Πρόβλημα 3.5 όπως παράδειγμα 3.1

$$x(s) = \frac{1}{s(s+1)(0.5s+1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{0.5\cdot s+1} \quad (1)$$

$$(1) \xRightarrow{\times(s)} \frac{\cancel{s}}{\cancel{s}(s+1)(0.5\cdot s+1)} = A + \frac{B\cdot s}{s+1} + \frac{C\cdot s}{0.5\cdot s+1}$$

$$\xRightarrow{s=0} \quad 1 = A$$

$$(1) \xRightarrow{\times(s+1)} \frac{s+1}{s(s+1)(0.5\cdot s+1)} = \frac{A(s+1)}{s} + B + \frac{C(s+1)}{0.5\cdot s+1}$$

$$\xRightarrow{s=-1} \frac{1}{-1(0.5(-1)+1)} = B \Rightarrow B = -\frac{1}{0.5} = -2$$

$$(1) \xRightarrow{\times(0.5\cdot s+1)} \frac{0.5\cdot s+1}{s(s+1)(0.5\cdot s+1)} = \frac{A(0.5s+1)}{s} + \frac{B\cdot(0.5\cdot s+1)}{s+1} +$$

$$\xRightarrow{s=-2} \frac{1}{-2(-2+1)} = C = \frac{1}{+2} = +0.5$$

Άρα $x(s) = \frac{1}{s} - \frac{2}{s+1} + \frac{0.5}{0.5\cdot s+1}$

$$L^{-1}\{1/s\} = 1$$

$$L^{-1}\left\{\frac{1}{s+a}\right\} = e^{-a\cdot t} = e^{-t}$$

$a=1$

$$\frac{0.5/0.5}{(0.5\cdot s+1)/0.5} = \frac{1}{s+2}$$

$a=2$ $\quad \rightarrow e^{-2t}$

$$x(t) = 1 - 2e^{-t} + e^{-2t}$$

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Problem 3.1c

$$\frac{d^2x}{dt^2} + \frac{3dx}{dt} + x = 1 \quad x(0) = x'(0) = 0$$

$$L\left\{\frac{d^2x}{dt}\right\} = s^2x(s) - s\,x(0) - x'(0) = s^2x(s)$$

$$L\left\{\frac{dx}{dt}\right\} = s\,x(s) - x(0) = s\,x(s)$$

$$L\{x\} = x(s)$$

$$L\{1\} = 1/s$$

Άρα $s^2x(s) + 3s\,x(s) + x(s) = \frac{1}{s} \Rightarrow (s^2+3\cdot s+1)\cdot x(s) = 1/s \Rightarrow$

$$\Rightarrow x(s) = \frac{1}{s(s^2+3s+1)}$$

Ρίζες τριωνύμου $\quad \Delta = 3^2 - 4 = 9-4 = 5 \quad p_1 = \frac{-3+\sqrt{5}}{2} = -0.382$

$p_2 = \frac{-3-\sqrt{5}}{2} = -2.618$

$$\frac{1}{s(s^2+3s+1)} = x(s) = \frac{A}{s} + \frac{B}{(s+0.382)} + \frac{C}{s+2.618} \quad (1)$$

$$\boxed{s^2+3s+1 = (s-p_1)(s-p_2)}$$

$$= \frac{1}{s(s+0.382)(s+2.618)}$$

όπως παράδειγμα 3.1

$$(1) \xrightarrow{\times s} \frac{1}{s^2+3s+1} = A + \frac{B\cdot s}{s+0.382} + \frac{C\cdot s}{s+2.618} \xrightarrow{s=0} A = 1$$

$$(1) \xrightarrow{(s+0.382)} \frac{(s+0.382)}{s(s+2.618)(s+0.382)} = \frac{A(s+0.382)}{s} + B + C\cdot\frac{(s+0.382)}{s+2.618}$$

$$\xrightarrow{s=-0.382} \frac{1}{-0.382(-0.382+2.618)} = B = -1.17$$

$$(1) \xrightarrow{s+2.618} \frac{(s+2.618)}{s(s+2.618)(s+0.382)} = \frac{A(s+2.618)}{s} + \frac{B(2.618+s)}{(s+0.382)} + C$$

$$\xrightarrow{s=-2.618} \frac{1}{-2.618(-2.618+0.382)} = C = +0.171$$

$$(1) \Rightarrow x(s) = \frac{1}{s} + \frac{-1.17}{s+0.382} + \frac{0.171}{s+2.618}$$

$$L^{-1}\left\{\frac{1}{s+a}\right\} = e^{-at}$$

$a = 0.382$ ni 2.618

$$x(t) = 1 - 1.17\,e^{-0.382t} + 0.171\,e^{-2.618t}$$

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Problems

3.1a $\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = 1 \qquad x(0) = x'(0) = 0$

