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# ANALYSIS OF POWER ELECTRONIC CONVERTERS

## DC/AC

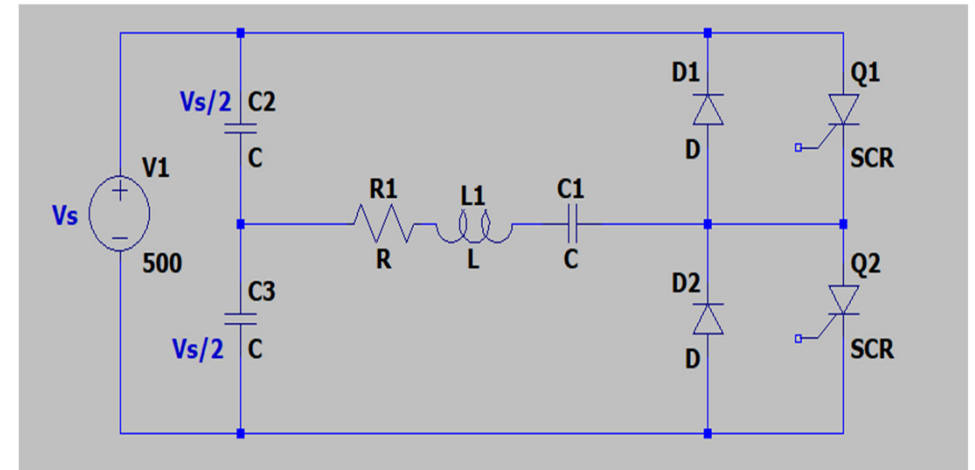
EXERCISES 32-33

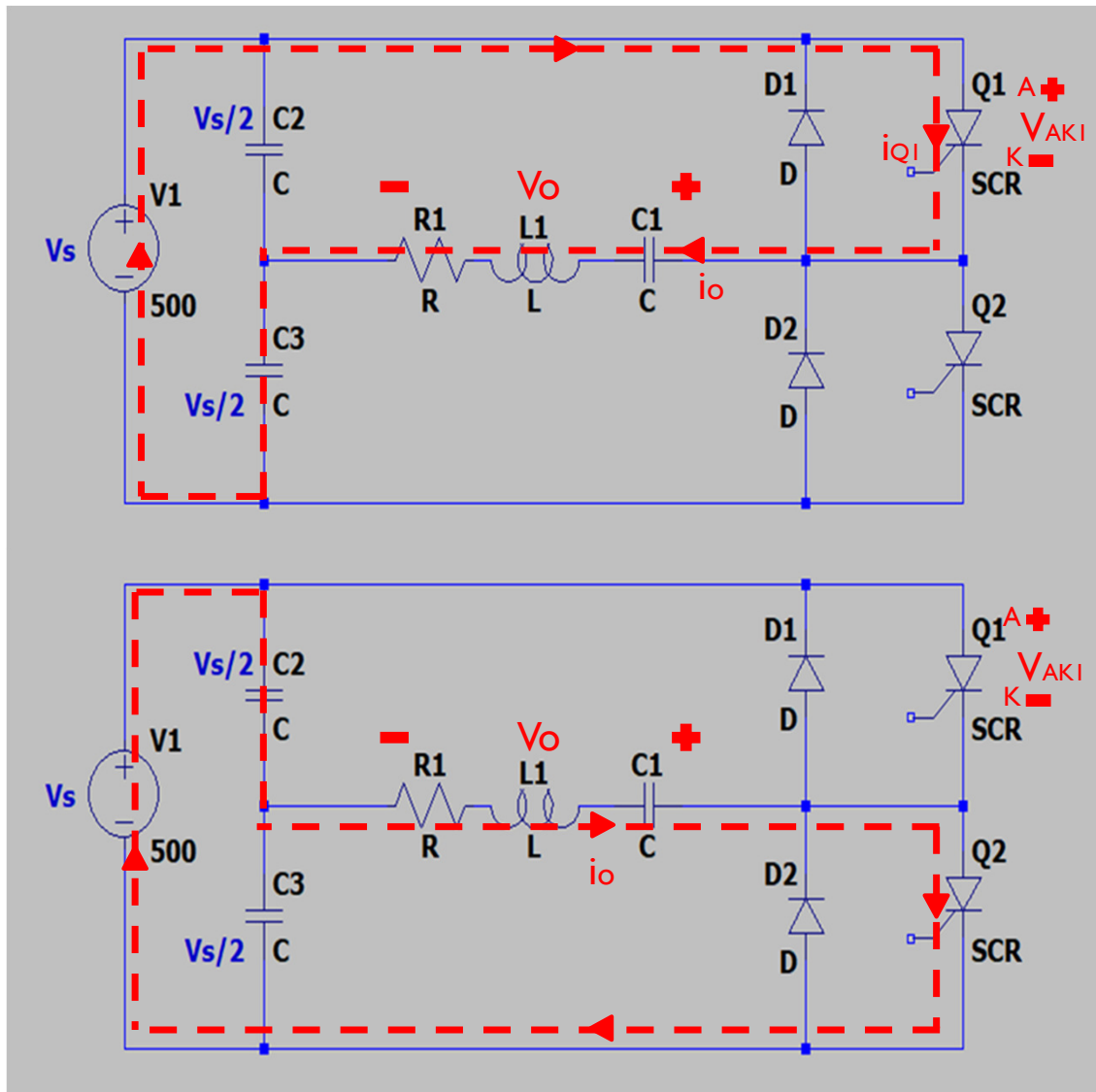
Kiosi Anastasia 57590  
Tihalas Andreas 57498

