QY4. Qubit Devices

- QDs
- Semiconducting Qubits

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Lesson #3

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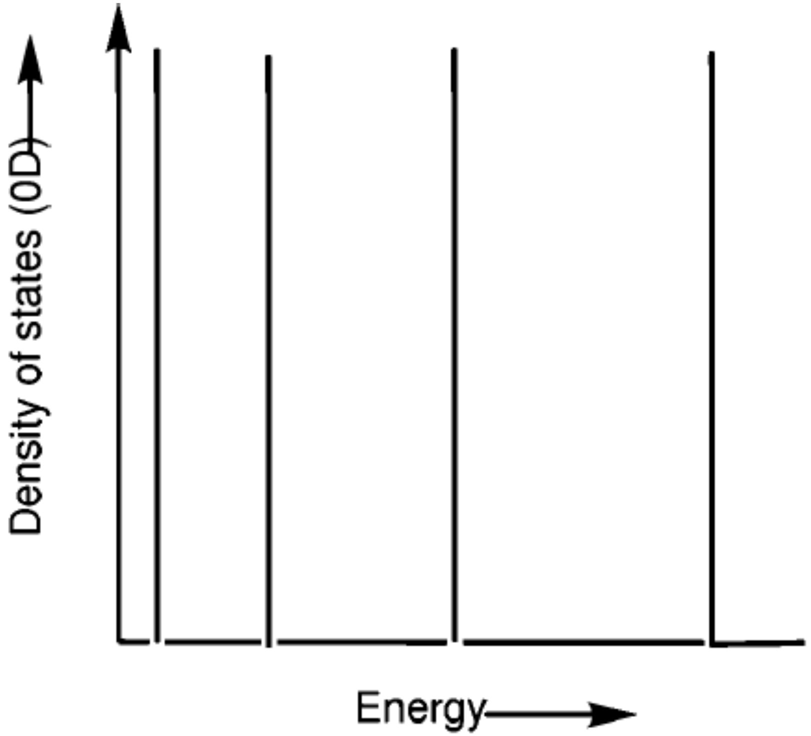
MSc in QUANTUM COMPUTING AND QUANTUM TECNNOLOGIES

NATIONAL CENTRE FOR SCIENTIFIC RESEARCH "DEMOKRITOS"

Energy Levels in QDs

$$E = \frac{\hbar^2\pi^2}{2m_e}\left(\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2\right)$$

$$= E_{n_x,n_y,n_z}$$



Zero-point energy (zpe)

$L_x=L_y=L_z=l$

$n_x=n_y=n_z=1$

Box

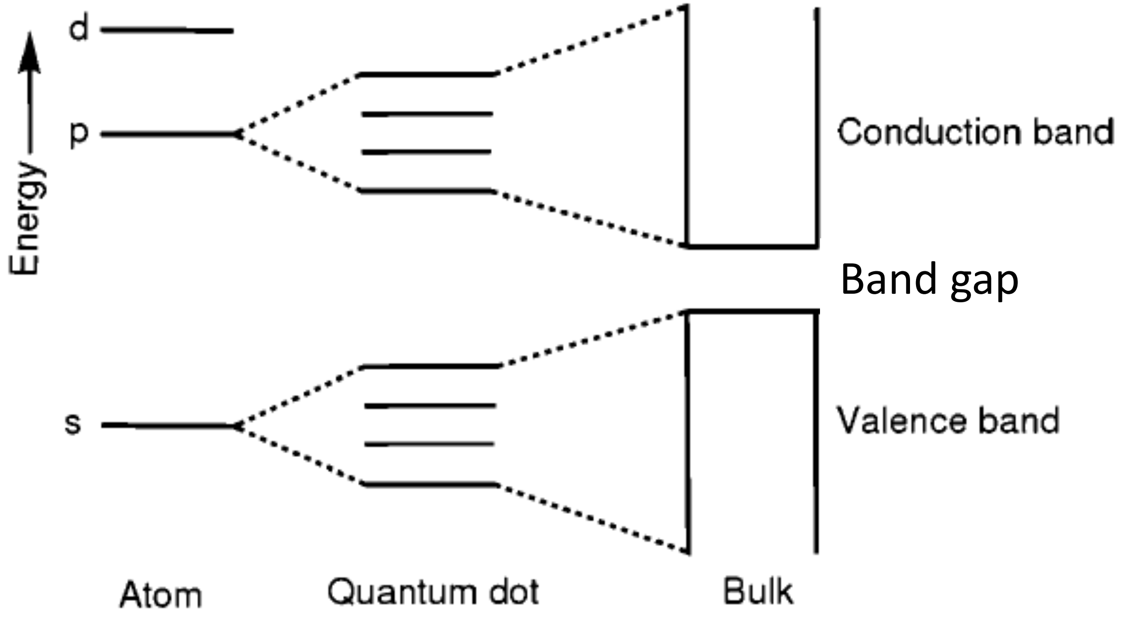
$$E_{zpe} = \frac{3h^2}{8m_e^* l^2}$$

Sphere

$$E_{zpe} = \frac{h^2}{2m_e^* d^2}$$

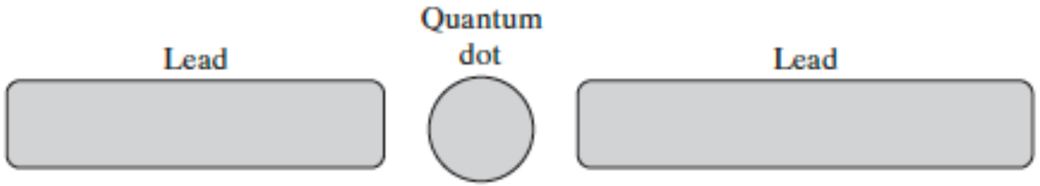
QD Energy Bandgap

QD Energy band diagram

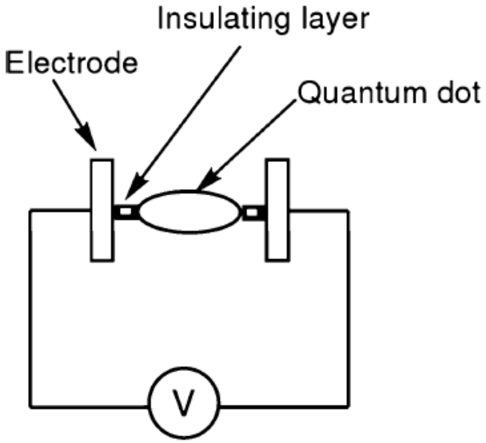


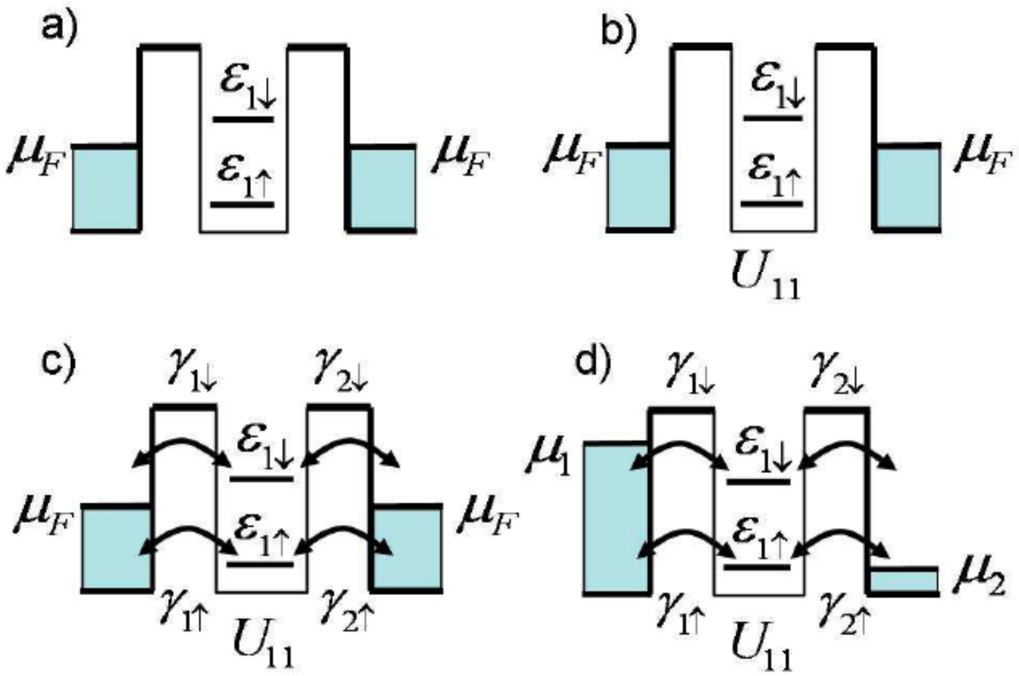
$$E_{g(dot)} = E_{g(bulk)} + E_{conf.} + E_{Coulomb} = E_{g(bulk)} + \frac{h^2}{2m^* l^2} - \frac{1.8e^2}{2\pi\varepsilon\varepsilon_o l}$$

