- AR(1) ή ARMA(1,0)

$$(1-\varphi_1 B)z_t = a_t$$

$$z_t - \varphi_1 z_{t-1} = a_t$$

$$z_t = \varphi_1 z_{t-1} + a_t$$

↑ λ. θόρυβος.

σε AR(1): $\varphi_1 = \rho_1$

δεν υπάρχει σταθερός όρος (φ_0) $\Rightarrow \bar{z}_t = 0$

- AR(2) ή ARMA(2,0)

$$(1-\varphi_1 B - \varphi_2 B^2)z_t = a_t$$

$$z_t - \varphi_1 z_{t-1} - \varphi_2 z_{t-2} = a_t$$

$$z_t = \varphi_1 z_{t-1} + \varphi_2 z_{t-2} + a_t$$

Εξ. Yule-Walker:

$$\begin{cases} \rho_1 = \varphi_1 + \varphi_2 \rho_1 \\ \rho_2 = \varphi_1 \rho_1 + \varphi_2 \end{cases}$$

ρ_1, ρ_2 = γν. από δείγμα.

άγνωστοι $\varphi_1, \varphi_2 = ?$

Μοντέλο ARMA(1,1): (μεικτό)

$$(1-\varphi_1 B) z_t = (1-\theta_1 B) a_t.$$

$$\Rightarrow z_t - \varphi_1 z_{t-1} = a_t - \theta_1 a_{t-1} \Rightarrow$$

$$\Rightarrow z_t = \underbrace{\varphi_1 z_{t-1}}_{\text{αυτοπ/μο}} + a_t - \underbrace{\theta_1 a_{t-1}}_{\text{κινητού μέσου όρου}}$$

ARIMA (p, d, q)

$$\phi(B)(1-B)^d Y_t = \theta(B) e_t$$

πολυώνυμο τάξης p

διαφορά

πολυώνυμο τάξης q

π.χ.

ARIMA(0,1,1) = IMA(1,1)

1. $(1-B)^1 Y_t = (1-\theta_1 B)a_t \Rightarrow$

$\Rightarrow Y_t - BY_t = a_t - \theta_1 B a_t \Rightarrow$

$\Rightarrow Y_t - Y_{t-1} = a_t - \theta_1 a_{t-1} \Rightarrow$

$$\Rightarrow \boxed{Y_t = Y_{t-1} - \theta_1 a_{t-1} + a_t}$$

συντελεστής μονάδα, όχι αυτοπαλίνδρομος όρος

Εποχικά μόνο μοντέλα.

Πχ1 $AR(1)_{12}$ $(= SARMA(1,0)_{12})$

$$Y_t = \phi Y_{t-12} + a_t \quad , \quad -1 < \phi < 1$$

Πχ 2: $SARIMA(1,1,1)_{12}$.

$$\Phi_2(B^{12})(1-B^{12})^1 Y_t = \Theta_2(B^{12}) a_t$$

κεφαλαία.

$$(1-\Phi_1 B^{12})(1-B^{12}) Y_t = (1-\Theta_1 B^{12}) a_t$$

ARIMA-(S)ARIMA (όπως στο θέμα);

$$\left\{ \begin{aligned} \varphi_p(B)\,\Phi_P(B^{12})\,(1-B)^d\,(1-B^{12})^D\,Y_t = \\ = \theta_q(B)\,\Theta_Q(B^{12})\,a_t \end{aligned} \right\}$$

π.χ

$$ARIMA(0,1,0)\times ARIMA(3,1,0)_{12}$$

$\Phi_1 = -0{,}9133,\ \Phi_2 = -0{,}8146,\ \Phi_3 = -0.6002$

$$1\cdot\left(1+0{,}9133B^{12}+0{,}8146B^{24}+0.6002B^{36}\right)\cdot$$
$$(1-B)(1-B^{12})Y_t = 1\cdot 1\cdot e_t$$

ARIMA(0,0,1) x ARIMA(1,0,0)$_{12}$.

$$1 \cdot (1 - \phi_1 B^{12}) \cdot 1 \cdot 1 \cdot Y_t = (1 - \theta_1 B) \cdot 1 \cdot a_t$$

$$\Rightarrow \quad Y_t - \phi_1 Y_{t-12} = a_t - \theta_1 a_{t-1}$$

$$\boxed{Y_t = \phi_1 Y_{t-12} + a_t - \theta_1 a_{t-1}}$$