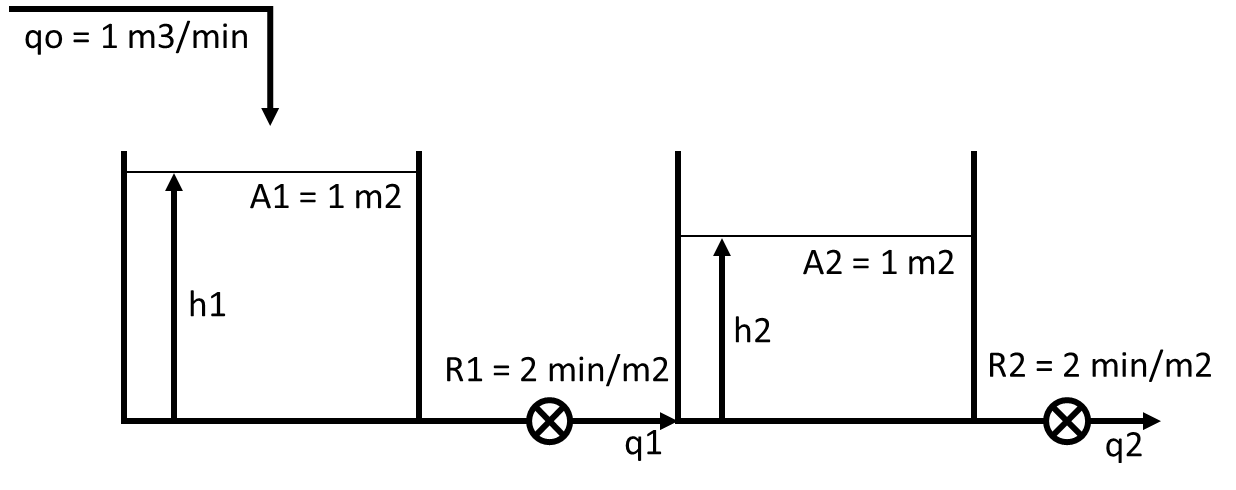
1η Πρόοδος Δυναμική και Έλεγχος Διεργασιών 14 Νοεμβρίου 2017

ΘΕΜΑ1 (10μονάδες)



Σε χρόνο 0 προστίθεται στιγμιαία 1 m3. Να υπολογιστεί και να παρασταθεί γραφικά (σε χρόνο 0, 1, 5, 10, 20, 30 min) η απόκριση της στάθμης στην 1η και τη 2η δεξαμενή.

@SS 1 – (h1s – h2s)/2 = 0 ⬄ (h1s – h2s) = 2 ⬄ h1s = 4

(h1s – h2s)/2 – h2s/2 = 0 ⬄ h2s = 2

Ισοζύγιο μάζας στην 1η δεξαμενή

qo(t) – (h1(t) – h2(t))/R1 = A1\*dh1(t)/dt ⬄ Qo(t) – H1(t)/2 + H2(t)/2 = 1\*dH1(t)/dt

⬄ 2\*Qo(t) – H1(t) + H2(t) = 2\*dH1(t)/dt ⬄ 2Qo(s) – H1(s) + H2(s) = 2sH1(s) (1)

Ισοζύγιο μάζας στην 2η δεξαμενή

(h1(t) – h2(t))/R1 – h2(t)/R2 = A2\*dh2(t)/dt ⬄ h1(t)/2 – h2(t)/2 – h2(t)/2 = 1\*dh2(t)/dt ⬄

⬄ h1(t) – 2\*h2(t) = 2\*dh2(t)/dt ⬄ H1(t) – 2\*H2(t) = 2dH2(t)/dt ⬄ H1(s) – 2H2(s) = 2sH2(s) ⬄

⬄ H1(s) = (2s + 2)H2(s) ⬄ H2(s) = H1(s)/(2s + 2)

(1) 2Qo(s) – H1(s) + H1(s)/(2s + 2) = 2sH1(s) ⬄ 2Qo/H1 – 1 + 1/(2s + 2) = 2s ⬄ Qo/H1 = s – 0,5/(2s + 2) + 0,5 ⬄