QD measurements



$$Q = CV, \qquad E = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$





Single Electron Capacitor

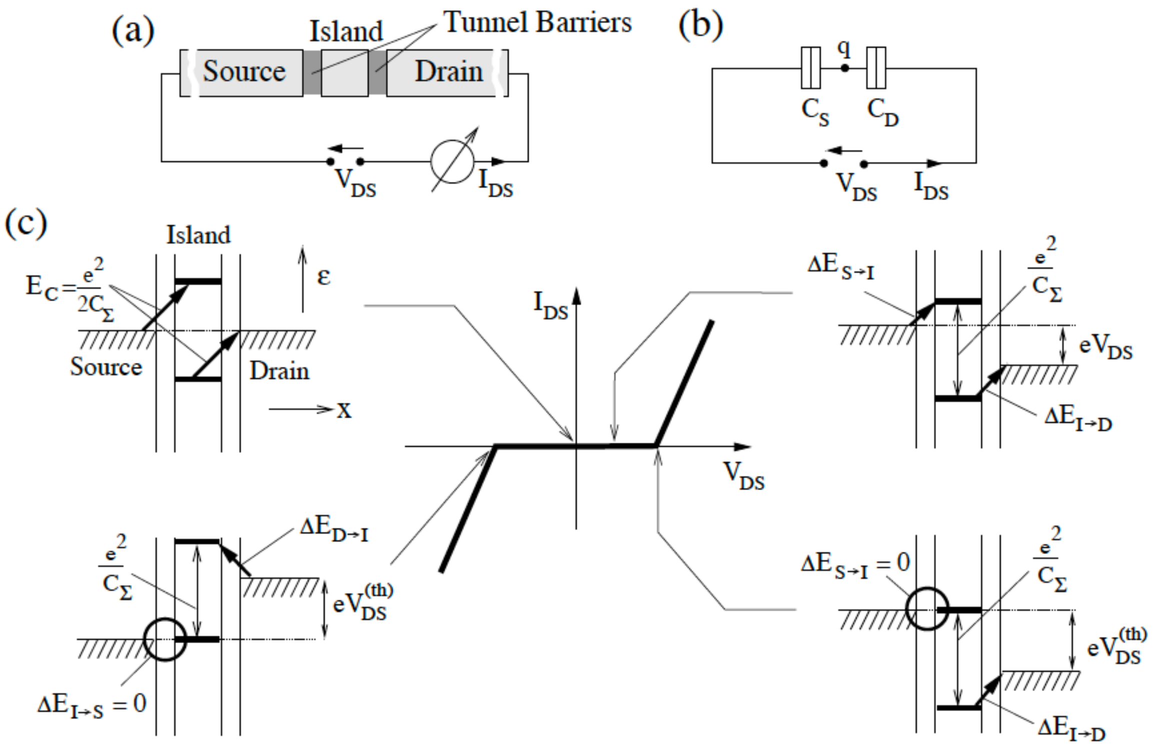


Fig. 1.2: (a) Two-terminal arrangement for discussing the Coulomb blockade effect in electrical transport. (b) The respective capacitance circuit. Note $C_\Sigma = C_S + C_D$. (c) Sketch of the expected non-linear $I_{DS}(V_{DS})$ characteristic with energy schemes for distinct V_{DS} values reflecting the energetical position of the Fermi levels of the island for charge states $q = -e$ and $q = e$ relatively to the Fermi level of source and drain.

Electrostatic Barriers, $V_{DS}>0$

$$\Delta E_{S\to I} = E_C - e\,\frac{C_D}{C_\Sigma}\,V_{DS}$$

$$\Delta E_{I\to D} = E_C + e\,\frac{C_D}{C_\Sigma}\,V_{DS} - e\,V_{DS}$$

Ecurrent suppression

$$|V_{DS}| \geq V_{DS}^{(th)} \equiv \min\left(\frac{e}{2\,C_S};\,\frac{e}{2\,C_D}\right)$$

Single Electron Transistor – SET

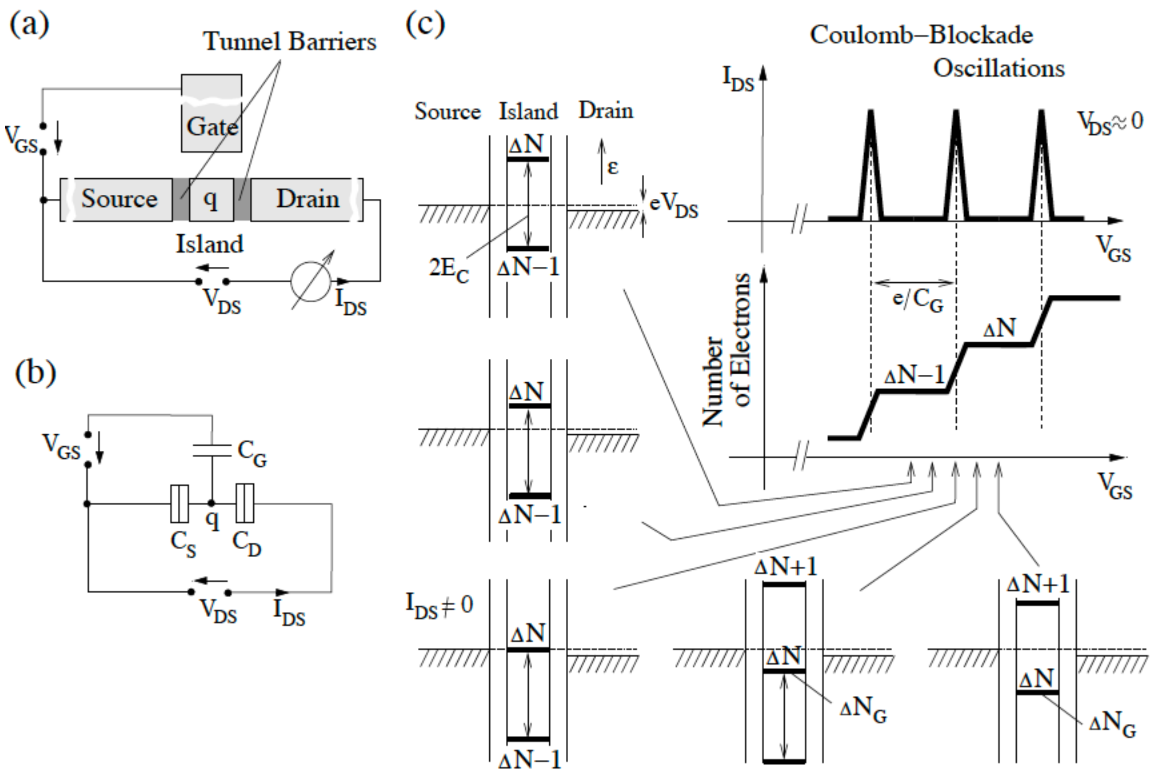


Fig. 1.3: (a) Three-terminal arrangement of a single-electron transistor. (b) The respective capacitance circuit. Note $C_\Sigma = C_S + C_D + C_G$. (c) With increasing gate voltage V_{GS}, electrons are accumulated on the island. Whenever the charge state can energetically fluctuate by e, i.e., the energy for two charge states is degenerate, current I_{DS} flows for small applied V_{DS} through the island, leading to a periodically modulated $I_{DS}(V_{GS})$-characteristic – the Coulomb blockade oscillations. For distinct V_{GS} values, the respective energy schemes are given.

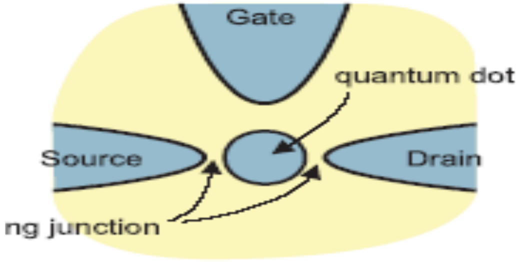
Electrostatic Barrier, Single electron charging

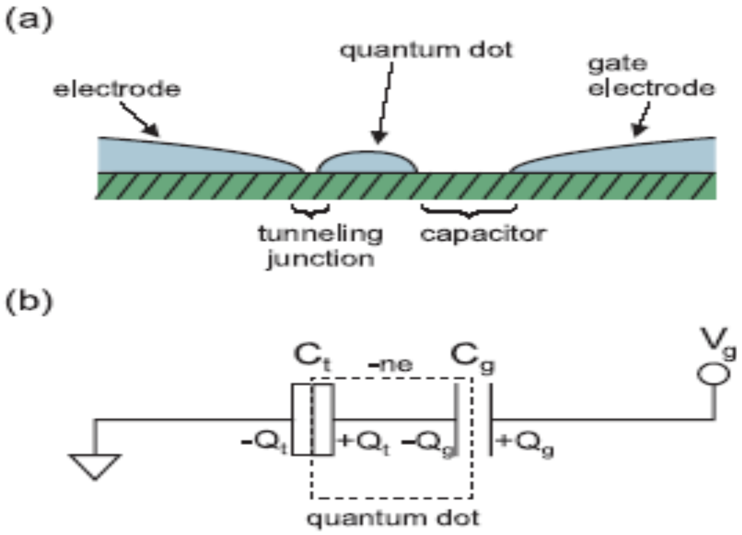
$$\Delta E_{S\to I} = E_C - e\,\frac{C_G}{C_\Sigma}\,V_{GS} \stackrel{!}{=} 0$$

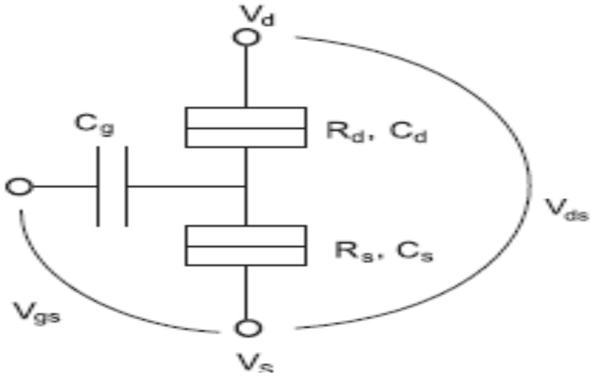
$$V_{GS}^{(th)} = \frac{E_C}{e\,C_G/C_\Sigma} = \frac{e}{2\,C_G}$$

Single Electron Transistor – SET

- Conditions for observing single electron tunneling phenomena
  - $E_c > k_b T$
    - $E_c = e^2/2C_\Sigma$
  - $R_t > R_k$
    - $R_k = h/e^2$ (25.8 KOhms)







Example

Calculate the size of a sphere shaped quantum dot of Si that would produce observable single electron effect at room temperature.

