Φυλλάδιο 5 , ΑΣΚΗΣΗ 7, 13-05-2024

2. Use Romberg integration to find an approximation to $\int_1^3 e^x \sin x dx$. Complete the table until $R_{n,n-1}$ and $R_{n,n}$ agree to within 10^{-4}. Compare your answer to the exact result $y(x) = \frac{1}{2}e^x(\sin x - \cos x)$.

Exact result is: $\frac{1}{2}e^x(\sin x - \cos x) = |_1^3 \frac{1}{2}e^x(\sin x - \cos x) = 10.95017$.

$n_1 = 1$: $h_1 = \frac{3-1}{1} = 2$

$$\begin{aligned}
R_{1,1} &= \frac{h_1}{2}(f(a)+f(b)) = \frac{2}{2}(e^1 \sin 1 + e^3 \sin 3) = 5.1218264 \\
h_2 &= h_1/2 = 1 \\
R_{2,1} &= \frac{h_2}{2}(f(a)+f(b)+2f(a+h_2)) = \frac{1}{2}R_{1,1} + \frac{1}{2}(2 * e^2 \sin 2) = 9.2797629 \\
R_{2,2} &= \frac{4R_{2,1}-R_{1,1}}{4-1} = 10.665742 \quad |R_{2,2} - R2,1| > 10^{-6} \\
h_3 &= h_2/2 = 1/2 \\
R_{3,1} &= \frac{h_3}{2}(f(a)+f(b)+2(f(a+h_3)+f(a+2h_3)+f(a+3h_3))) \\
&= \frac{1}{2}R_{2,1} + \frac{1/2}{2}(2*(e^{1.5}\sin 1.5 + e^{2.5}\sin 2.5) = 10.520554 \\
R_{3,2} &= \frac{4R_{3,1}-R_{2,1}}{4-1} = 10.934151 \\
R_{3,3} &= \frac{4^2R_{3,2}-R_{2,2}}{4^2-1} = 10.952045 \quad |R_{3,2} - R3,2| \simeq 1.8\ 10^{-2} \\
h_4 &= h_3/2 = 1/4 \\
R_{4,1} &= \frac{R_{3,1}}{2} + \frac{1}{2}\frac{1}{4}2(f((1.25)+f(1.75)+f(2.25)+f(2.75)) = 10.842043 \\
R_{4,2} &= \frac{4R_{4,1}-R_{3,1}}{4-1} = 10.949206 \\
R_{4,3} &= \frac{4^2R_{4,2}-R_{3,2}}{4^2-1} = 10.95021 \\
R_{4,4} &= \frac{4^3R_{4,3}-R_{3,3}}{4^3-1} = 10.950181 \quad |R_{4,4} - R4,3| \simeq 2.9\ 10^{-5}
\end{aligned}$$

|Exact solution-$R_{4,4}$| $= 1.1\ 10^{-5}$.