DEMOCRITUS UNIVERSITY OF THRACE
DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING

THE INSTANTANEOUS POWER THEORY

Advanced techniques for active and reactive power control

Special Applications of Power Electronics

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Introduction

- Converters have non linear behavior and generate reactive power
- They add up heavy harmonic components on their outputs
- Need of finding a new approach to the electrical power analysis
- It is defined in the time domain(Clarke Transformation), not in the frequency domain (FFT).
- It can be applied to three-phase systems with or without a neutral wire.
- It is valid not only in the steady state, but also in the transient state.
- Considers the three-phase system as a unit(3 coordinates).
- It is efficient and flexible.

The Clarke Transformation

- The p-q Theory uses the αβ0 transformation which consists of instantaneous stationary states of 3-phase voltages and currents**.**

$$\begin{bmatrix} v_0 \\ v_\alpha \\ v_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}$$

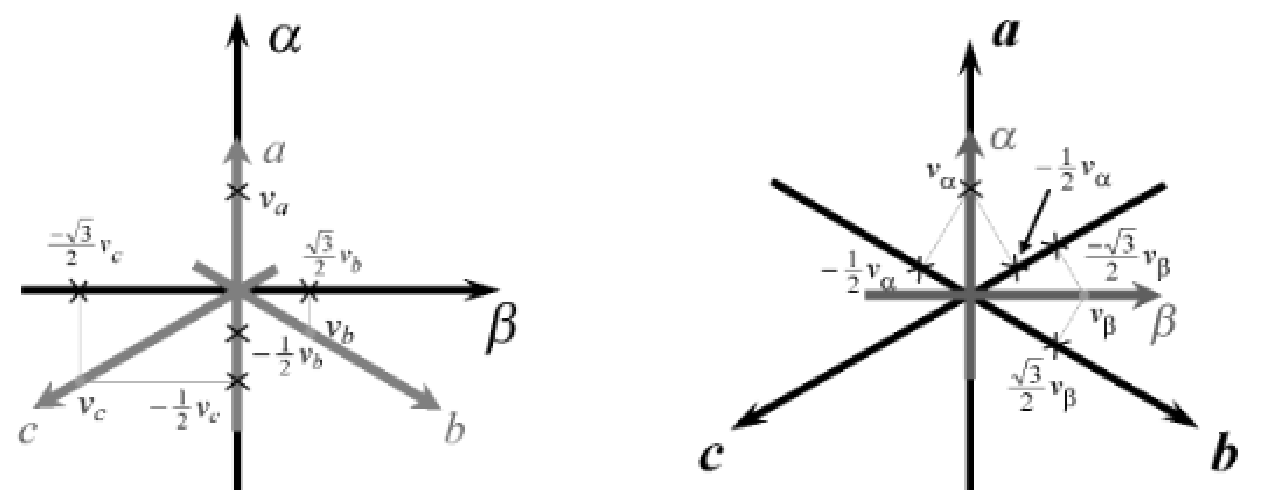
$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & 1 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_0 \\ v_\alpha \\ v_\beta \end{bmatrix}$$

Advantages of the αβ0 transformation

- Separate zero-sequence components from the abc-phase components

(No zero-sequence current exists in a three phase, three-wire system)

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}$$



Voltage Vectors

$$\begin{cases} v_a(t) = \sqrt{2}V\cos(\omega t + \phi_V) \\ v_b(t) = \sqrt{2}V\cos\left(\omega t + \phi_V - \dfrac{2\pi}{3}\right) \\ v_c(t) = \sqrt{2}V\cos\left(\omega t + \phi_V + \dfrac{2\pi}{3}\right) \end{cases}$$

$$\begin{cases} v_\alpha = \sqrt{3}V\cos(\omega t + \phi_V) \\ v_\beta = \sqrt{3}V\sin(\omega t + \phi_V) \end{cases}$$

$$\mathbf{e} = v_\alpha + jv_\beta \Rightarrow \left| \begin{array}{l} \mathbf{e} = \sqrt{3}V[\cos(\omega t + \phi_V) + j\sin(\omega t + \phi_V)] \\ \mathbf{e} = \sqrt{3}V\, e^{j(\omega t + \phi_V)} \end{array} \right.$$

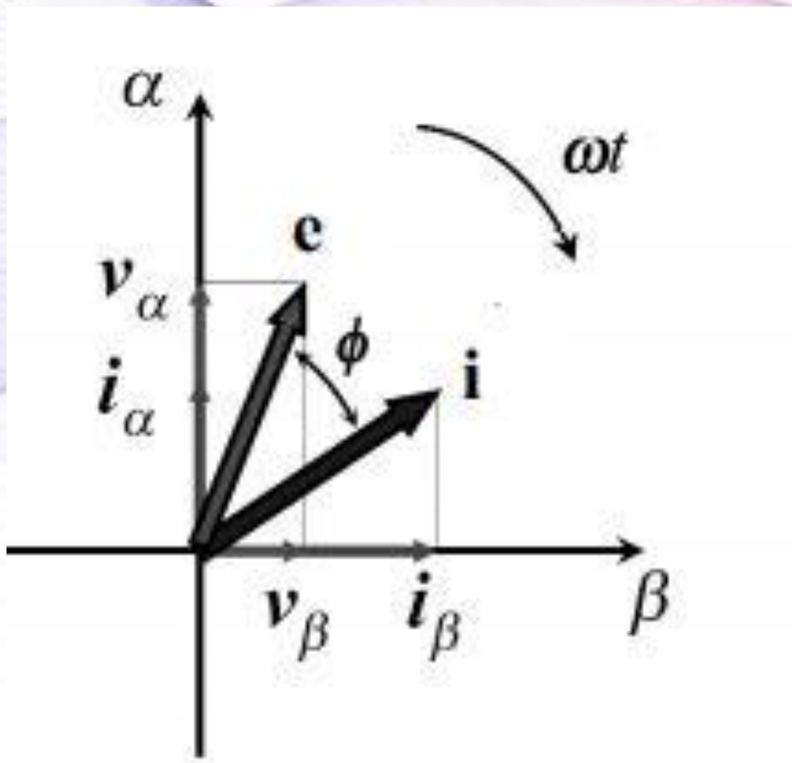
Currents Vectors

$$\begin{cases} i_a(t) = \sqrt{2}I\cos(\omega t + \phi_I) \\ i_b(t) = \sqrt{2}I\cos\left(\omega t + \phi_I - \dfrac{2\pi}{3}\right) \\ i_c(t) = \sqrt{2}I\cos\left(\omega t + \phi_I + \dfrac{2\pi}{3}\right) \end{cases}$$