Solution. The energy change on charging of the quantum dot capacitor should be much larger than kT in order to observe the single electron effects.

At 300 K, $kT = 1.38 \times 10^{-23}$ J K^{-1} x 300 K = 414 x 10^{-23} J

Taking 1 eV = 1.602 x 10^{-19} J, kT = 258.43 x10^{-4} eV = 25.84 meV.

The energy change on charging of the quantum dot by a single electron

$$= q^2/2C = e^2/2C$$

The capacitance of the *sphere* shaped capacitor, $C = 4\pi\varepsilon\varepsilon_0 r = 4\pi \times 11.5 \times 8.85 \times 10^{-12} \times r$

(taking the dielectric constant of silicon to be 11.5 and the permittivity of vacuum , $\varepsilon_0 = 8.85 \times 10^{-12}$ F.m^{-1}.)

$C = 1278.294 \times r \times 10^{-12}$ F = $1.278 \times r \times 10^{-18}$ F, when r is taken in nm.

Energy change on charging by a single electron

$$= e^2/2C = (1.6 \times 10^{-19})^2/2 \times 1.278 \times r\ 10^{-18} \text{ J} = = 0.0626/r \text{ eV}$$

This energy should be much larger than kT for the single electron effect to be observable

i.e. $0.0626/r$ eV >> 0.02584 eV at 300K

or $0.0626/r \approx 0.5 \times 0.02584 = 0.1292$ (say) or $r \approx 0.5$ nm

Hence the quantum dot should have a radius of the order of 0.5 nm for this effect to be observable at room temperature.

Single Electron Transistor – SET

- Conditions for observing single electron tunneling phenomena
  - $E_c > k_b T$
    - $E_c = e^2/2C_\Sigma$
  - $R_t > R_k$
    - $R_k = h/e^2$ (25.8 KOhms)

Another requirement for observing the single electron effects is that the fluctuations in the number of electrons in the quantum dot should be negligible. The time constant for an R-C circuit is RC. The time taken by an electron to move in or out of a junction should be of this order.

According to the Heisenberg uncertainty principle, the product of the energy change accompanying this transfer and the time taken should be larger than h, the Planck's constant

$$\Delta E . \Delta t > h$$

$$\text{or } (e^2/2C).RC > h$$

$$\text{or } R > 2\, h/e^2 = 51.6 \text{ k}\Omega.$$

Single electron charging (II)

Electrostatic Energy, ΔN electron charging

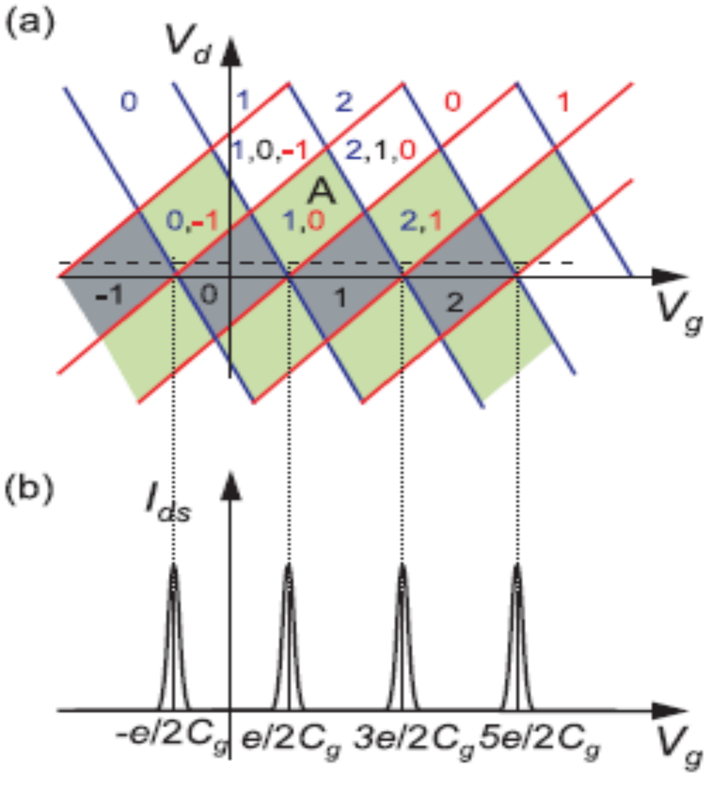
$$E_{\text{elst}}(\Delta N; V_{\text{GS}}, V_{\text{DS}}) = -\Delta N\, e \left(\frac{C_{\text{G}}}{C_{\Sigma}} V_{\text{GS}} + \frac{C_{\text{D}}}{C_{\Sigma}} V_{\text{DS}} \right) + \frac{(\Delta N\, e)^2}{2\, C_{\Sigma}}$$

Electrostatic Barriers, ΔN + 1 electron charging

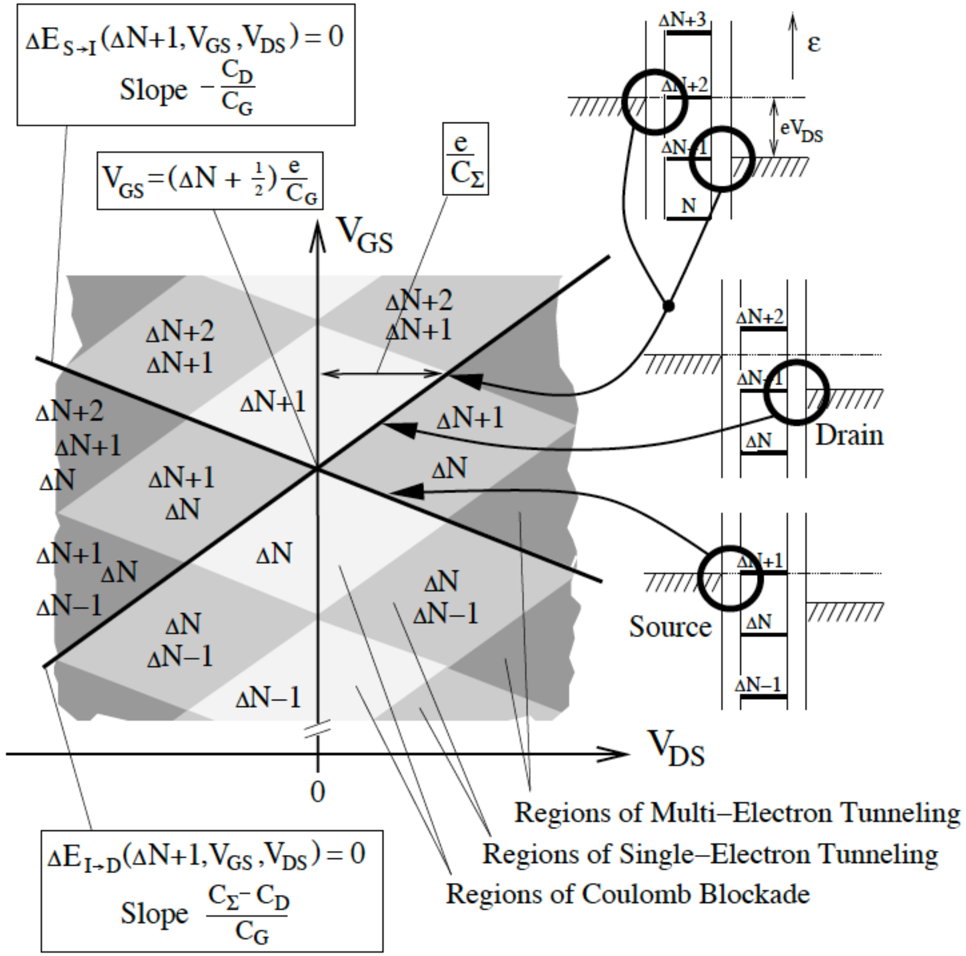
$$\begin{aligned} \Delta E_{\text{S}\to\text{I}}(\Delta N + 1; V_{\text{GS}}, V_{\text{DS}}) &= E_{\text{elst}}(\Delta N + 1; V_{\text{GS}}, V_{\text{DS}}) - E_{\text{elst}}(\Delta N; V_{\text{GS}}, V_{\text{DS}}) \\ &= (\Delta N + \tfrac{1}{2}) \frac{e^2}{C_{\Sigma}} - e \frac{C_{\text{G}}}{C_{\Sigma}} V_{\text{GS}} - e \frac{C_{\text{D}}}{C_{\Sigma}} V_{\text{DS}} \,. \end{aligned}$$

$$\begin{aligned} \Delta E_{\text{I}\to\text{D}}(\Delta N; V_{\text{GS}}, V_{\text{DS}}) &= E_{\text{elst}}(\Delta N - 1; V_{\text{GS}}, V_{\text{DS}}) - e\, V_{\text{DS}} - E_{\text{elst}}(\Delta N; V_{\text{GS}}, V_{\text{DS}}) \\ &= -(\Delta N - \tfrac{1}{2}) \frac{e^2}{C_{\Sigma}} + e \frac{C_{\text{G}}}{C_{\Sigma}} V_{\text{GS}} - e \left(1 - \frac{C_{\text{D}}}{C_{\Sigma}}\right) V_{\text{DS}} \,. \end{aligned}$$