## EXERCISE 32

For the single-phase half-bridge inverter of the figure on the right, the following parameters are given :  
 $V = 500\text{V}$ ,  $T = \text{trigger pulse period} = 2000\mu\text{sec}$ ,  $\omega L = 10\Omega$  and  $1/\omega C = 10\Omega$ .

- A) Draw the waveforms of  $v_0$ ,  $i_0$ ,  $i_{Q1}$ ,  $i_{D1}$  and  $V_{AK1}$ . Ignore the higher harmonic components of the output current.
- B) Calculate the RMS and average value of the thyristor and diode current.
- C) Given that the thyristors' turn-off time is  $50\mu\text{sec}$ , determine if the thyristors need forced commutation to turn-off and calculate the value of their current when they turn-off.





Note : Due to the load characteristics,  $\omega L = 10\Omega = 1/\omega C$ , current will not flow through the FWDs, thus their rms and average current values are zero.

State 1 : 0 – T/2

- Q1 ON
- $V_o = V_s/2 = 250V$
- $V_{AK1} = 0$
- $i_o$  positive
- $i_{Q1} = i_o$
- $i_{D1} = 0$

State 2 : T/2 – T

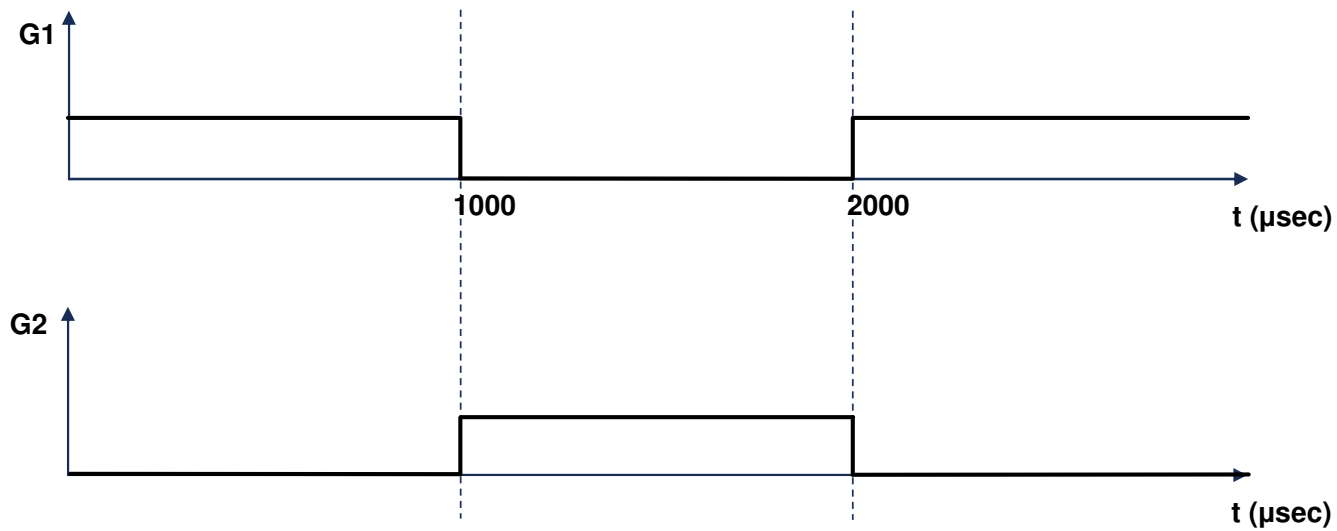
- Q2 ON
- $V_o = -V_s/2 = -250V$
- $V_{AK1} = V_s/2 + V_s/2 = V_s = 500V$
- $i_o$  negative
- $i_{Q1} = 0$
- $i_{D1} = 0$