Qo/H1 = (2s2 + 2s – 0,5 + s + 1)/(2s + 2) ⬄ H1/Qo = (2s + 2)/(2s2 + 3s + 0,5) ⬄

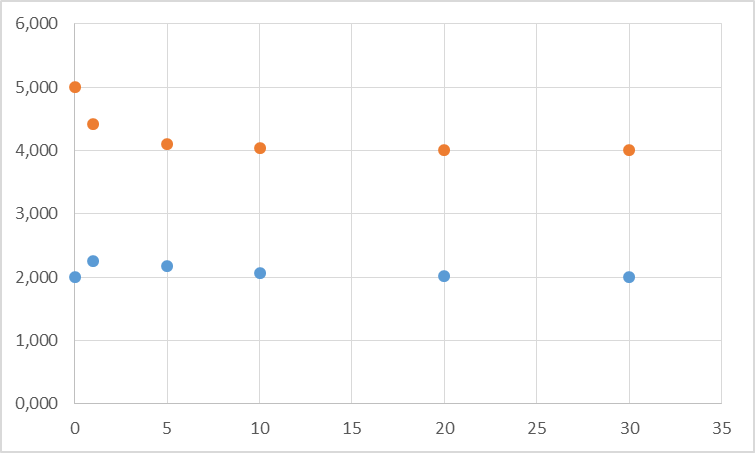
**H1/Qo = 2s/(2s2 + 3s + 0,5) + 2/(2s2 + 3s + 0,5)**

(1) 2Qo(s) – (2s + 2)H2(s) + H2(s) = 2s(2s + 2)H2(s) ⬄ 2Qo/H2 = (2s + 2) – 1 + 2s(2s + 2) ⬄

Qo/H2 = s + 1 – 0,5 + 2s2 + 2s = 2s2 + 3s + 0,5 ⬄ **H2/Qo = 1/(2s2 + 3s + 0,5)**

Κρουστική διαταραχή της παροχής: qo(t) = qos + δ(t) ⬄ qo(t) – qos = δ(t) ⬄ Qo(t) = δ(t) ⬄ Qo(s) = 1

Λύση τριωνύμου Δ = 9-4\*2\*0,5 = 5 x1 = (-3+5^0,5)/4 = -0,191 x2 = (-3-5^0,5)/4 = -1,309



HEAVYSIDE

2s/2(s + 0,191)(s + 1,309) = A/(s + 0,191) + B/(s + 1,309)

A = (-0,191)/(-0,191+1,309) = -0,171

B = (-1,309)/(-1,309+0,191) = 1,171

1/2(s + 0,191)(s + 1,309) = Γ/(s + 0,191) + E/(s + 1,309)

Γ = 0,5/(-0,191+1,309) = 0,447

E =0,5/(-1,309+0,191) = -0,447

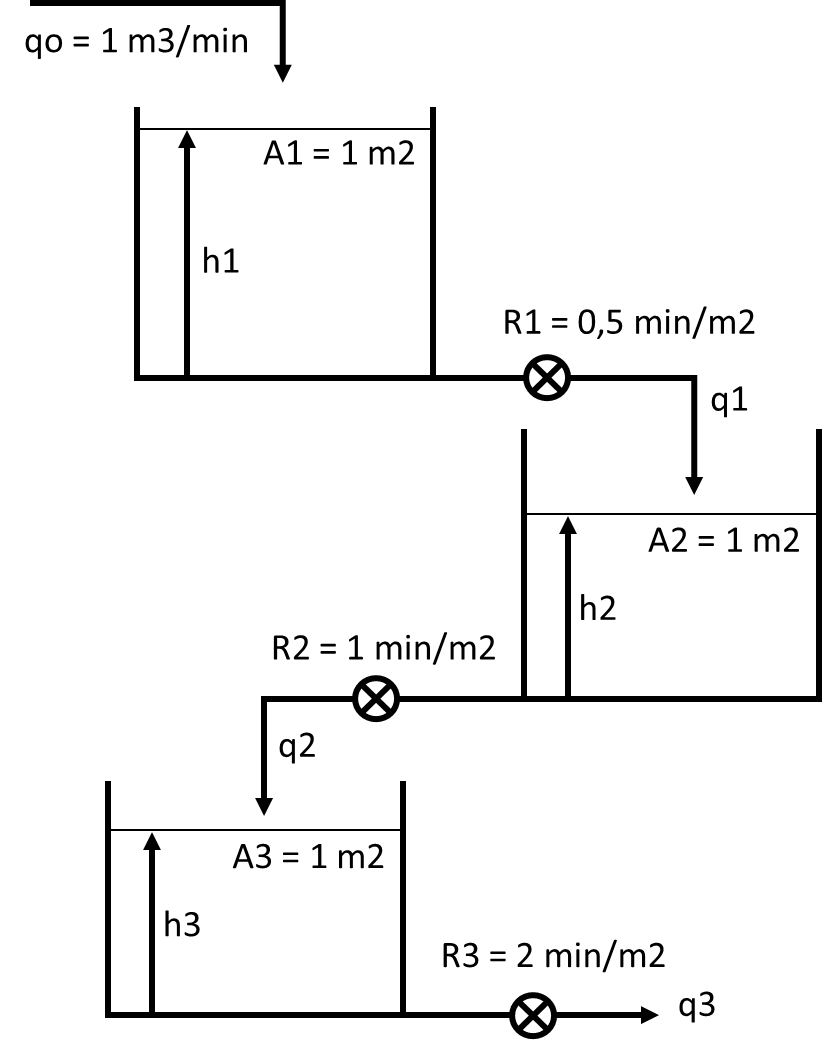
H1 = -0,171/(s + 0,191) + 1,171/(s + 1,309) + 0,894/(s + 0,191) - 0,894/(s + 1,309) ⬄