SET Measurements

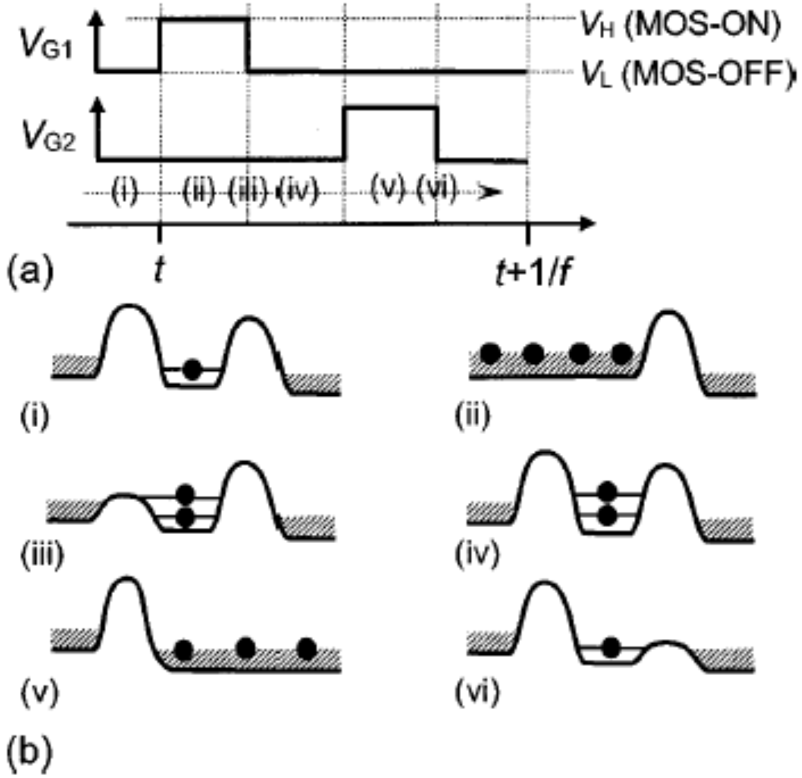


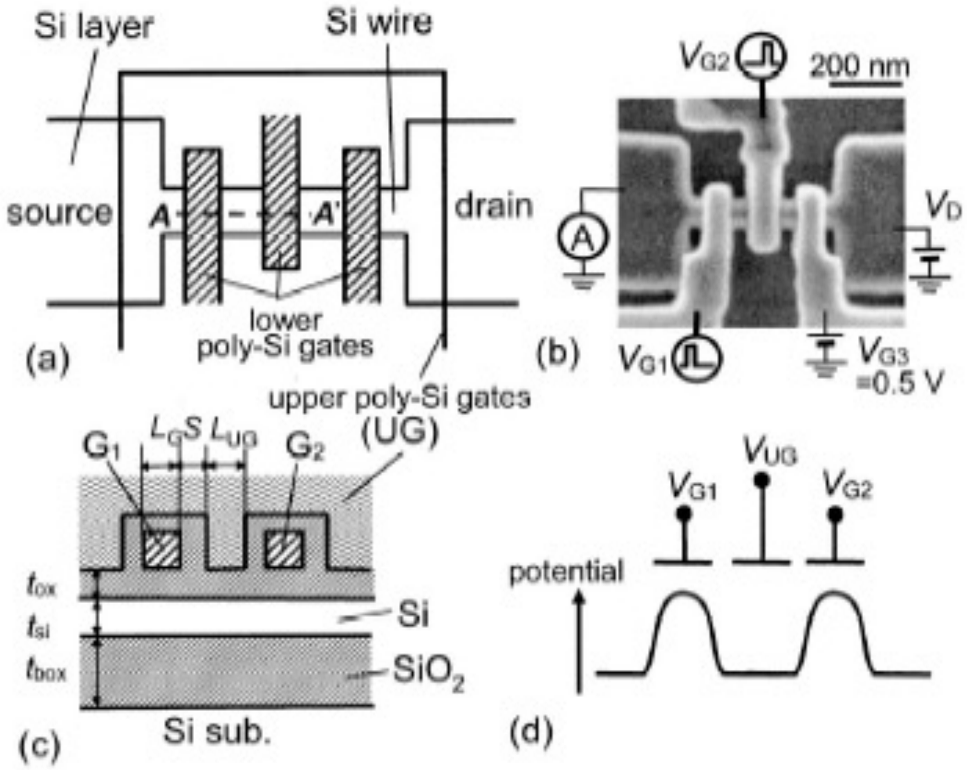
Current flows when $V_g = ne/2C_g$



Example

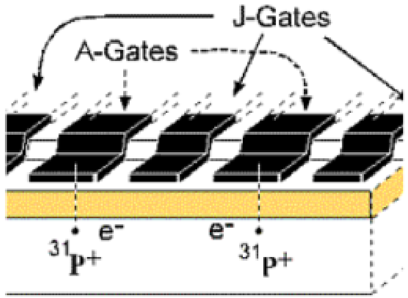
30 nm wide Si-wire channel and poly-Si gates defined by E-beam lithography



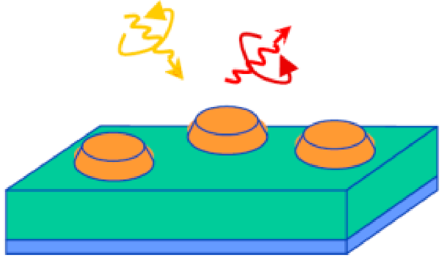


Current quantization due to single electron transfer in Si-wire charge coupled device, Applied Physics Letters, Vol 84 (8), 23 Feb 2004, 1323

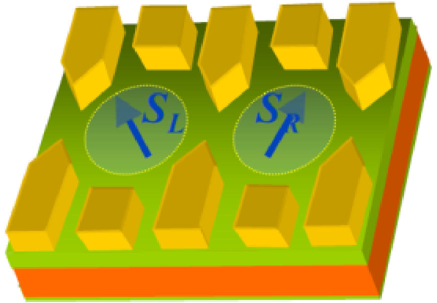
Examples of quantum dots: Different Approaches