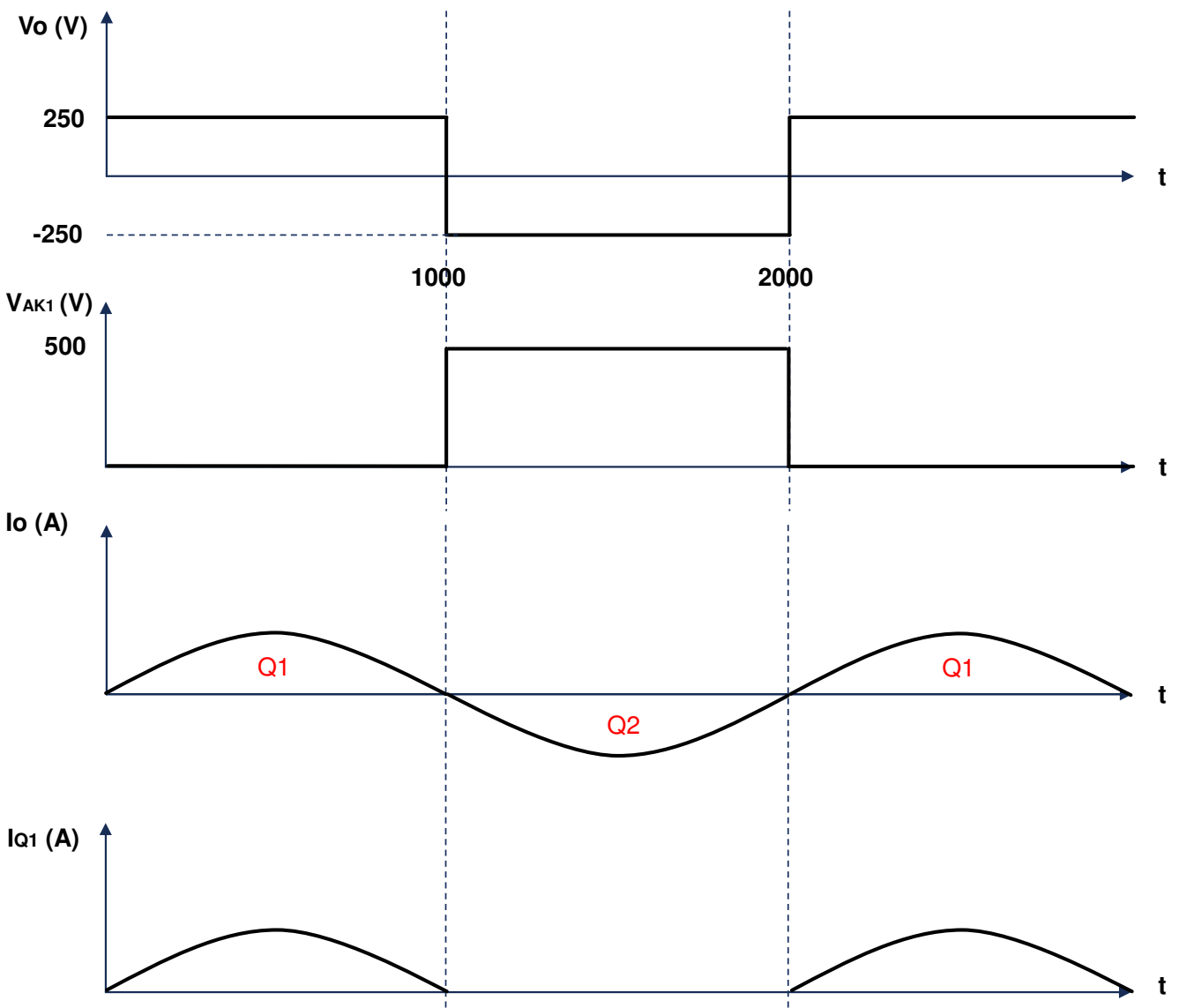
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$$A) \varphi = \tan^{-1} \left( \frac{\omega L - 1/\omega C}{R} \right) = 0$$

The output current's fundamental harmonic is in phase with output voltage's fundamental harmonic.