$$\begin{cases} i_\alpha = \sqrt{3}I\cos(\omega t + \phi_I) \\ i_\beta = \sqrt{3}I\sin(\omega t + \phi_I) \end{cases}$$

$$\mathbf{i} = i_\alpha + ji_\beta \Rightarrow \left| \begin{array}{l} \mathbf{i} = \sqrt{3}I[\cos(\omega t + \phi_I) + j\sin(\omega t + \phi_I)] \\ \mathbf{i} = \sqrt{3}I\, e^{j(\omega t + \phi_I)} \end{array} \right.$$

- In the case of a three-phase balanced sinusoidal system the voltage and current vectors have constant amplitudes and rotate in the clockwise direction



$$\mathbf{e}_{abc} = v_a e^{j0} + v_b e^{j2\pi/3} + v_c e^{-j2\pi/3}$$

$$\mathbf{e}_{abc} = \frac{3\sqrt{2}}{2} V[\cos(\omega t + \phi_V) + j \sin(\omega t + \phi_V)] = \frac{3\sqrt{2}}{2} V e^{j(\omega t + \phi_V)}$$

$$\mathbf{e} = v_\alpha + j v_\beta \Rightarrow \left| \begin{array}{l} \mathbf{e} = \sqrt{3} V[\cos(\omega t + \phi_V) + j \sin(\omega t + \phi_V)] \\ \mathbf{e} = \sqrt{3} V e^{j(\omega t + \phi_V)} \end{array} \right.$$

$$\mathbf{e} = \sqrt{\frac{2}{3}} \mathbf{e}_{abc}$$

The vector representation of three-phase instantaneous voltages and currents has been used increasingly in the field of power electronics. For instance, it has been used in vector control of ac motor drives, in space-vector pulse-width-modulation(PWM) of power converters, as well as in control of power conditioners.

Active Power in Terms of Clarke Components

- The three-phase instantaneous active power p3φ(t) describes the total instantaneous energy flow per second between two subsystems.

$$p_{3\phi}(t) = v_a(t)i_a(t) + v_b(t)i_b(t) + v_c(t)i_c(t)$$
$$\Updownarrow$$
$$p_{3\phi} = v_a i_a + v_b i_b + v_c i_c$$

- In n-wire systems you can peak a 0 voltage reference point which makes it possible to use n-1 wattometers to measures the total power.

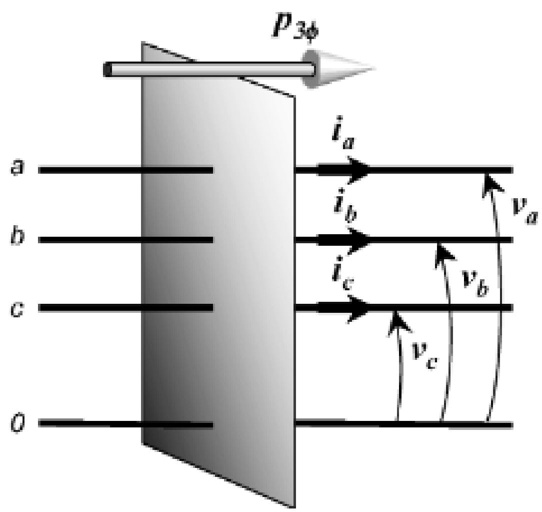
$$p_{3\phi} = (v_a - v_b)i_a + (v_b - v_b)i_b + (v_c - v_b)i_c = v_{ab}i_a + v_{cb}i_c$$

The Instantaneous Powers of the p-q Theory

- There are 3 instantaneous powers

1. Zero-sequence power
2. Real power
3. Imaginary power

$$\begin{bmatrix} p_0 \\ p \\ q \end{bmatrix} = \begin{bmatrix} v_0 & 0 & 0 \\ 0 & v_\alpha & v_\beta \\ 0 & v_\beta & -v_\alpha \end{bmatrix} \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix}$$



The three phase, three wire system

- The conventional concept of the complex power uses a voltage phasor and the conjugate of a current phasor. (steady f)
- The instantaneous complex power **s** is defined as the product of the voltage vector **e** and the conjugate of the current vector **i***.

$$\mathbf{s} = \mathbf{e} \cdot \mathbf{i}^* = (v_\alpha + jv_\beta)(i_\alpha - ji_\beta) = \underbrace{(v_\alpha i_\alpha + v_\beta i_\beta)}_{p} + j\underbrace{(v_\beta i_\alpha - v_\alpha i_\beta)}_{q}$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \frac{1}{v_\alpha^2 + v_\beta^2}\begin{bmatrix} v_\alpha & v_\beta \\ v_\beta & -v_\alpha \end{bmatrix}\begin{bmatrix} p \\ 0 \end{bmatrix} + \frac{1}{v_\alpha^2 + v_\beta^2}\begin{bmatrix} v_\alpha & v_\beta \\ v_\beta & -v_\alpha \end{bmatrix}\begin{bmatrix} 0 \\ q \end{bmatrix}$$

$$\triangleq \begin{bmatrix} i_{\alpha p} \\ i_{\beta p} \end{bmatrix} + \begin{bmatrix} i_{\alpha q} \\ i_{\beta q} \end{bmatrix}$$