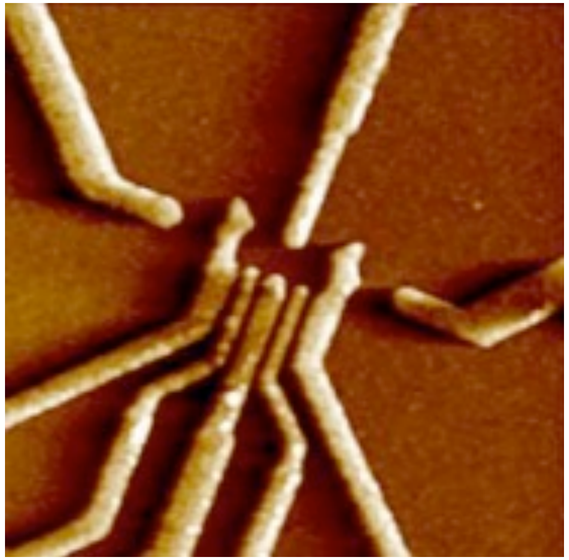
Kane, Nature 1998



Imamoglu *et al*, PRL 1999



Loss & DiVincenzo
PRA 1998

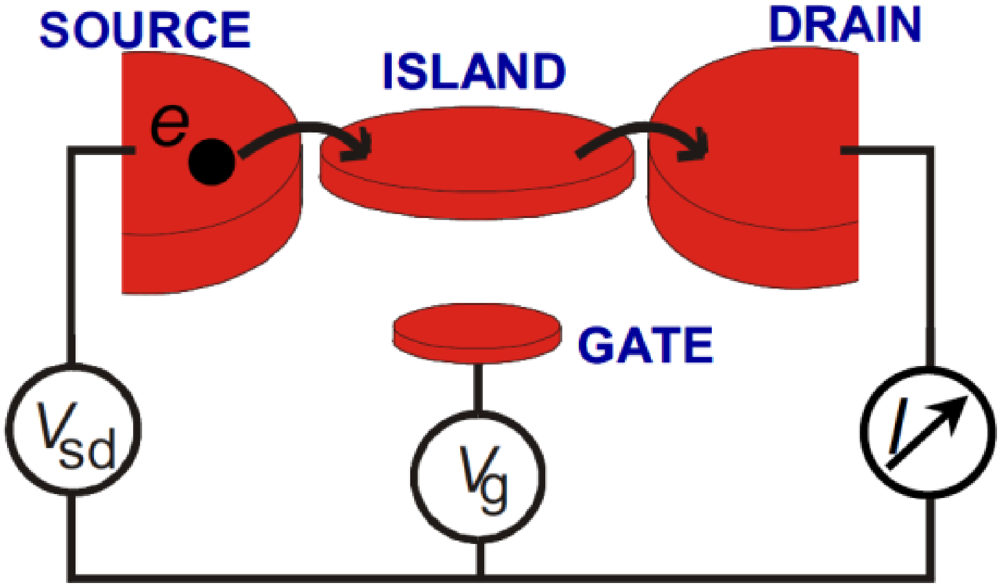


Electrically controlled and measured quantum dots

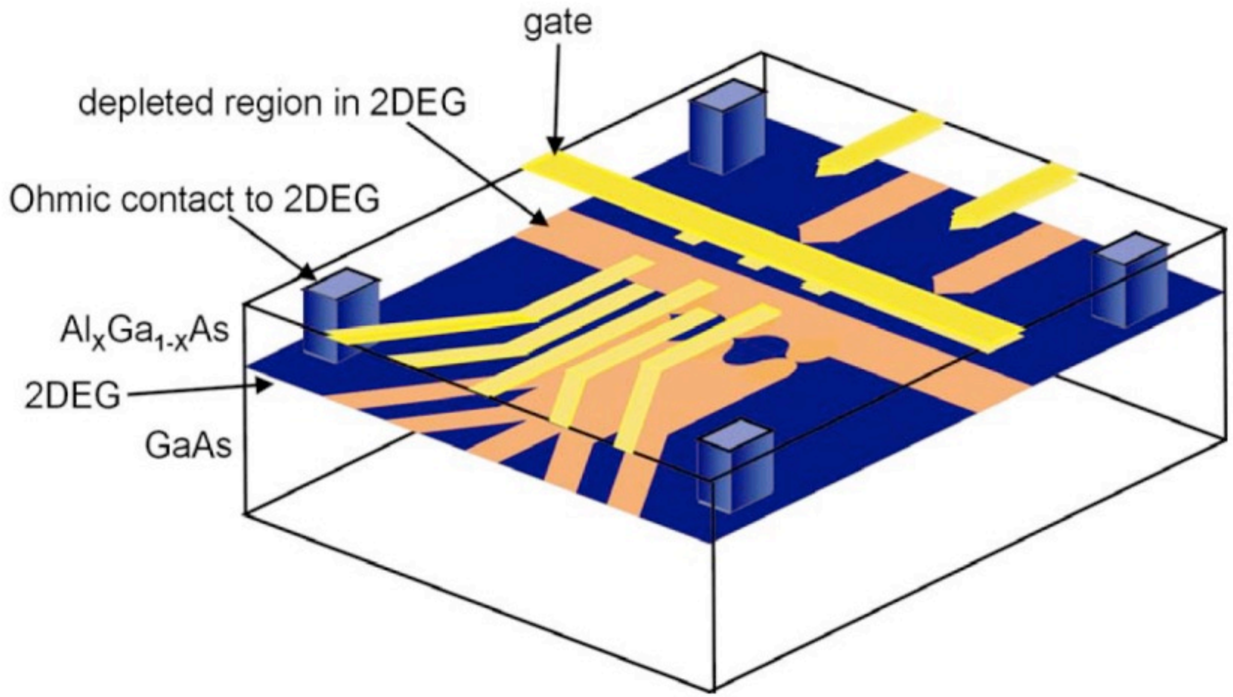
A small semiconducting (or metallic) island where electrons are confined, giving a discrete level spectrum

• Coupled via tunnel barriers to source and drain reservoirs

• Coupled capacitively to gate electrode, to control # of electrons

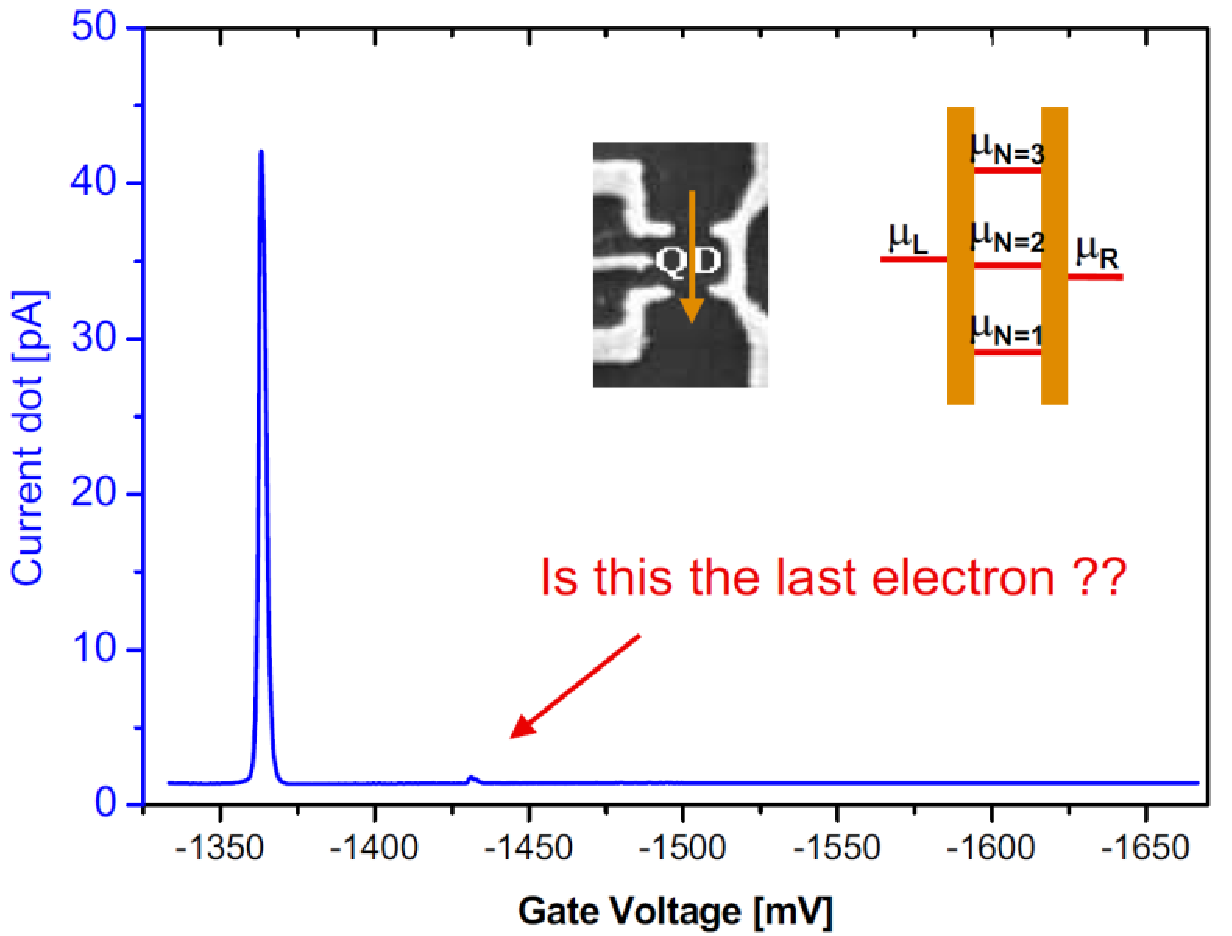


Electrostatically defined quantum dots

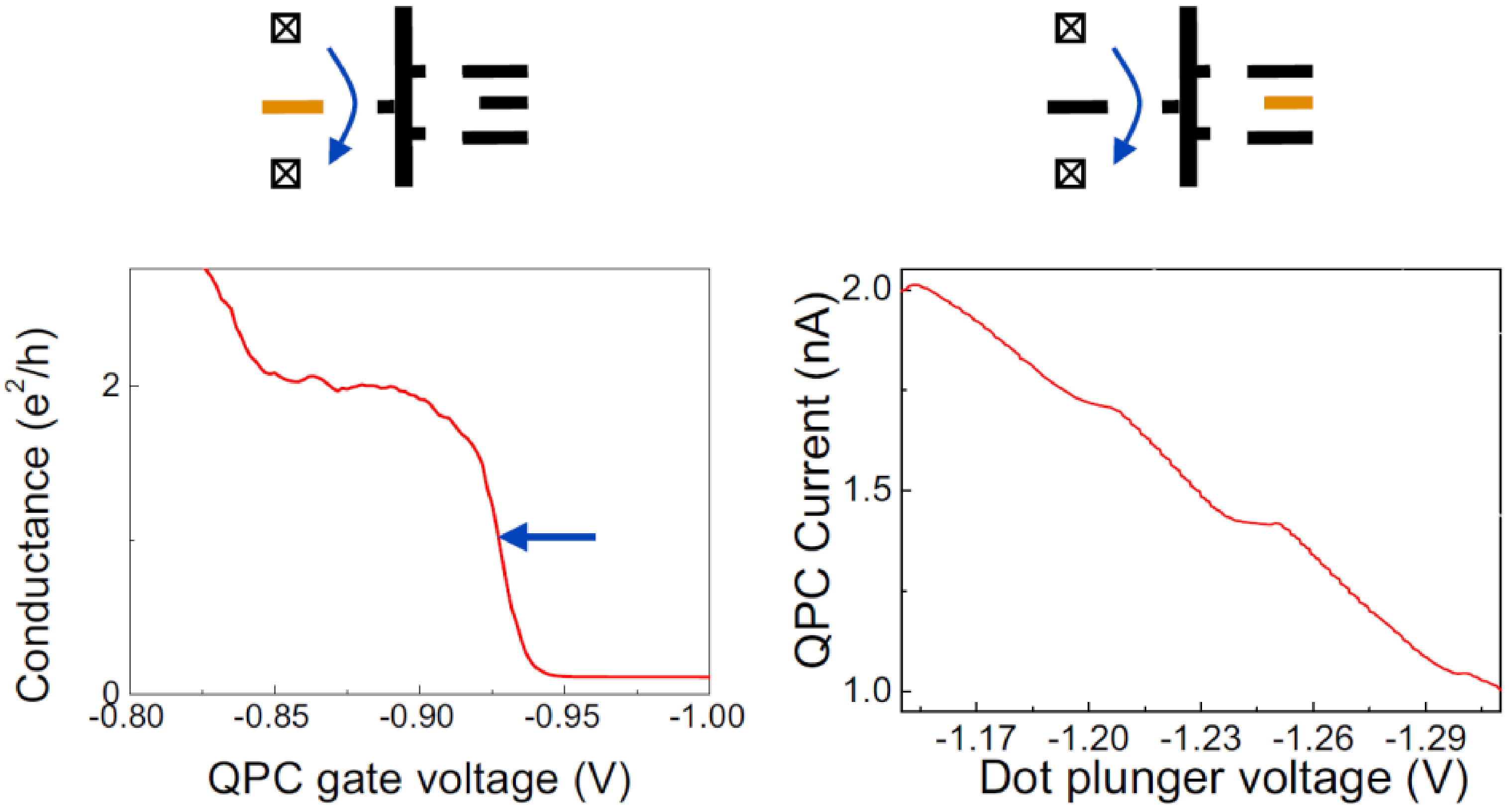


- Electrically measured (contact to 2DEG)
- Electrically controlled number of electrons
- Electrically controlled tunnel barriers