The circuit's waveforms follow:





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B) According to the waveforms and by using Fourier series analysis, we have :

Output voltage

$$v_0 = \sum_{n=1,3,5,\dots}^{\infty} \frac{2V}{n\pi} \sin(n\omega t)$$

Output current

$$i_0 = \sum_{n=1,3,5,\dots}^{\infty} \frac{2V}{n\pi|Z_n|} \sin(n\omega t - \varphi_n)$$

Where:

- $\varphi_n$ : the phase angle between current's and voltage's  $n_{nth}$  harmonic components.  
and  $\varphi_1 = \tan^{-1} \left( \frac{\omega L - 1/\omega C}{R} \right) = 0$
- $Z_n = \sqrt{R^2 + (nL\omega - 1/n\omega C)^2}$  : the load's impedance on frequency of the  $n_{nth}$  harmonic component.  
 $Z_1 = \sqrt{1.2^2 + (10 - 10)^2} = 1.2\Omega$

- Current's first harmonic component:

$$i_{o,1} = \frac{2 \times 500}{1.2 \times \pi} \sin(\omega t)$$

- Thyristor average current value :

$$\bar{I}_Q = \frac{1}{2\pi} \int_0^\pi \frac{1000}{1.2\pi} \sin \omega t d\omega t = \frac{1}{2\pi} \frac{1000}{1.2\pi} (-\cos \pi + \cos 0) = 84.43A$$

- Thyristor rms current value :

$$\begin{aligned} \tilde{I}_Q &= \left[ \frac{1}{2\pi} \int_0^\pi \left( \frac{1000}{1.2\pi} \sin \omega t \right)^2 d\omega t \right]^{1/2} = \left[ \frac{1}{2\pi} \frac{1000^2}{1.2^2 \pi^2} \int_0^\pi \frac{1}{2} (1 - \cos 2\omega t) d\omega t \right]^{1/2} = \\ &= \left[ \frac{1}{2\pi} \frac{1000^2}{1.2^2 \pi^2} \left[ \frac{\pi}{2} - \frac{1}{2} (\sin \pi - \sin 0) \right] \right]^{1/2} = \left[ \frac{1}{2\pi} \frac{1000^2}{1.2^2 \pi^2} \frac{\pi}{2} \right]^{1/2} = 132.6A \end{aligned}$$

- rms output current value :

$$\tilde{I}_o = \frac{1000}{\sqrt{2} \times 1.2\pi} = 187.56 \text{ A}$$

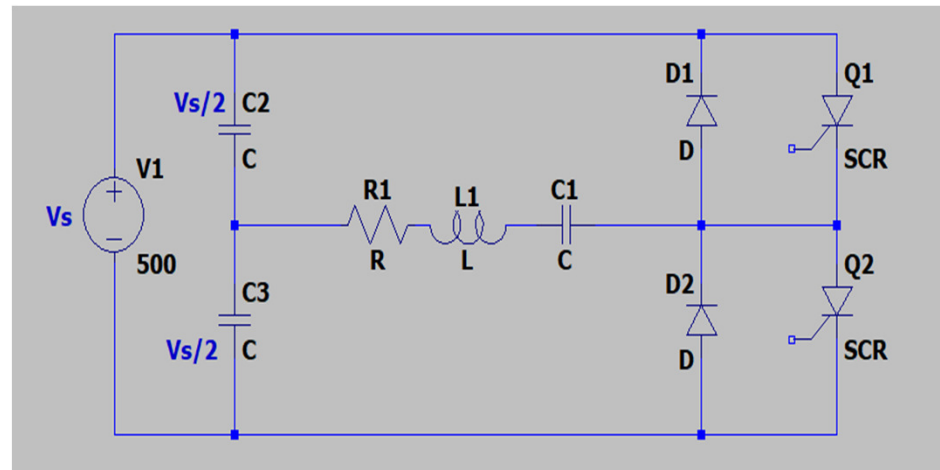
C)

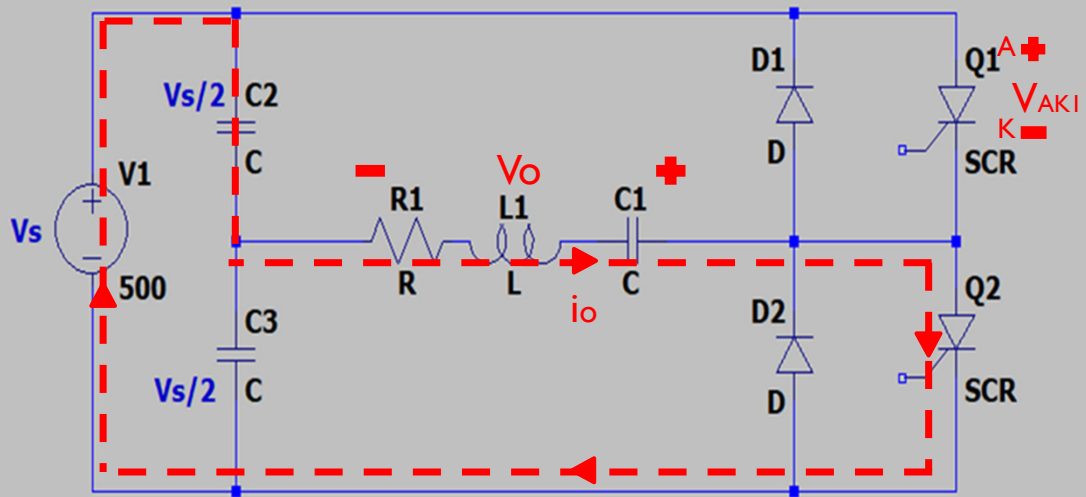
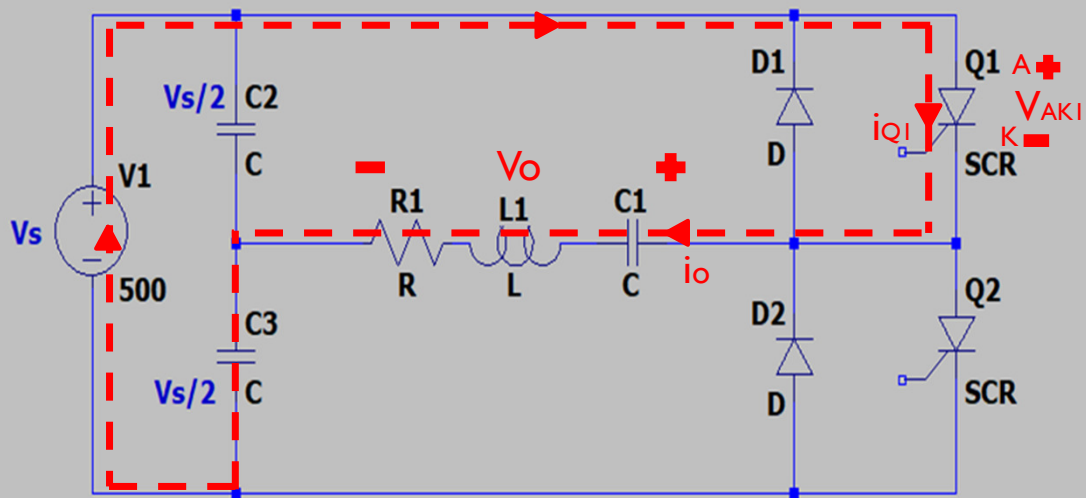
As there is no time interval between the moment when thyristor Q1's current becomes zero and the moment when current develops through thyristor Q2, forced commutation is required. The currents will be equal to zero when commutation ends.



## EXERCISE 33

Repeat exercise 32 while the load is consisted of  $R=1.2\Omega$ ,  $\omega L=7.92\Omega$  and  $1/\omega C=10\Omega$ .