$$i_{\alpha p} = \frac{v_\alpha}{v_\alpha^2 + v_\beta^2}\, p \qquad i_{\alpha q} = \frac{v_\beta}{v_\alpha^2 + v_\beta^2}\, q \qquad i_{\beta p} = \frac{v_\beta}{v_\alpha^2 + v_\beta^2}\, p \qquad i_{\beta q} = \frac{-v_\alpha}{v_\alpha^2 + v_\beta^2}\, q$$

$$p = v_\alpha i_{\alpha p} + v_\beta i_{\beta p} + v_\alpha i_{\alpha q} + v_\beta i_{\beta q}$$
$$= \frac{v_\alpha^2}{v_\alpha^2 + v_\beta^2} p + \frac{v_\beta^2}{v_\alpha^2 + v_\beta^2} p + \frac{v_\alpha v_\beta}{v_\alpha^2 + v_\beta^2} q + \frac{-v_\alpha v_\beta}{v_\alpha^2 + v_\beta^2} q$$

$$v_\alpha i_{\alpha p} + v_\beta i_{\beta p} = p_{\alpha p} + p_{\beta p} = p$$

$$v_\alpha i_{\alpha q} + v_\beta i_{\beta q} = p_{\alpha q} + p_{\beta q} = 0$$

It should be noted that the watt [W] can be used as the unit of all the powers ,$p_{\alpha p}$, $p_{\alpha q}$, $p_{\beta p}$, and $p_{\beta q}$, because each power is defined by the product of the instantaneous voltage on one axis and a part of the instantaneous current on the same axis.

Conclusions

- The instantaneous current i_α is divided into the instantaneous active component $i_{\alpha p}$ and the instantaneous reactive component $i_{\alpha q}$. This same division is made for the currents on the β axis.

- The sum of the α axis instantaneous active power $p_{\alpha p}$ and the β axis instantaneous active power $p_{\beta p}$ corresponds to the instantaneous real power p.

- The sum of $p_{\alpha q}$ and $p_{\beta q}$ is always zero. Therefore, they neither make a contribution to the instantaneous nor average energy flow between the source and the load in a three-phase circuit. This is the reason that they were named instantaneous reactive power on the α and β axes. The instantaneous imaginary power q is a quantity that gives the magnitude of the powers $p_{\alpha q}$ and $p_{\beta q}$.

- Because the sum of pαq and pβq is always zero, there is no need for a reactive compensation source.

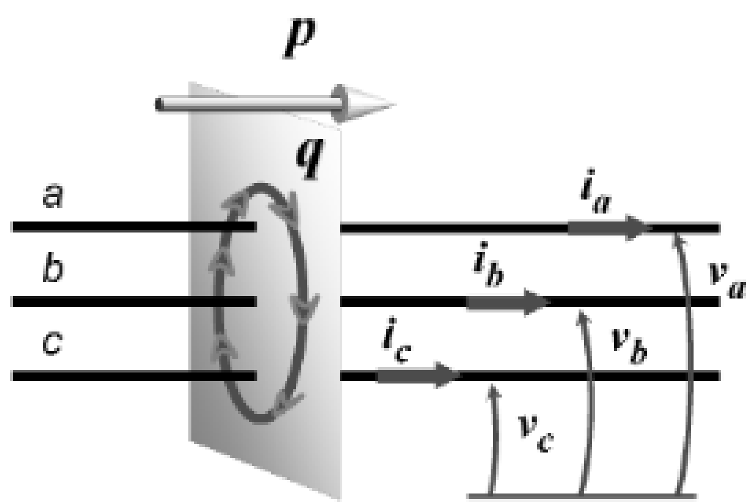
A new kind of reactive power

$$q = v_\beta i_\alpha - v_\alpha i_\beta = \frac{1}{\sqrt{3}}[(v_a - v_b)i_c + (v_b - v_c)i_a + (v_c - v_a)i_b]$$

$$= \frac{1}{\sqrt{3}}(v_{ab}i_c + v_{bc}i_a + v_{ca}i_b)$$

The αβ reference frames, is defined as the sum of products of voltages and currents on different axes, the same rule applies on the abc system with the line voltages and currents. Some measuring instruments for reactive power use that principal. The difference is that this principal is based on phasors(f), but the preshown values are based on the time domain. (As was shown, the imaginary power q does not contribute to the total energy flow between the source and the load, and vice-versa.) And that is why this reactive power is called imaginary reactive power and it is measured in volt-ampere imaginary (vai).

The imaginary power q is proportional to the quantity of energy that is being ex-changed between the phases of the system. It does not contribute to the energy transfer between the source and the load at any time.



p : *instantaneous total energy flow per time unit;*
q : *energy exchanged between the phases without transferring energy.*

Example 1

Suppose a three-phase ideal voltage source supplying power to a three-phase balanced impedance. The phase voltages and line currents can be expressed as

$$\begin{cases} v_a(t) = \sqrt{2}V\sin(\omega t) \\ v_b(t) = \sqrt{2}V\sin\left(\omega t - \dfrac{2\pi}{3}\right) \\ v_c(t) = \sqrt{2}V\sin\left(\omega t + \dfrac{2\pi}{3}\right) \end{cases} \quad \text{and} \quad \begin{cases} i_a(t) = \sqrt{2}I\sin(\omega t + \phi) \\ i_b(t) = \sqrt{2}I\sin\left(\omega t - \dfrac{2\pi}{3} + \phi\right) \\ i_c(t) = \sqrt{2}I\sin\left(\omega t + \dfrac{2\pi}{3} + \phi\right) \end{cases} \tag{3.36}$$

The $\alpha\beta$ transformation of the above voltages and currents are:

$$\begin{cases} v_\alpha = \sqrt{3}V\sin(\omega t) \\ v_\beta = -\sqrt{3}V\cos(\omega t) \end{cases} \quad \text{and} \quad \begin{cases} i_\alpha = \sqrt{3}I\sin(\omega t + \phi) \\ i_\beta = -\sqrt{3}I\cos(\omega t + \phi) \end{cases} \tag{3.37}$$

The above two equations make it possible to calculate the real and imaginary powers:

$$\begin{cases} p = 3VI\cos\phi \\ q = -3VI\sin\phi \end{cases} \tag{3.38}$$

Example 2

To better explain the concepts behind the *p-q* Theory, the following two situations are examined: (i) a three-phase balanced voltage with a three-phase balanced capacitive load (capacitance C), and (ii) an unbalanced load (just one capacitor connected between two phases).