Laplace transformation

$$L\left\{\frac{d^2x}{dt^2}\right\} = s^2 x(s) - s\,x(0) - x(0)' = s^2 x(s)$$

$$L\left\{\frac{dx}{dt}\right\} = s\,x(s) - x(0) = s\,x(s)$$

$$L\{x\} = x(s)$$

$$L\{1\} = \frac{1}{s}$$

Άρα $s^2 x(s) + s\,x(s) + x(s) = \frac{1}{s} \Rightarrow x(s)(s^2+s+1) = \frac{1}{s}$

$$\Rightarrow x(s) = \frac{1}{s(s^2+s+1)} = \frac{A}{s} + \frac{B}{s^2+s+1}$$

ρίζα δευτεροβάθμια συνάρτησης (τριωνύμου 2ου βαθμού)
$\Delta = 1 - 4 = -3 < 0 \Rightarrow$ μιγαδικές ρίζα

Άρα $x(s) = \frac{A}{s} + \frac{B\cdot s + C}{s^2+s+1} = \frac{1}{s(s^2+s+1)}$ (1)

$$(1) \xRightarrow{\times s} A + \frac{(B\cdot s + C)\cdot s}{s^2+s+1} = \frac{1}{s^2+s+1} \xRightarrow{s=0} A = 1$$

$$(1) \Rightarrow \frac{1}{s} + \frac{B\cdot s + C}{s^2+s+1} = \frac{1}{s(s^2+s+1)} \Rightarrow$$

$$\Rightarrow s^2 + s + \cancel{1} + B\cdot s^2 + C\cdot s = \cancel{1} \Rightarrow$$

$$\Rightarrow s^2(1+B) + s(1+C) = 0$$

ισχύς για κάθε $s \Rightarrow 1 + B = 0 \Rightarrow B = -1$
$1 + C = 0 \Rightarrow C = -1$

Άρα $x(s) = \frac{1}{s} + \frac{-s-1}{s^2+s+1} = \frac{1}{s} - \frac{s+1}{s^2+s+1}$

$L^{-1}\left(\frac{1}{s}\right) = 1$

$$\frac{s+1}{\underbrace{s^2+2\cdot\frac{1}{2}\cdot s+\left(\frac{1}{2}\right)^2}_{(s+1/2)^2} \underbrace{-\frac{1}{4}+1}_{3/4}} = \frac{s+1}{(s+1/2)^2+3/4} = \frac{s+1/2-1/2+1}{(s+1/2)^2+3/4}$$

$$= \frac{s+1/2}{(s+1/2)^2+3/4} + \frac{1/2}{(s+1/2)^2+3/4} \frac{\times\sqrt{3/4}}{\sqrt{3/4}} \qquad a,\ a,\ k^2$$

$$\frac{1}{2}\frac{\sqrt{4}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$= \frac{s+1/2}{(s+1/2)^2+3/4} + \frac{\sqrt{3/4}}{(s+1/2)^2+3/4} \qquad \frac{\sqrt{4}}{2\times\sqrt{3}} \rightarrow 1/\sqrt{3}$$

$$L^{-1}\left\{\frac{s+1/2}{(s+1/2)^2+3/4}\right\} = e^{-t/2}\cos(\sqrt{3}t/2)$$

$a=1/2,\ k=\sqrt{3/4}$

$$L^{-1}\left\{\frac{\sqrt{3/4}}{(s+1/2)^2+3/4}\right\} = e^{-t/2}\sin(\sqrt{3}t/2)$$

$k=\sqrt{3}/2,\ a=1/2$

Άρα $x(t) = 1 - \left[e^{-t/2}\cos(\sqrt{3}t/2) + \frac{1}{\sqrt{3}}e^{-t/2}\sin(\sqrt{3}t/2)\right]$

$$= 1 - e^{-t/2}\left(\cos(\sqrt{3}t/2) + \frac{1}{\sqrt{3}}e^{-t/2}\sin(\sqrt{3}\,t/2)\right]$$

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

SKAG

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