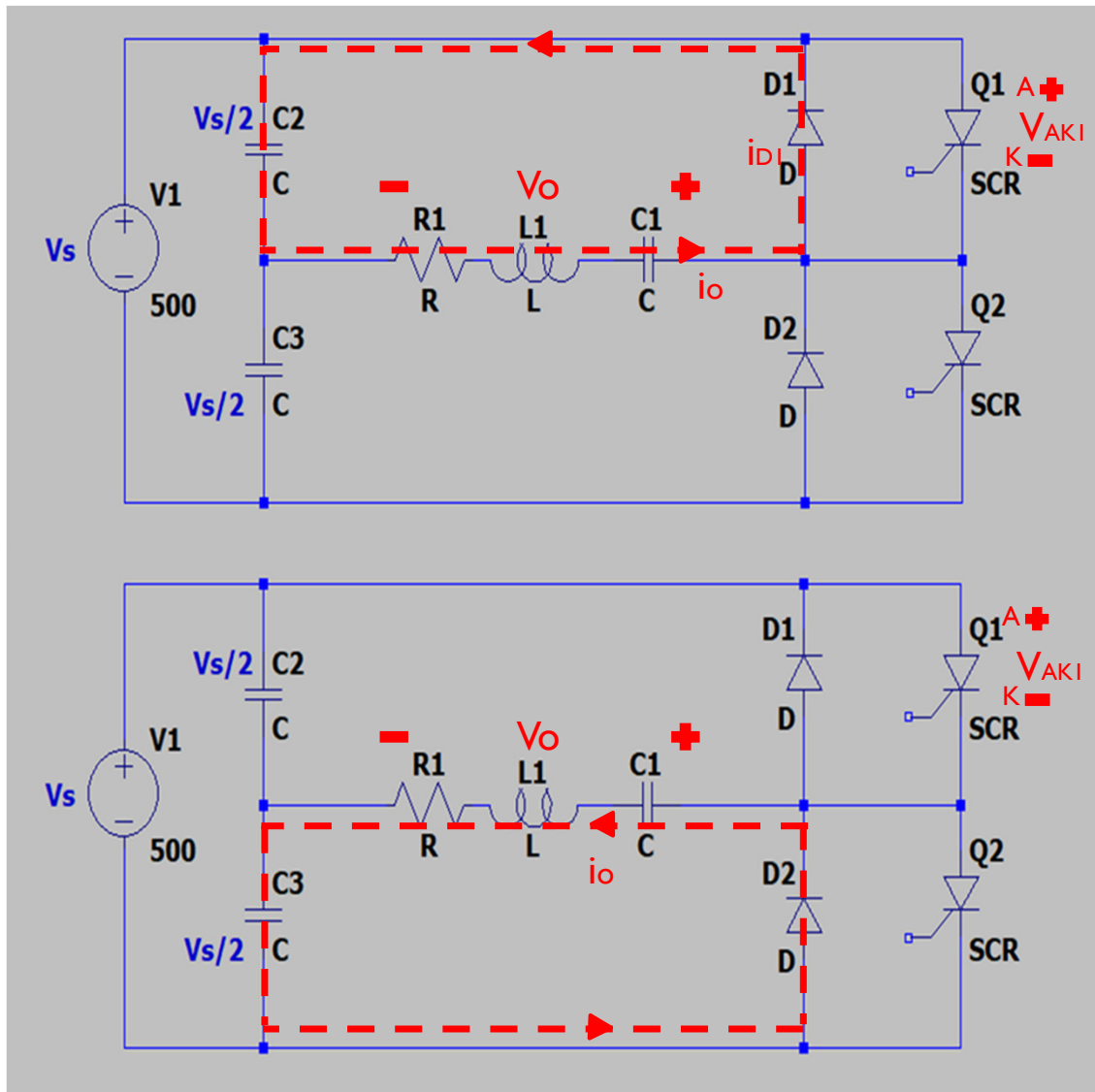
Note : In this case, due to the load characteristics (capacitive), current will flow through the diodes.

State 1 :  $0 - T/2 - t_g$

- Q1 ON
- $V_o = V_s/2 = 250V$
- $V_{AK1} = 0$
- $i_o$  positive
- $i_{Q1} = i_o$
- $i_{D1} = 0$

State 2 :  $T/2 - T - t_g$

- Q2 ON
- $V_o = -V_s/2 = -250V$
- $V_{AK1} = V_s/2 + V_s/2 = V_s = 500V$
- $i_o$  negative
- $i_{Q1} = 0$
- $i_{D1} = 0$



As output current precedes voltage, the current of thyristor Q1 becomes zero before Q2 is fired. The circuit's conditions cause the diode D1 to free wheel. As soon as Q2 is fired, D1 is reverse-biased and the current commutates from D1 to Q2. This process repeats for Q2 and D2 and so on.

State 3 :  $T/2 - t_g - T/2$

- D1 ON
- $V_o = V_s/2 = 250V$
- $V_{AK1} = 0$
- $i_o$  negative
- $i_{Q1} = 0$
- $i_{D1} = i_o$