In the first case, the load is balanced under steady-state conditions. The following real and imaginary powers are obtained:

$$\begin{cases} p = 0 \\ q = -3\dfrac{V^2}{X_C} \end{cases} \tag{3.39}$$

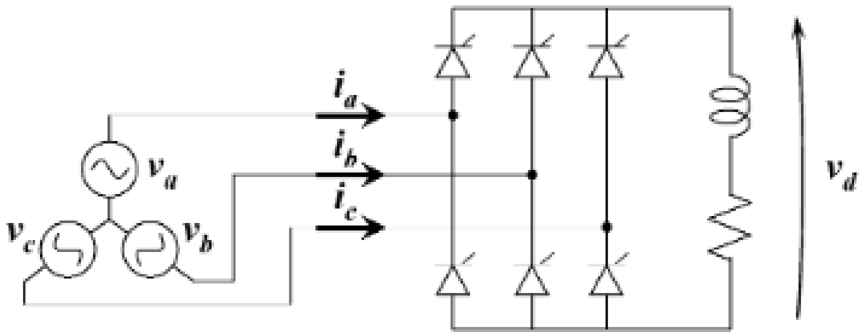
The term X_C represents the reactance of the capacitor. As expected, in this case there is no power flowing from the source to the load. Moreover, the imaginary power is constant and coincident with the conventional three-phase reactive power.

In the second case, a capacitor (capacitance C) is connected between phases *a* and *b*. The instantaneous real and imaginary powers are given by

$$\begin{aligned} p &= \frac{3V^2}{X_C}\sin\left(2\omega t + \frac{\pi}{3}\right) \\ q &= -\frac{3V^2}{X_C}\left[1 + \cos\left(2\omega t + \frac{\pi}{3}\right)\right] \end{aligned} \tag{3.40}$$

Example 3

$$
\begin{cases}
i_a(t) = \sqrt{2}I_1 \sin\left(\omega t - \frac{\pi}{6}\right) - \sqrt{2}I_5 \sin\left(5\omega t - \frac{\pi}{6}\right) + \sqrt{2}I_7 \sin\left(7\omega t - \frac{\pi}{6}\right) - \dots \\
i_b(t) = \sqrt{2}I_1 \sin\left(\omega t - \frac{2\pi}{3} - \frac{\pi}{6}\right) - \sqrt{2}I_5 \sin\left(5\omega t + \frac{2\pi}{3} - \frac{\pi}{6}\right) \\
\qquad + \sqrt{2}I_7 \sin\left(7\omega t - \frac{2\pi}{3} - \frac{\pi}{6}\right) - \dots \\
i_c(t) = \sqrt{2}I_1 \sin\left(\omega t + \frac{2\pi}{3} - \frac{\pi}{6}\right) - \sqrt{2}I_5 \sin\left(5\omega t - \frac{2\pi}{3} - \frac{\pi}{6}\right) \\
\qquad + \sqrt{2}I_7 \sin\left(7\omega t + \frac{2\pi}{3} - \frac{\pi}{6}\right) - \dots
\end{cases}
\tag{3.41}
$$



firing angle: 30°

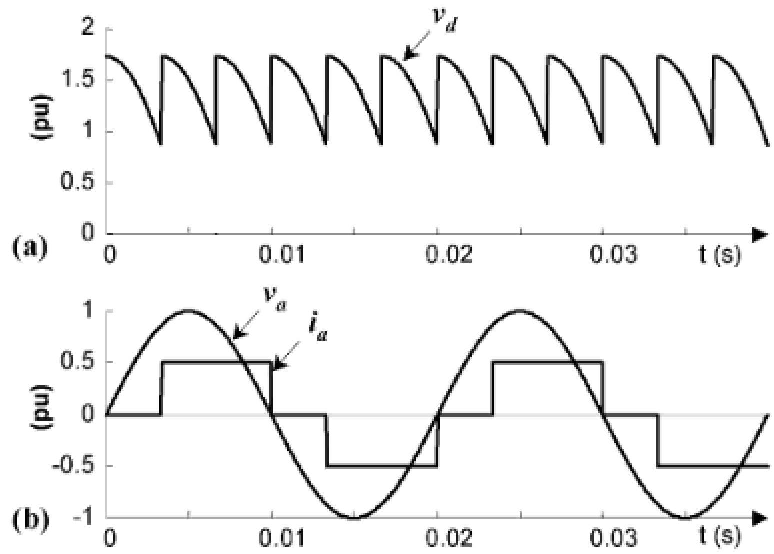


Figure 3-6. (a) Rectifier dc output voltage v_d and (b) *a*-phase voltage and current waveforms.

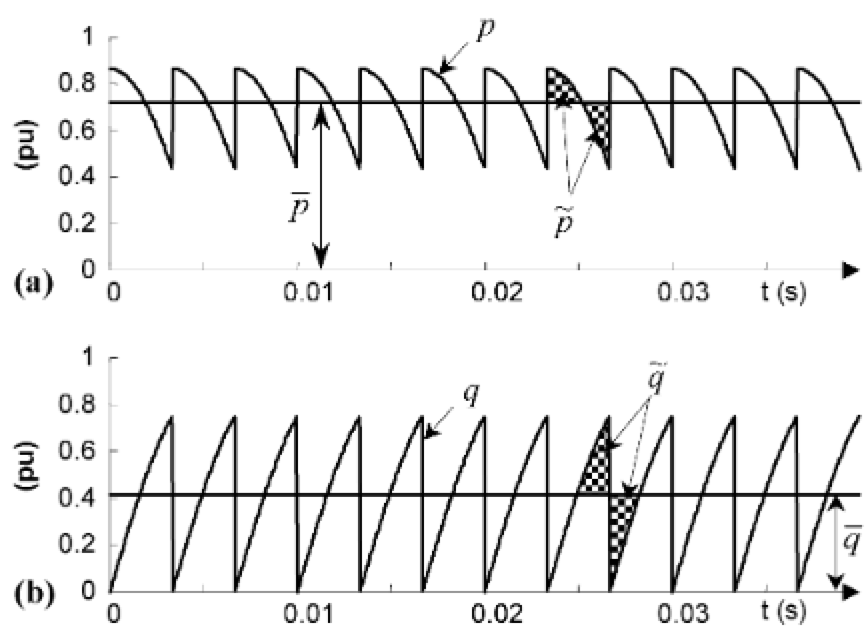


Figure 3-7. (a) Real power p and (b) imaginary power q.

Real power: $p = \bar{p} + \tilde{p}$

Imaginary power: $q = \bar{q} + \tilde{q}$

Average powers — Oscillating powers

$$\begin{cases} \bar{p} = 3VI_1 \cos \dfrac{\pi}{6} \\ \bar{q} = 3VI_1 \sin \dfrac{\pi}{6} \end{cases}$$

After calculation i_{ap} is in phase with Va but i_{aq} delays exactly 90º in relation to Va and that is why thyristor rectifiers are considered inductive loads.

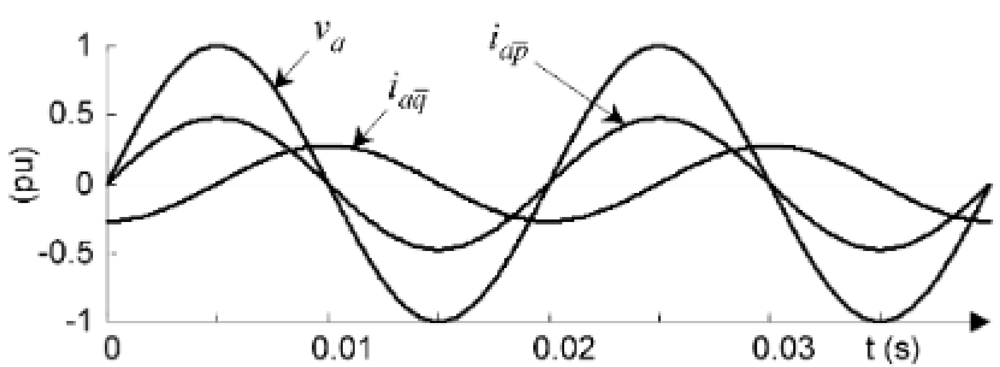


Figure 3-8. Voltage in *a* phase; currents $i_{a\bar{p}}$ and $i_{a\bar{q}}$.

Active filtering

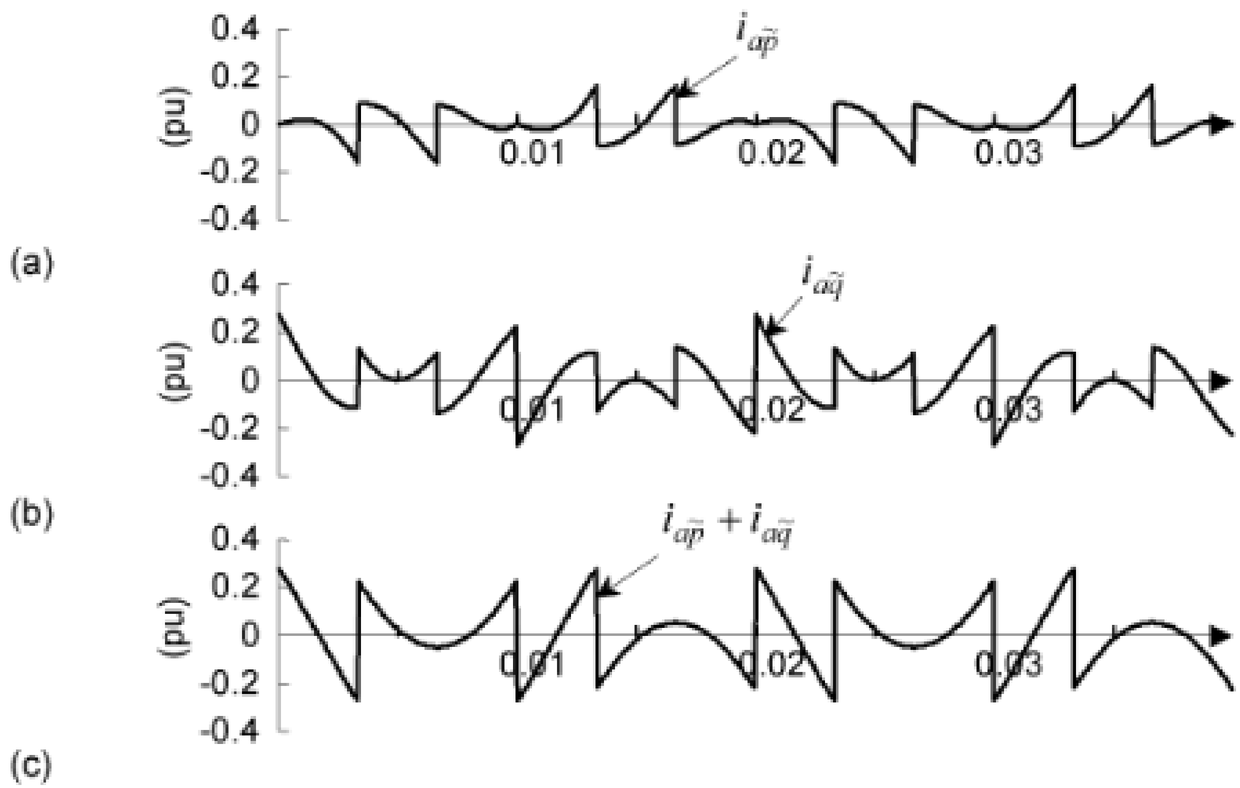


Figure 3-9. Currents $i_{a\tilde{p}}$, $i_{a\tilde{q}}$, and the sum ($i_{a\tilde{p}} + i_{a\tilde{q}}$).

