x}_i$ thc monodiˆstathc mh-grammik c exÐswshc $f_i(x^k_1; x^k_2; :::; x^k_{i \quad 1}; x; x^k_{i+1}; :::; x^k_n) = 0$ wc proc x, jètoume $x_i^{k+1} = x_i^k + !_k(\hat{x}_i \quad x_i^k)$ kai p gaine sto b ma 6

Elègqoume an $kx_i^{k+1} \quad x_i^k k \quad 1$
An den isqÔei, suneqÐzoume sto b ma 3. Diaforetikˆ, phgaÐnoume sto b ma 9

Kajorismìc eisìdwn mejìdou

Jètoume k = 1

Elègqoume an k < MIT
An den isqÔei, phgaÐnoume sto b ma 9. Diaforetikˆ, auxˆnoume to k kai suneqÐzoume sto eplomeno b ma

Elègqoume an $kF_n(x^k)k$ 2
An den isqÔei, suneqÐzoume sto epìmeno b ma. Diaforetikˆ, phgaÐnoume sto b ma 9

Jètoume i = 0

An isqÔei i < n, auxˆnoume to i kai phgaÐnoume sto epìmeno b ma. Diaforetikˆ, phgaÐnoume sto b ma 8

BrÐskoume th lÔsh $\hat{x}_i$ thc monodiˆstathc mh-grammik c exÐswshc $f_i(x_1^k; x_2^k; :::; x_{i-1}^k; x; x_{i+1}^k; :::; x_n^k) = 0$ wc proc x, jètoume $x_i^{k+1} = x_i^k + !_k(\hat{x}_i - x_i^k)$ kai p gaine sto b ma 6

Elègqoume an $kx_i^{k+1} - x_i^k k$ 1
An den isqÔei, suneqÐzoume sto b ma 3. Diaforetikˆ, phgaÐnoume sto b ma 9

Termatismìc mejìdou kai epistrof exìdwn

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ΔΗΜΟΚΡΙΤΕΙΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΘΡΑΚΗΣ