

## Φυλλάδιο 5 , ΑΣΚΗΣΗ 11, 13-05-2024

Integrate the data using Simpson's-1/3 and Simpson's-3/8 Rules.

| $i$   | 0     | 1     | 2     | 3     | 4     | 5     | 6     | 7     |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $x_i$ | 1.0   | 1.1   | 1.2   | 1.3   | 1.4   | 1.5   | 1.6   | 1.7   |
| $f_i$ | 1.543 | 1.669 | 1.811 | 1.971 | 2.151 | 2.352 | 2.577 | 2.828 |

NOTE: 8 abscissas  $\Rightarrow n=7$  subintervals. So we cannot use Simpson's-1/3 rule alone ( $n$  is not divisible by 2) or Simpson's-3/8 rule alone ( $n$  is not divisible by 3).

However, in this problem we can combine the methods by appropriately dividing the interval:

1. We'll use Simpson's-1/3 rule on interval  $[1.0, 1.4]$  (4 subintervals is divisible by 2), and
2. we'll use Simpson's-3/8 rule on interval  $[1.4, 1.7]$  (3 subintervals is divisible by 3).

This way we obtain consistent accuracy  $O(h^4)$  on the entire interval  $[1.0, 1.7]$ .

1. Use Simpson's-1/3 rule on interval  $[1.0, 1.4]$ .  $h = 0.1$

$$\begin{aligned} \int_{1.0}^{1.4} f(x) dx &\approx \frac{h}{3} [f_0 + 4f_1 + 2f_2 + 4f_3 + f_4] \\ &= \frac{0.1}{3} [1.543 + 4(1.669) + 2(1.811) + 4(1.971) + 2.151] \\ &= 0.729200. \end{aligned}$$

2. Use Simpson's-3/8 rule on interval  $[1.4, 1.7]$ .  $h = 0.1$

$$\begin{aligned} \int_{1.4}^{1.7} f(x) dx &\approx \frac{3h}{8} [f_4 + 3f_5 + 3f_6 + f_7] \\ &= \frac{3(0.1)}{8} [2.151 + 3(2.352) + 3(2.577) + 2.828] \\ &= 0.741225. \end{aligned}$$

3. Now add the results.

$$\begin{aligned} \int_{1.0}^{1.7} f(x) dx &\approx \int_{1.0}^{1.4} f(x) dx + \int_{1.4}^{1.7} f(x) dx \\ &= 1.470425. \end{aligned}$$

NOTE: Alternatively, we could have used Simpson's-3/8 rule on interval  $[1.0, 1.3]$  and Simpson's-1/3 rule on interval  $[1.3, 1.7]$ . We would obtain a different approximation:  $1.470441\bar{6}$ .

BE CAREFUL:

- We could use Simpson's-1/3 rule on interval  $[1.0, 1.6]$  (6 subintervals is divisible by 2) and the Trapezoidal rule on interval  $[1.6, 1.7]$ , **but why wouldn't we?**
- We could also use Simpson's-3/8 rule on interval  $[1.0, 1.6]$  (6 subintervals is divisible by 3) and the Trapezoidal rule on interval  $[1.6, 1.7]$ , **but why wouldn't we?**