ΦΥΛΛΑΔΙΟ 5, ΑΣΚΗΣΗ 6, ΕΑΡΙΝΟ 2024

The standard normal probability density function is $[\exp(-x^2/2)]/(2\pi)^{1/2}$. The probability that the random variable X lies between 0 and 1 is:

$$P = \Pr(0 \le X \le 1) = \frac{1}{\sqrt{2\pi}} \int_0^1 e^{-x^2/2} dx$$

Application of the trapezoidal rule with different numbers of segments over this interval gives the results tabulated below. Using these results, calculate the estimate of P with the highest possible order, carrying 7 decimal places. Arrange your work in tabular form, and show all formulas you use. What is the order of error of your result?

h	0.5	0.25	0.125
#	2	4	8
P	0.3362609	0.3400818	0.3410295

Solution:

Use Romberg Integration starting with the $O(h^2)$ trapezoidal results given above and tabulate in the appropriate column of the typical Romberg integration table:

# Trap. segments	Trapezoidal $O(h^2)$	Simpson's 1/3 $O(h^4)$	Boole 5-point $O(h^6)$	$O(h^8)$
2	0.3362609	0.3413554	**0.3413447**	
4	0.3400818	0.3413454		
8	0.3410295			

To find $O(h^4)$ from $O(h^2)$, we use:

$$\frac{4 \times P_{better} - 1 \times P_{poorer}}{3} = P_{better} + \frac{P_{better} - P_{poorer}}{3}$$

To find $O(h^6)$ from $O(h^4)$, we use:

$$\frac{16 \times P_{better} - 1 \times P_{poorer}}{15} = P_{better} + \frac{P_{better} - P_{poorer}}{15}$$

To find $O(h^8)$ from $O(h^6)$, we would use (not needed here):

$$\frac{64 \times P_{better} - 1 \times P_{poorer}}{63} = P_{better} + \frac{P_{better} - P_{poorer}}{63}$$

The best order results that we can obtain here is that **P = 0.3413447**. From its position in the above table, we see that this is an $O(h^6)$ estimate.