Use of the p-q Theory for Shunt Current Compensation

- One important application of the p-q Theory is the compensation of undesirable currents.
- A kind of shunt compensator is the active filter, it is assumed that the shunt compensator behaves as a three-phase, controlled current source that can draw any set of arbitrarily chosen current references i^*_{Ca}, i^*_{Cb}, and i^*_{Cc}.
- Undesired portions of the real and imaginary powers of the load that should be compensated are selected.
- The powers to be compensated are represented by $-p^*_c$ and $-q^*_c$.

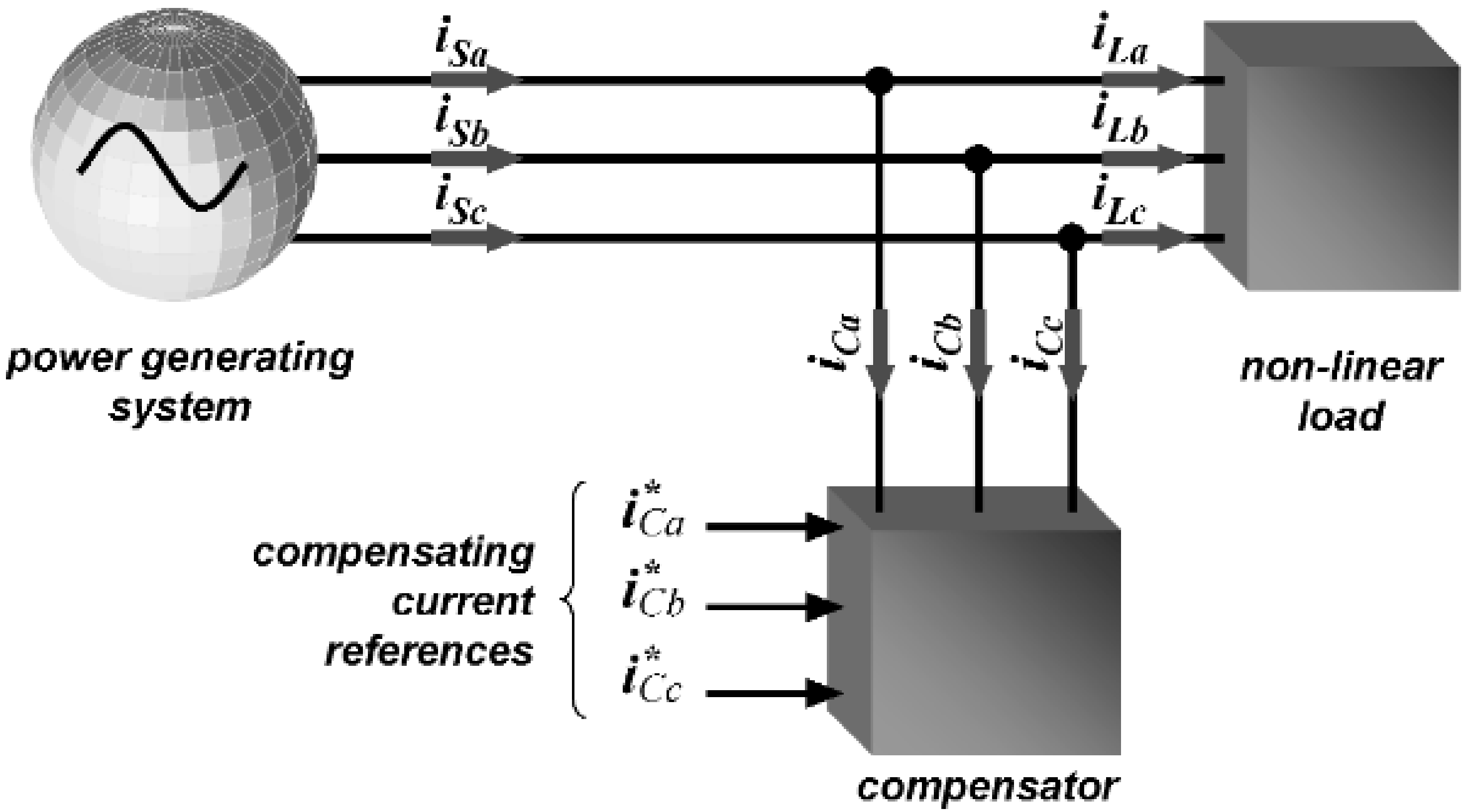


Figure 3-10. Basic principle of shunt current compensation.