H1(t) = 1,171\*exp(-1,309\*t)+0,894\*exp(-0,191\*t)-0,171\*exp(-0,191\*t)-0,894\*exp(-1,309\*t)

H2 = 0,447/(s + 0,191) - 0,447/(s + 1,309) ⬄ H2(t) = 0,447\*exp(-0,191\*t)-0,447\*exp(-1,309\*t)

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **t** | **H1** | **h1** | **H2** | **h2** | **t** | **H1** | **h1** | **H2** | **h2** |
| **0** | 1,000 | 5,000 | 0,000 | 2,000 | **10** | 0,041 | 4,041 | 0,066 | 2,066 |
| **1** | 0,424 | 4,424 | 0,249 | 2,249 | **20** | 0,006 | 4,006 | 0,010 | 2,010 |
| **5** | 0,107 | 4,107 | 0,171 | 2,171 | **30** | 0,001 | 4,001 | 0,001 | 2,001 |

ΘΕΜΑ 2 (10 μονάδες)



Στο σύστημα δεξαμενών η παροχή αυξάνεται βηματικά κατά 1 m3/min, για 5 λεπτά και στη συνέχεια ελαττώνεται στην αρχική της τιμή. Να υπολογιστεί η στάθμη των τριών δεξαμενών σε χρόνο 4 και 6 λεπτά.

Βηματική μεταβολή της qo: qo(t) = qos + 1u(t) – 1u(t-5) ⬄ qo(t) – qos = 1u(t) – 1u(t-5) ⬄

⬄ Qo(t) = 1u(t) – 1u(t-5) ⬄ **Qo(s) = 1/s – exp(-5s)/s**

**Ισοζύγιο μάζας στην 1η δεξαμενή**

@SS 1 – h1s/0,5 = 0 ⬄ h1s = 0,5

qo(t) – h1(t)/R1 = A1\*dh1(t)/dt ⬄ Qo(t) – H1(t)/0,5 = 1\*dH1(t)/dt

⬄ 0,5\*Qo(t) – H1(t) = 0,5\*dH1(t)/dt ⬄ 0,5Qo(s) – H1(s) = 0,5sH1(s) ⬄

⬄ 0,5 – Η1/Qo = 0,5sH1/Qo ⬄ H1/Qo\*(0,5s + 1) = 0,5 ⬄ H1/Qo = 1/(s + 2) (1)

H1 = 1/s(s + 2) – exp(-5s)/s(s +2) (2)

HEAVYSIDE 1/s(s + 2) = A/s + B/(s + 2) A = 0,5 B = -0,5

(2) H1 = 0,5/s – 0,5/(s + 2) – 0,5\*exp(-5s)/s + 0,5\*exp(-5s)/(s + 2) ⬄

⬄H1(t) = 0,5\*(1 – exp(-2t))\*u(t) - 0,5\*(1-exp(-2\*(t-5)))\*u(t-5)

H1(4) = 0,5\*(1 – exp(-2\*4)) = 0,50 m h1(4) = 0,50+0,50 = 1 m

H1(6) = 0,5\*(1 – exp(-2\*6))-0,5\*(1-exp(-2\*(6-5))) = 0,07 h1(6) = 0,50+0,07 = 0,57 m

q1(t) = h1(t)/R1 ⬄ Q1(t) = H1(t)/0,5 ⬄ Q1(s) = H1(s)/0,5 ⬄ H1 = 0,5Q1

(1) 0,5Q1/Qo = 1/(s + 2) ⬄ Q1/Qo = 2/(s + 2)

**Ισοζύγιο μάζας στην 2η δεξαμενή** q1(t) – h2(t)/R2 = A1\*dh2(t)/dt ⬄ Q1(t) – H2(t)/1 = 1\*dH2(t)/dt ⬄

@SS 1 – h2s/1 = 0 ⬄ h2s = 1 ⬄ Q1(s) – H2(s) = sH2(s) ⬄ 1 – Η2/Q1 = sH2/Q1 ⬄

⬄ H2/Q1\*(s + 1) = 1 ⬄ H2/Q1 = 1/(s + 1) (3)