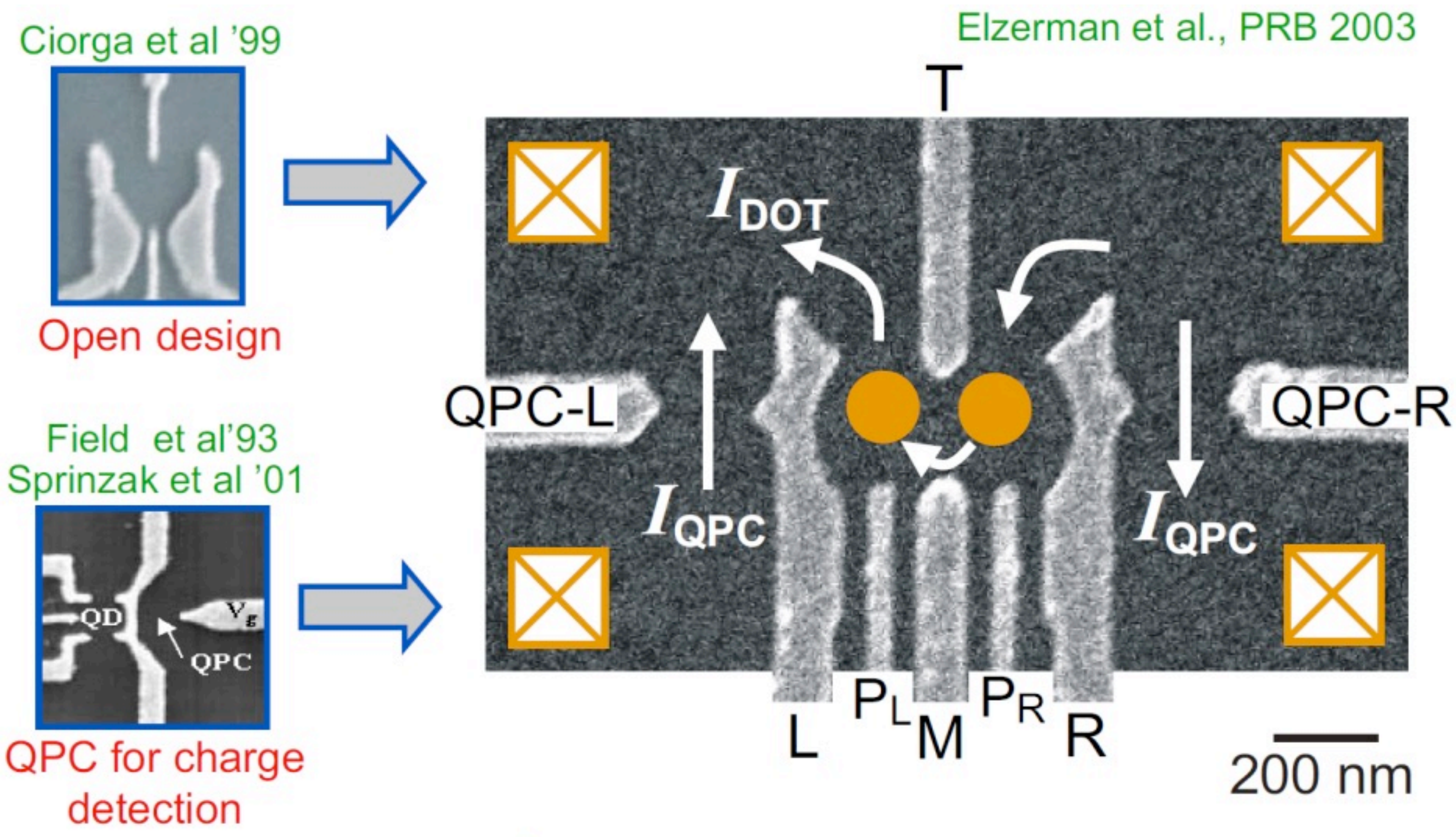
Transport through quantum dot - Coulomb blockade



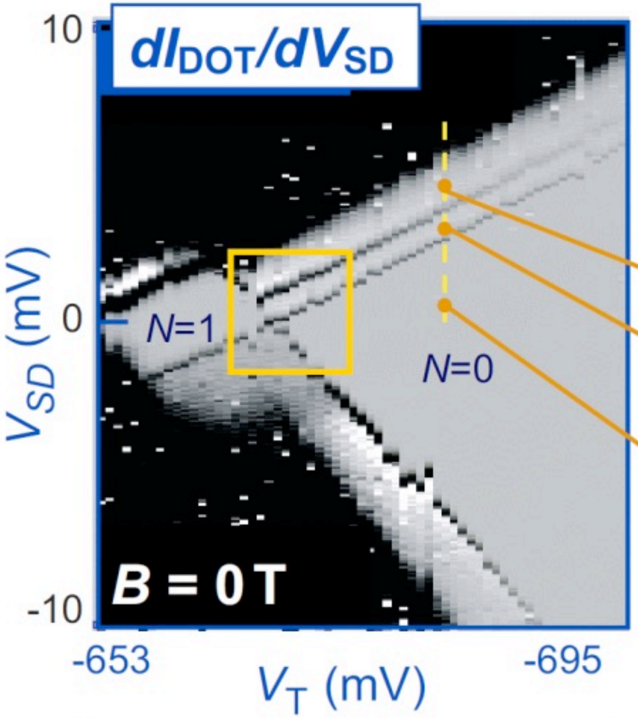
A quantum point contact (QPC) as a charge detector