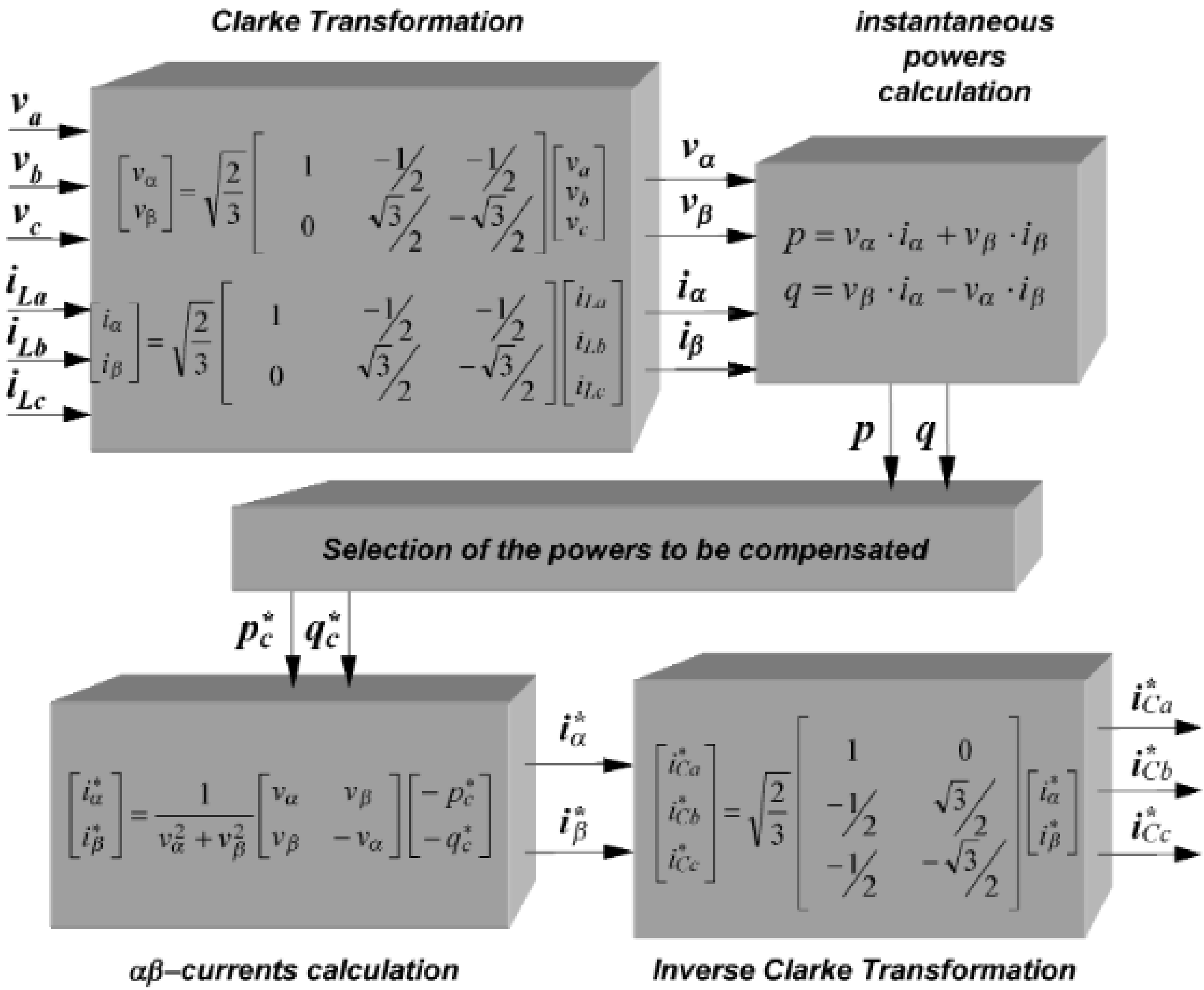


Figure 3-11. Control method for shunt current compensation based on the *p-q* Theory.

Compensation of the load imaginary power q

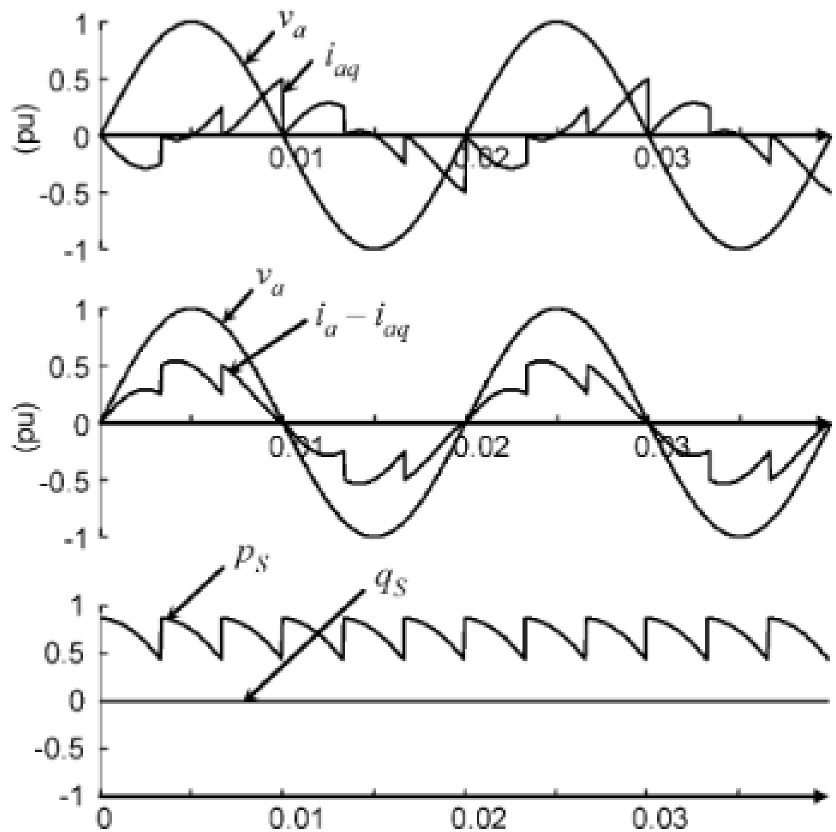


Figure 3-12. Eliminated current i_{aq}, compensated current $i_a - i_{aq}$, and the real and imaginary powers produced by the compensated current.

The real power p_S, produced by the compensated current, is equal to that produced by the load current, whereas the imaginary power q_S is zero in the source.