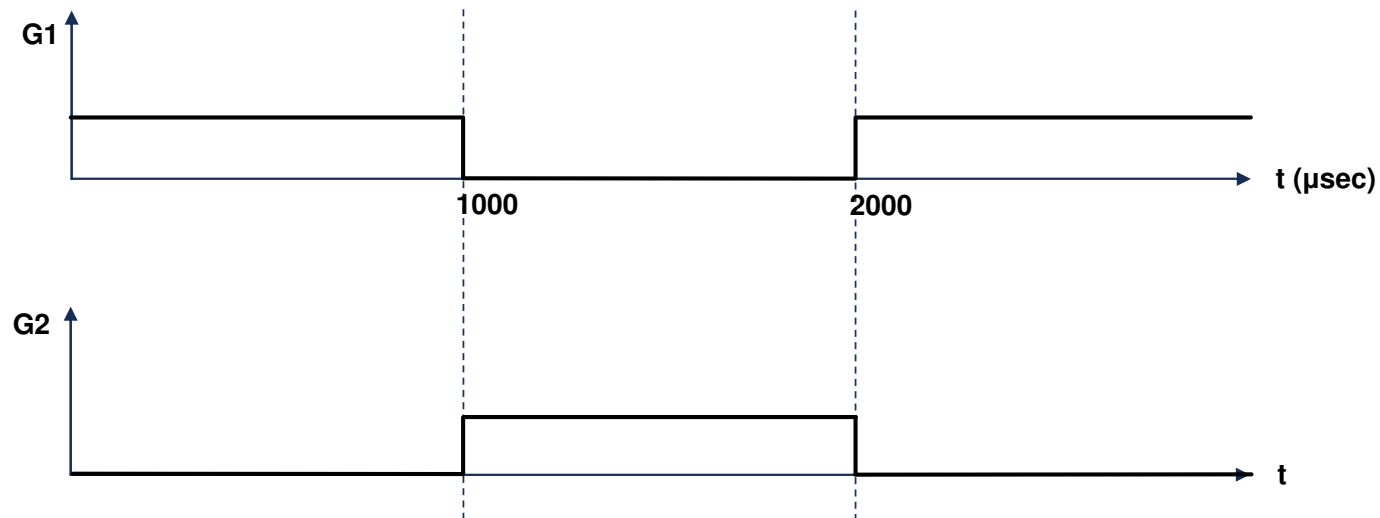
State 4 :  $T - t_g - T$

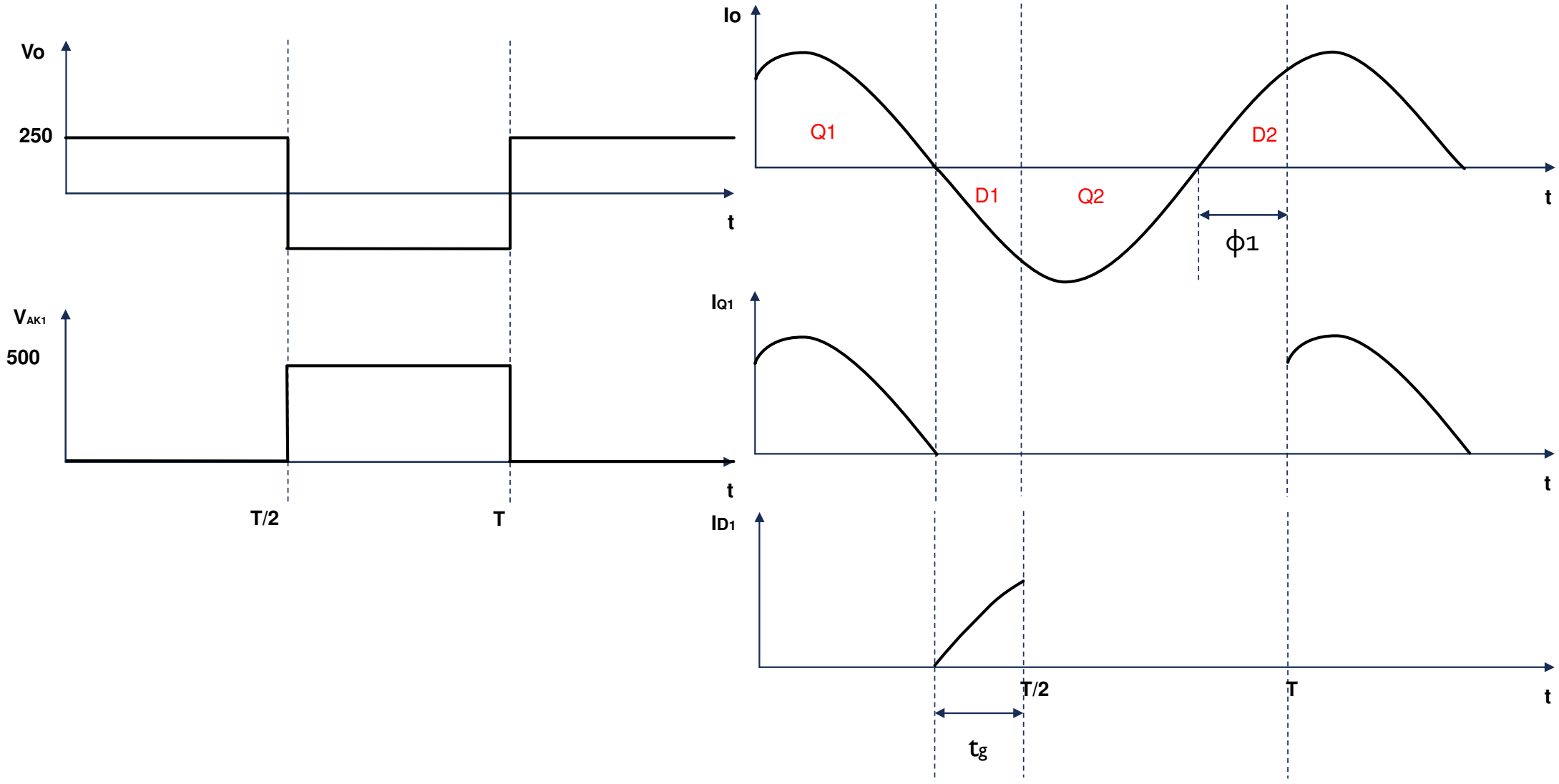
- D2 ON
- $V_o = -V_s/2 = -250V$
- $V_{AK1} = 500V$
- $i_o$  positive
- $i_{Q1} = 0$
- $i_{D1} = 0$