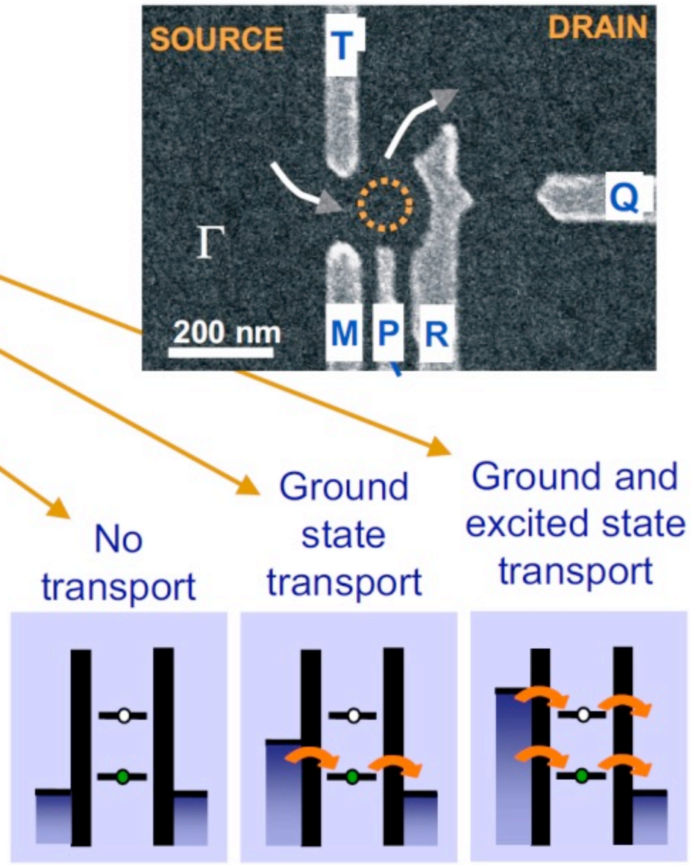
Few-electron double dot design



Energy level spectroscopy at B = 0

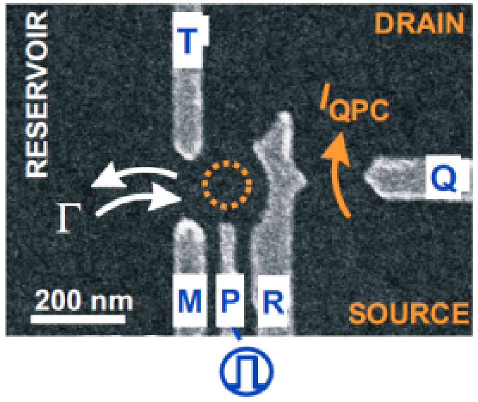


- ΔE ~ 1.1meV
- E_C ~ 2.5meV

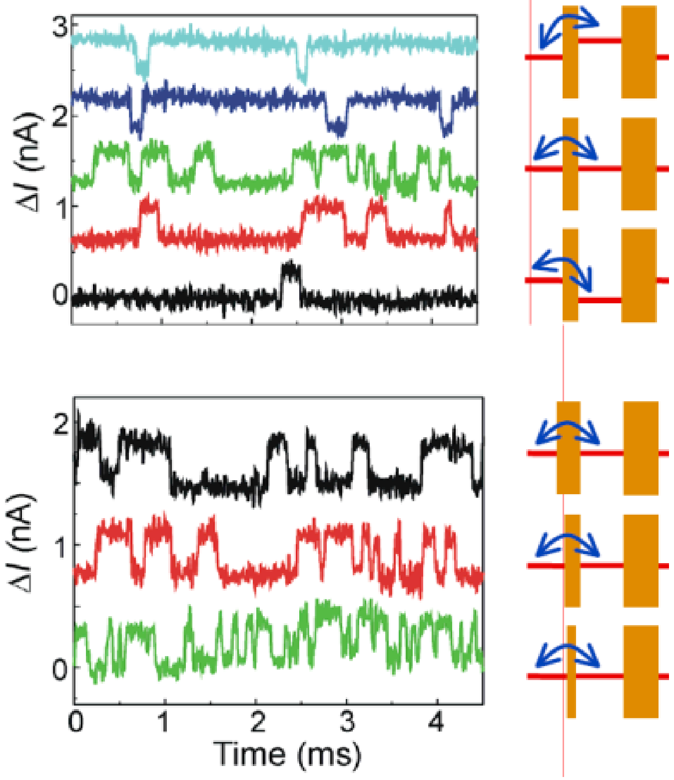


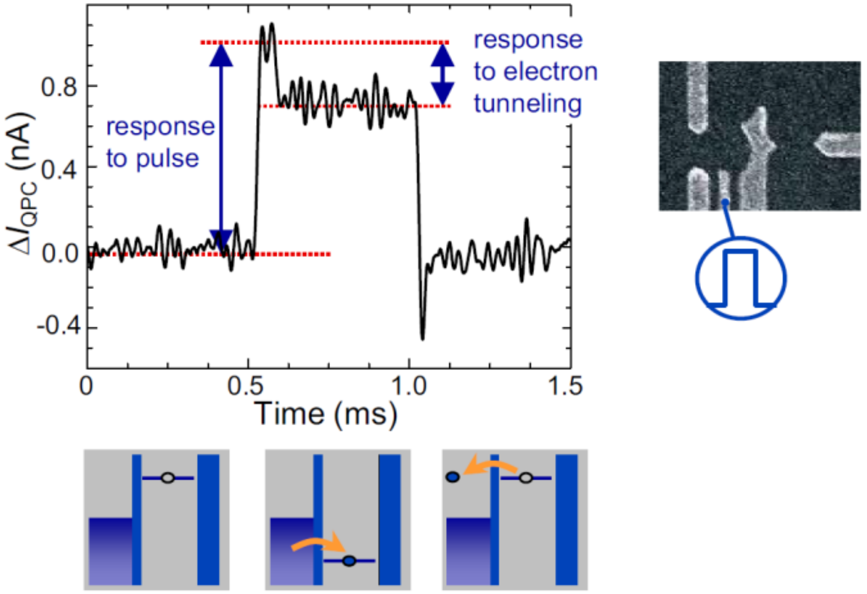
Observation of individual tunnel events

Vandersypen *et al*, APL 85, 4394, 2004
Also: Schlesser *et al*, 2004

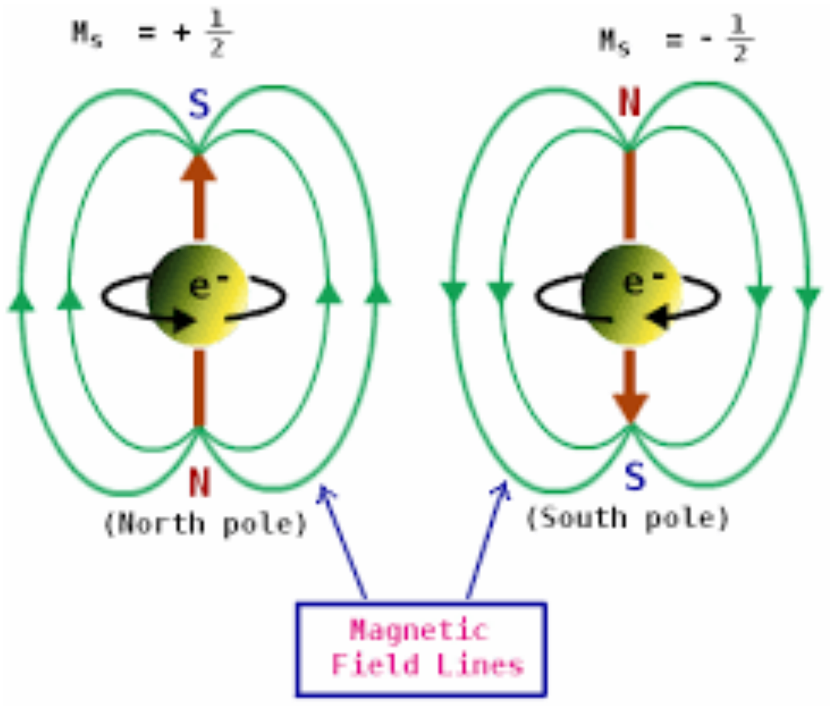


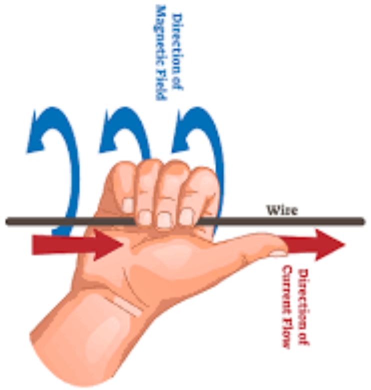
- V_{SD} = 1 mV
- I_{QPC} ~ 30 nA
- ΔI_{QPC} ~ 0.3 nA
- Shortest steps ~ 8 μs





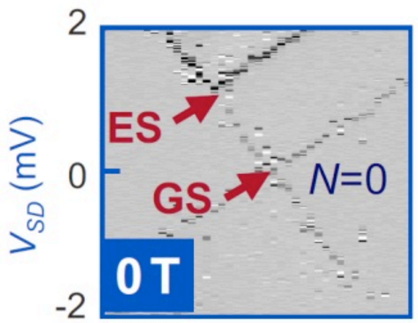
The effect of electron's spin and related interactions in semi-QB



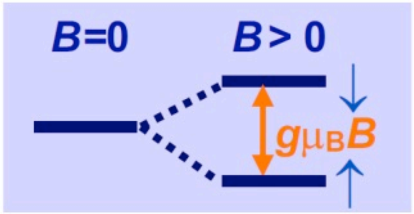


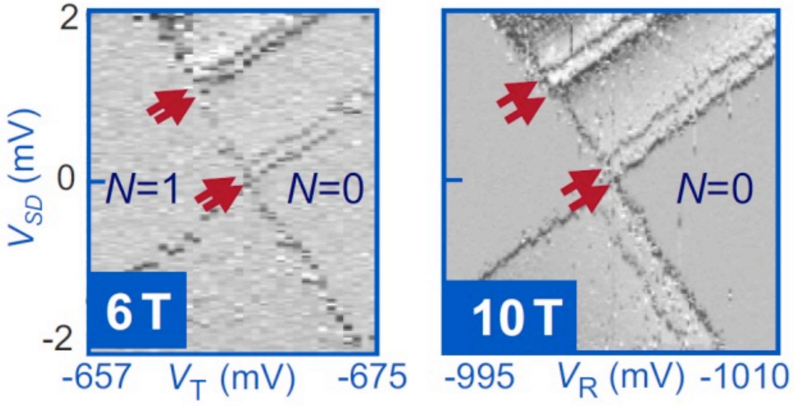
EXERCISE: We calculate that the magnetic field inside an electron is about $B = 8.3 \times 10^{13}$ T . This is about 8.3×10^{11} times bigger than the highest obtainable magnetic field in today's conditions. Therefore, is an unbreakable fundamental particle.

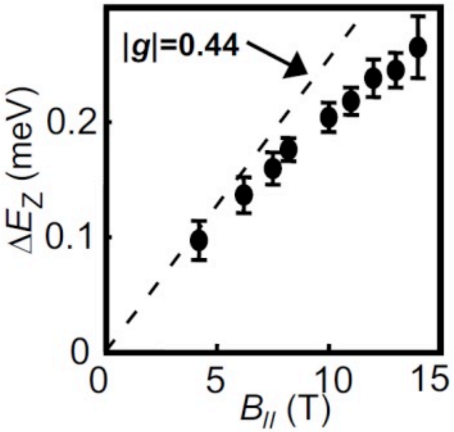
Single electron Zeeman splitting in $B_{//}$



Hanson et al, PRL 91, 196802 (2003)
Also: Potok et al, PRL 91, 016802 (2003)