H2/Qo = (H2/Q1)\*(Q1/Qo) = (1/(s + 1))\*(2/(s + 2)) = 2/(s + 1)(s + 2)

H2 = 1/s(s + 1)(s + 2) – exp(-5s)/s(s + 1)(s +2) (4)

HEAVYSIDE 1/s(s + 1)(s + 2) = A/s + B/(s + 1) + C/(s + 2) A = 0,5 B = -1 C = 1/(-2)((-1) = 0,5

(4) H2 = 0,5/s + 0,5/(s + 2) – 1/(s + 1) – 0,5\*exp(-5s)/s – 0,5\*exp(-5s)/(s + 2) + 0,5\*exp(-5s)/(s + 1) ⬄

⬄ H2(t) = 0,5\*(1+exp(-2\*t)-2\*exp(-t))\*u(t) – 0,5\*(1+exp(-2(t-5))-exp(-(t-5)))\*u(t-5)

H2(4) = 0,5\*(1+exp(-2\*4)-2\*exp(-4)) = 0,48 m h2(4) = 1,0+0,48 = 1,48 m

H2(6) = 0,5\*(1+exp(-2\*6)-2\*exp(-6))-0,5\*(1+exp(-2\*(6-5))-2\*exp(-(6-5))) = 0,30 m h2(6) = 1,0+0,30 = 1,30 m

q2(t) = h2(t)/R2 ⬄ Q2(t) = H2(t)/1 ⬄ Q2(s) = H2(s)

(3) Q2/Q1 = 1/(s + 1)

**Ισοζύγιο μάζας στην 3η δεξαμενή** q2(t) – h3(t)/R3 = A3\*dh3(t)/dt ⬄ Q2(t) – H3(t)/2 = 1\*dH3(t)/dt ⬄

@SS 1 – h3s/2 = 0 ⬄ h3s = 2 ⬄ 2Q2(s) – H3(s) = 2sH3(s) ⬄ 2 – Η3/Q2 = sH3/Q2 ⬄

⬄ H3/Q2\*(s + 1) = 2 ⬄ H3/Q2 = 2/(s + 1)

H3/Qo = (H3/Q2)\*(Q2/Q1)\*(Q1/Qo) = (2/(s + 1))\*(1/(s + 1))\*(2/(s + 2)) = 4/(s + 1)(s + 1)(s + 2)

H3 = 4/s(s + 1)2(s + 2) – 4exp(-5s)/s(s + 1)2(s +2) (5)

HEAVYSIDE 4/s(s + 1)2(s + 2) = A/s + B/(s + 1)2 + C/(s + 1) + D/(s + 2)

A = 4/(1)(2) = 2 B = 4/(-1)(1) = -4 D = 4/(-2)(1) = -2

4/s(s + 1)2(s + 2) = 2/s – 4/(s + 1)2 + C/(s + 1) – 2/(s + 2) ⬄ 4 = 2(s + 1)2(s + 2) – 4s(s + 2) + Cs(s + 1)(s + 2) – 2s(s + 1)2 =

= 2(s2 + 2s + 1)(s + 2) – 4s2 – 8s + C(s2 + s)(s + 2) – 2s(s2 + 2s + 1) =

= 2(s3 + 2s2 + s + 2s2 + 4s + 2) - 4s2 - 8s + C(s3 + s2 + 2s2 + 2s) – 2s3 – 4s2 – 2s =

= 2s3 + 4s2 + 2s + 4s2+ 8s + 4 - 4s2 - 8s + Cs3 + Cs2 + 2Cs2 + 2Cs – 2s3 – 4s2 – 2s ⬄