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$$A) \varphi = \tan^{-1} \left( \frac{\omega L - 1/\omega C}{R} \right) = \tan^{-1} \left( \frac{7.92 - 10}{1.2} \right) = -60^\circ$$

The output current's fundamental harmonic precedes output voltage's fundamental harmonic by  $60^\circ$ .  
The circuit's waveforms follow:





B) According to the waveforms and by using Fourier series analysis we have :

Output voltage

$$V_o = \sum_{n=1,3,5,\dots}^{\infty} \frac{2V}{n\pi} \sin(n\omega t)$$

Output current

$$I_o = \sum_{n=1,3,5,\dots}^{\infty} \frac{2V}{n\pi|Z_n|} \sin(n\omega t - \varphi_n)$$

- $\varphi_1 = \tan^{-1} \left( \frac{\omega L - 1/\omega C}{R} \right) = \tan^{-1} \left( \frac{7.92 - 10}{1.2} \right) = -60^\circ$
- $Z_1 = \sqrt{1.2^2 + (7.92 - 10)^2} = 2.4\Omega$

- Current's first harmonic component:

$$i_{o,1} = \frac{2 \times 500}{2.4 \times \pi} \sin(\omega t + 60^\circ)$$

- Thyristor average current value:

$$\bar{I}_Q = \frac{1}{2\pi} \int_0^{\frac{2\pi}{3}} \frac{1000}{2.4\pi} \sin \omega t d\omega t = \frac{1}{2\pi} \frac{1000}{2.4\pi} \left( -\cos \frac{2\pi}{3} + \cos 0 \right) = 21.1 * \frac{3}{2} = 31.66A$$

- Thyristor rms current value:

$$\begin{aligned} \tilde{I}_Q &= \left[ \frac{1}{2\pi} \int_0^{\frac{2\pi}{3}} \left( \frac{1000}{2.4\pi} \sin \omega t \right)^2 d\omega t \right]^{1/2} = \left[ \frac{1}{2\pi} \frac{1000^2}{2.4^2 \pi^2} \int_0^{\frac{2\pi}{3}} \frac{1}{2} (1 - \cos 2\omega t) d\omega t \right]^{1/2} = \\ &= \left[ \frac{1}{2\pi} \frac{1000^2}{2.4^2 \pi^2} \left[ \frac{\pi}{3} - \frac{1}{2} \left( \sin \frac{2\pi}{3} - \sin 0 \right) \right] \right]^{1/2} = 59.48A \end{aligned}$$

- Diode average current value:

$$\bar{I}_D = \frac{1}{2\pi} \int_0^{\frac{\pi}{3}} \frac{1000}{2.4\pi} \sin \omega t d\omega t = \frac{1}{2\pi} \frac{1000}{2.4\pi} \left( -\cos \frac{\pi}{3} + \cos 0 \right) = 21.1 * \frac{1}{2} = 10.55A$$

- Diode rms current value:

$$\begin{aligned} \tilde{I}_D &= \left[ \frac{1}{2\pi} \int_0^{\frac{\pi}{3}} \left( \frac{1000}{2.4\pi} \sin \omega t \right)^2 d\omega t \right]^{1/2} = \left[ \frac{1}{2\pi} \frac{1000^2}{2.4^2 \pi^2} \int_0^{\frac{\pi}{3}} \frac{1}{2} (1 - \cos 2\omega t) d\omega t \right]^{1/2} = \\ &= \left[ \frac{1}{2\pi} \frac{1000^2}{2.4^2 \pi^2} \left[ \frac{\pi}{6} - \frac{1}{2} \left( \sin \frac{\pi}{3} - \sin 0 \right) \right] \right]^{1/2} = 29.32A \end{aligned}$$





C)

As shown in the waveforms there's a time interval  $t_g = T/6 = 334 \mu\text{sec}$  between the moment when Q1's current becomes zero and when current develops through Q2. This time interval is long enough for thyristor Q1 to turn-off as it requires  $50 \mu\text{sec}$  (according to the data given) for its current to fall to zero. Therefore, there is no need for forced-commutation under these operational conditions (the thyristors turn off with natural commutation due to the load characteristics).