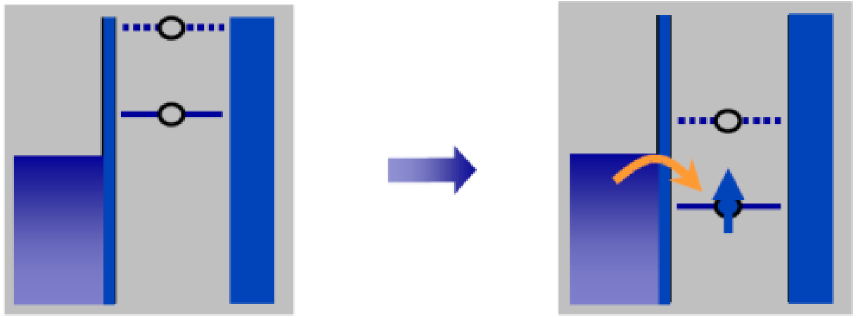




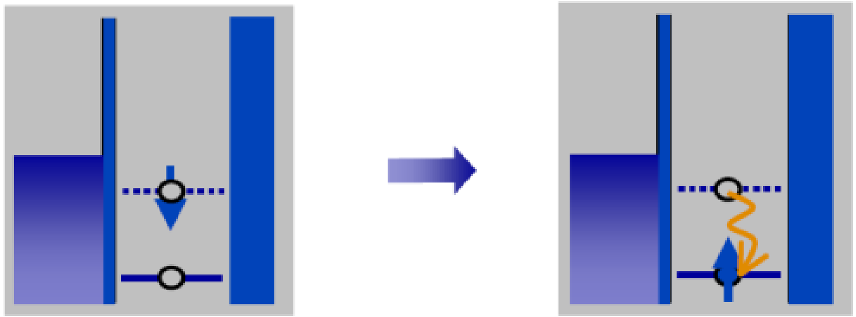


Initialization of a single electron spin

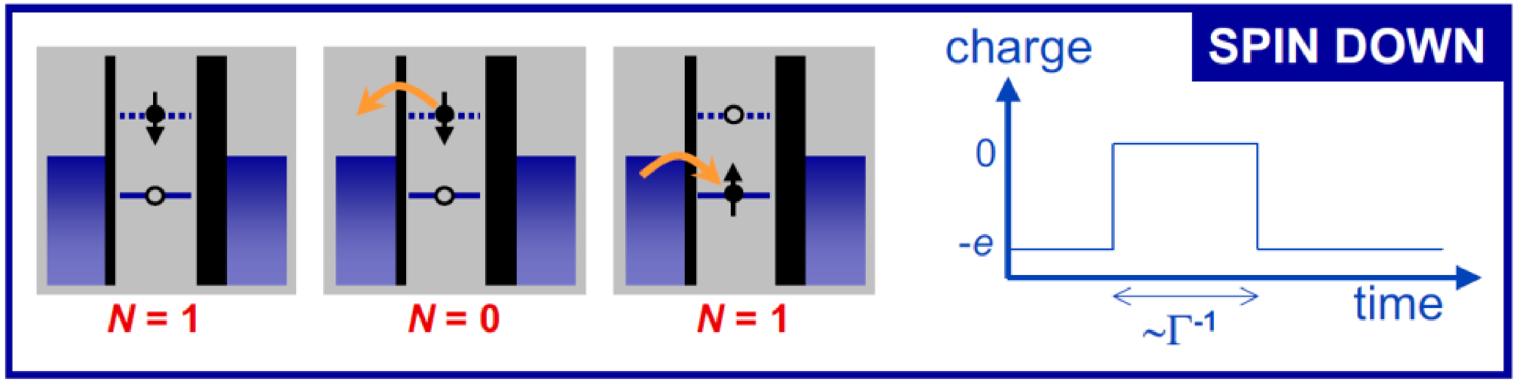
Method 1:
spin-selective tunneling



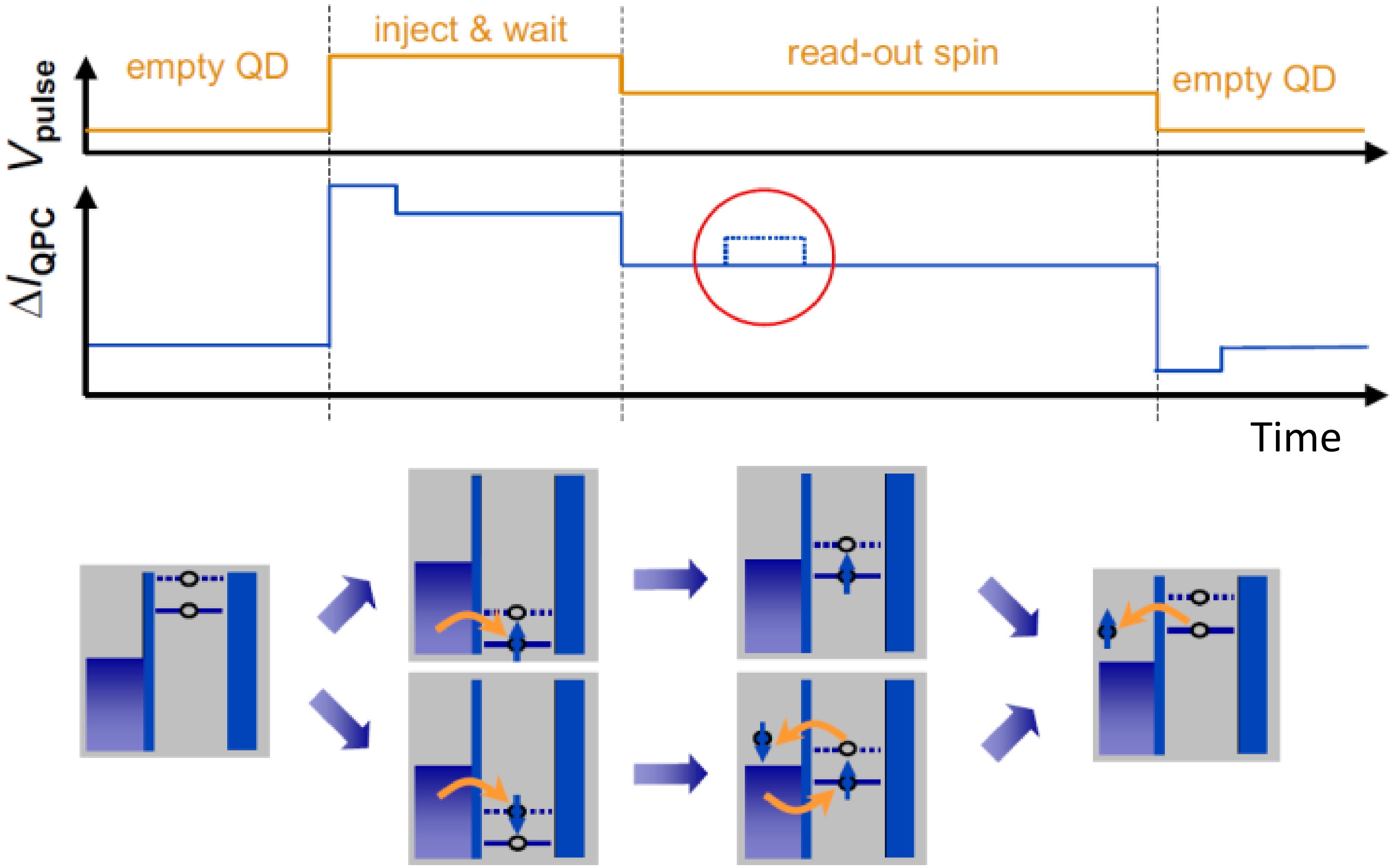
Method 2:
relaxation to ground state



Spin read-out principle: convert spin to charge

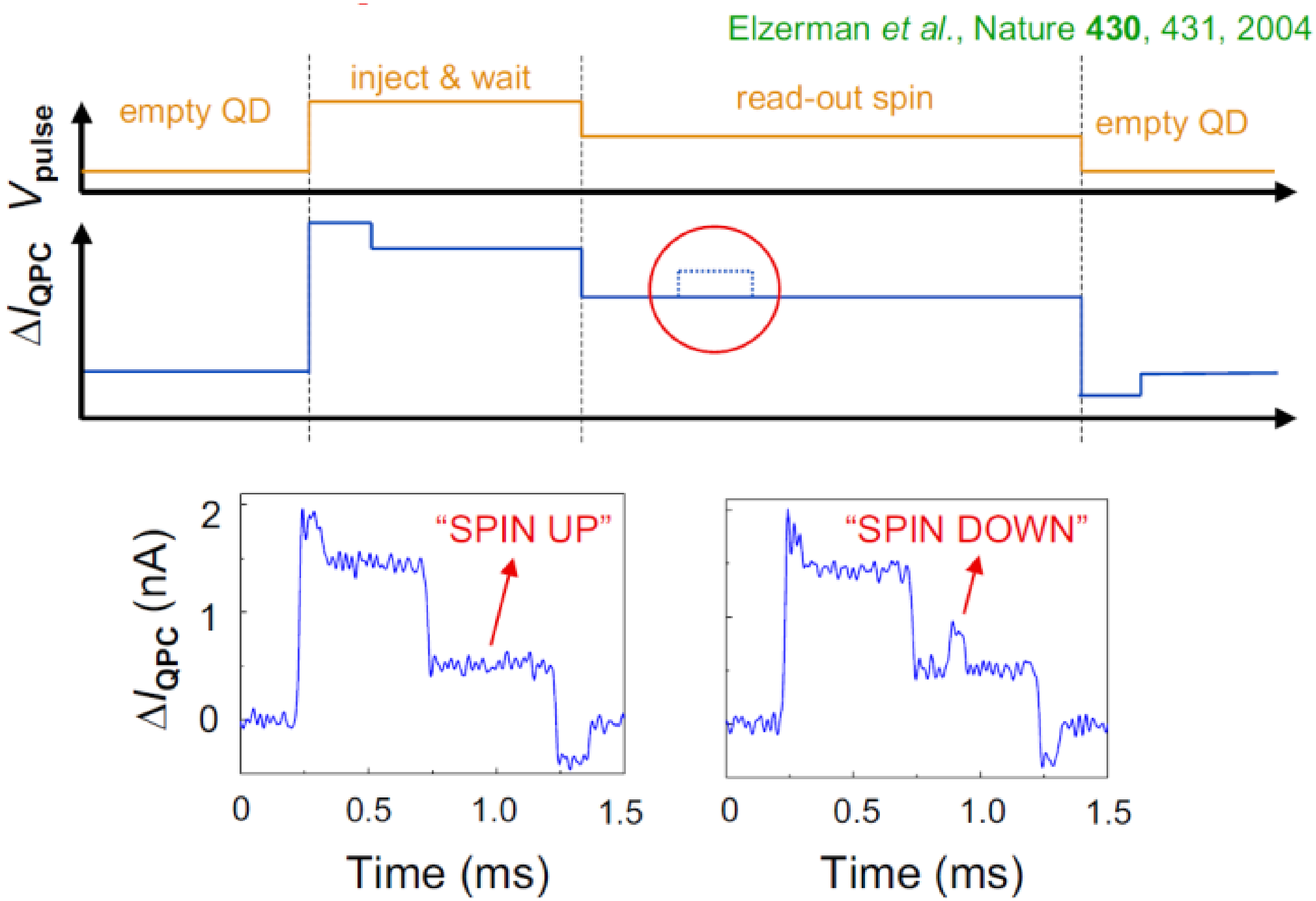


Spin read-out procedure



Inspiration: Fujisawa *et al.*, Nature **419**, 279, 2002

Spin read-out results



The end