s3(2 + C – 2) + s2(4 + 4 - 4 + C + 2C – 4) + s(2 + 8 - 8 + 2C – 2) = 0 ⬄ C = 0

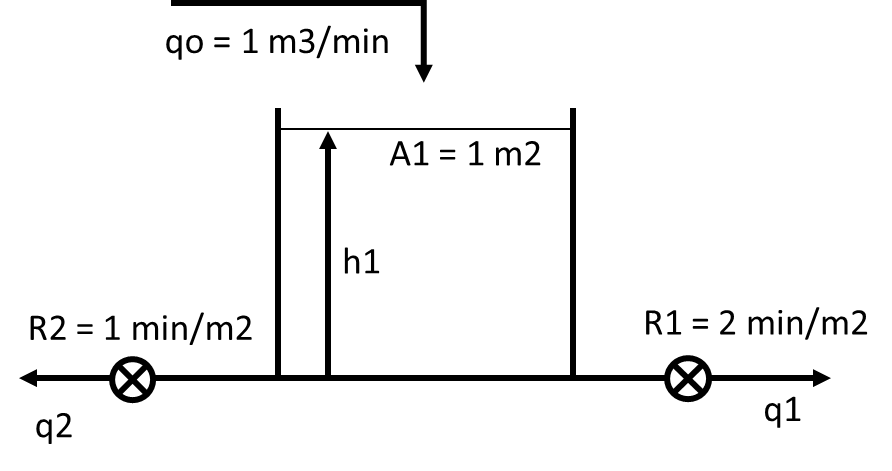
(5) H3 = 4/s – 4/(s + 1)2 – 2/(s + 2) – 4\*exp(-5s)/s + 4\*exp(-5s)/(s + 1)2 + 2\*exp(-5s)/(s + 2) ⬄

⬄ H3(t) = 4\*(1 + t\*exp(-t)-0,5\*exp(-2t))\*u(t) – 4\*(1 + (t-5)\*exp(-(t-5))-0,5\*exp(-2(t-5)))\*u(t-5)

H3(4) = 4\*(1 + 4\*exp(-4)-0,5\*exp(-2\*4)) = 4,29 m h3(4) = 2,0+4,29 = 5,29 m

H3(6) = 4\*(1+6\*exp(-6)-0,5\*exp(-2\*6))-4\*(1+(6-5)\*exp(-(6-5))-0,5\*exp(-2\*(6-5))) = -1,14 m h3(6) = 2,0-1,14 = 0,86 m

ΘΕΜΑ 3 (10 μονάδες)



Στη δεξαμενή επιβάλλεται η μεταβολή του σχήματος. Να υπολογιστεί η στάθμη στο 2ο, 3ο, 4ο και 5ο λεπτό.

Ισοζύγιο μάζας: @SS 1 – hs – hs/2 = 0 ⬄ 1 = 3hs/2 ⬄ hs = 2/3

q(t) – h(t)/R1 – h(t)/R2 = Adh(t)/dt ⬄ Q(t) – H(t)/2 – H(t) = dH(t)/dt ⬄ Q(s) – 3/2H(s) = sH(s) ⬄

⬄ 1 – 1,5H/Q = sH/Q ⬄ 1 = H/Q(s + 1,5) ⬄ H/Q = 1/(s + 1,5) (1)

q(t) = qs + 1\*u(t) + t\*u(t-2) ⬄ q(t) - qs = 1\*u(t) + t\*u(t-2) ⬄ Q(t) = 1\*u(t) + t\*u(t-2) ⬄Q(s) = 1/s + exp(-2s)/s2

(1) H = 1/s(s + 1,5) + exp(-2s)/s2(s + 1,5) (2)

HEAVYSIDE: 1/s(s + 1,5) = A/s + B/(s + 1,5) = (2/3)/s – (2/3)/(s + 1,5)

1/s2(s + 1,5) = C/s2 + D/s + E/(s + 1,5) C = 2/3 E = 1/(-3/2)^2 = 1/(9/4) = 4/9

1 = (2/3)\*(s + 3/2) + D\*s\*(s + 3/2) + (4/9)\*s2 = (2/3)s + 1 + D\*s2 + (3D/2)s + (4/9)\*s2

D = -4/9 2/3 = - 3D/2 ⬄ D = -4/9

(2) H = (2/3)/s – (2/3)/(s + 1,5) + exp(-2s)[(2/3)/s2 – (4/9)/s + (4/9)/(s + 1,5)] ⬄

H(t) = 2/3 – (2/3)exp(-1,5t) + [(2/3)(t – 2) – (4/9) + (4/9)exp(-1,5(t-2))]u(t – 2)

H(2) = 2/3-(2/3)\*exp(-1,5\*1) = 0,52 h(2) = 0,67 + 0,52 =1,19 m

H(3) = 2/3-(2/3)\*exp(-1,5\*2) = 0,63 h(3) = 0,67 + 0,63 =1,30 m

H(4) = 2/3-(2/3)\*exp(-1,5\*3) + (2/3)\*(3-2)-(4/9)+(4/9)\*exp(-1,5\*(3-2)) = 0,98 m h(4) = 0,67 + 0,98 =1,65 m

H(5) = 2/3-(2/3)\*exp(-1,5\*4) + (2/3)\*(4-2)-(4/9)+(4/9)\*exp(-1,5\*(4-2)) = 1,58 m h(5) = 0,67 + 1,58 =2,24 m