Compensation of the load average imaginary power q

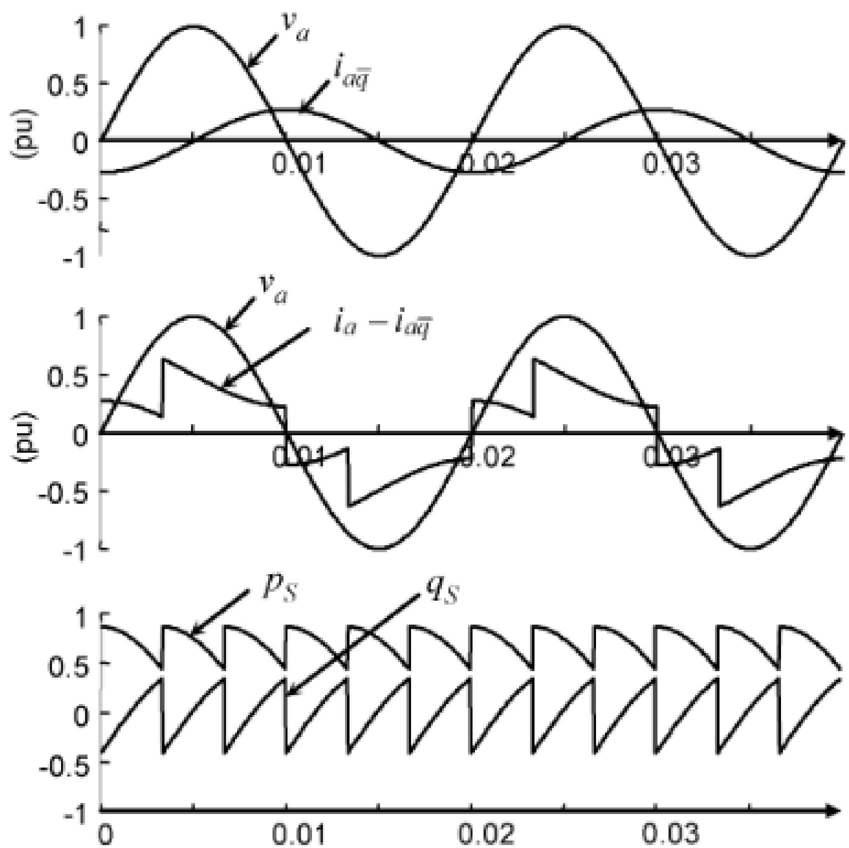


Figure 3-13. Eliminated current $i_{a\bar{q}}$, compensated current $i_a - i_{a\bar{q}}$, and the real and imaginary powers produced by the compensated current.

- Use only the average value of q for compensation.
- In this case, the compensating current i_{aq} has no harmonic components and, therefore, the compensator draws sinusoidal currents at the line frequency.

Compensation of the load oscillating real power p

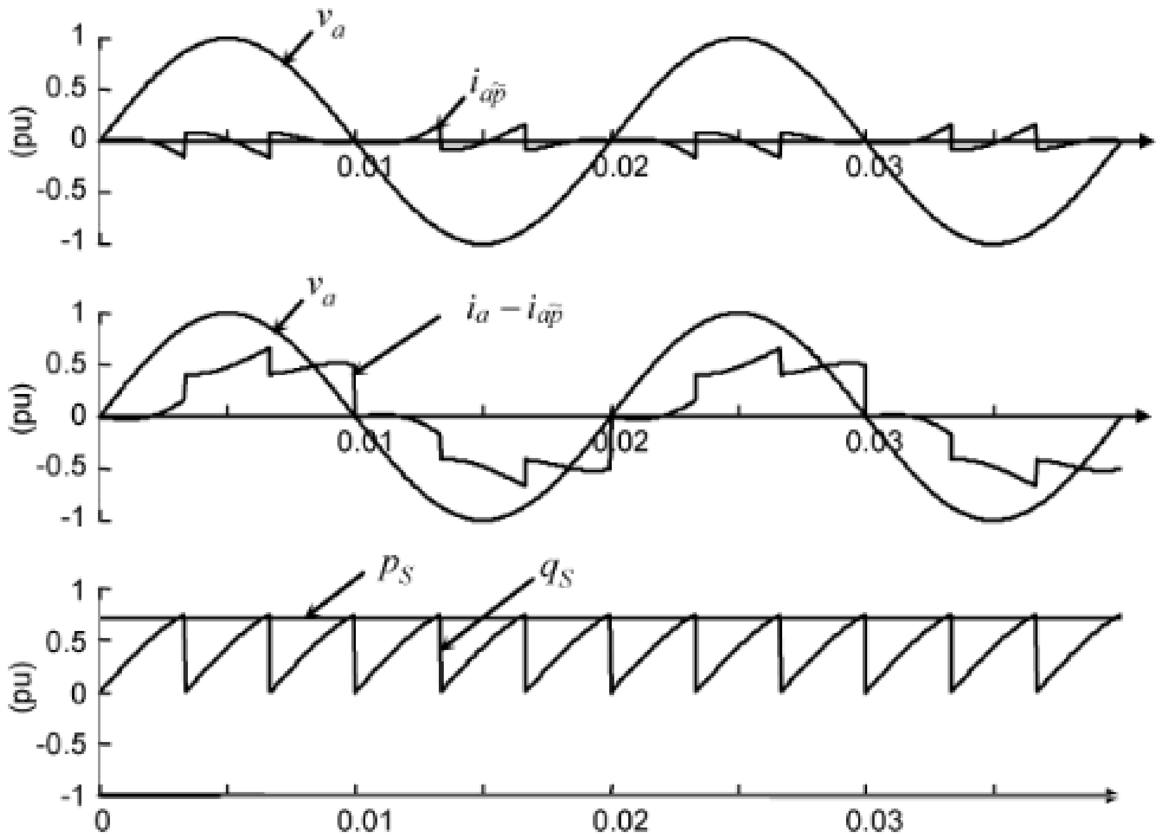


Figure 3-14. Eliminated current $i_{a\tilde{p}}$, compensated current $i_a - i_{a\tilde{p}}$, and the real and imaginary powers produced by the compensated current.

- Taking the current related to this power as the compensating current reference to the compensator makes constant (without ripple) the three-phase instantaneous real power that is equal to the calculated real power.
- The compensator must have the capability to supply and absorb energy, but with zero average value. Hence, it must be coupled with an energy storage system.

Compensation of the load oscillating imaginary power q and oscillating real power p

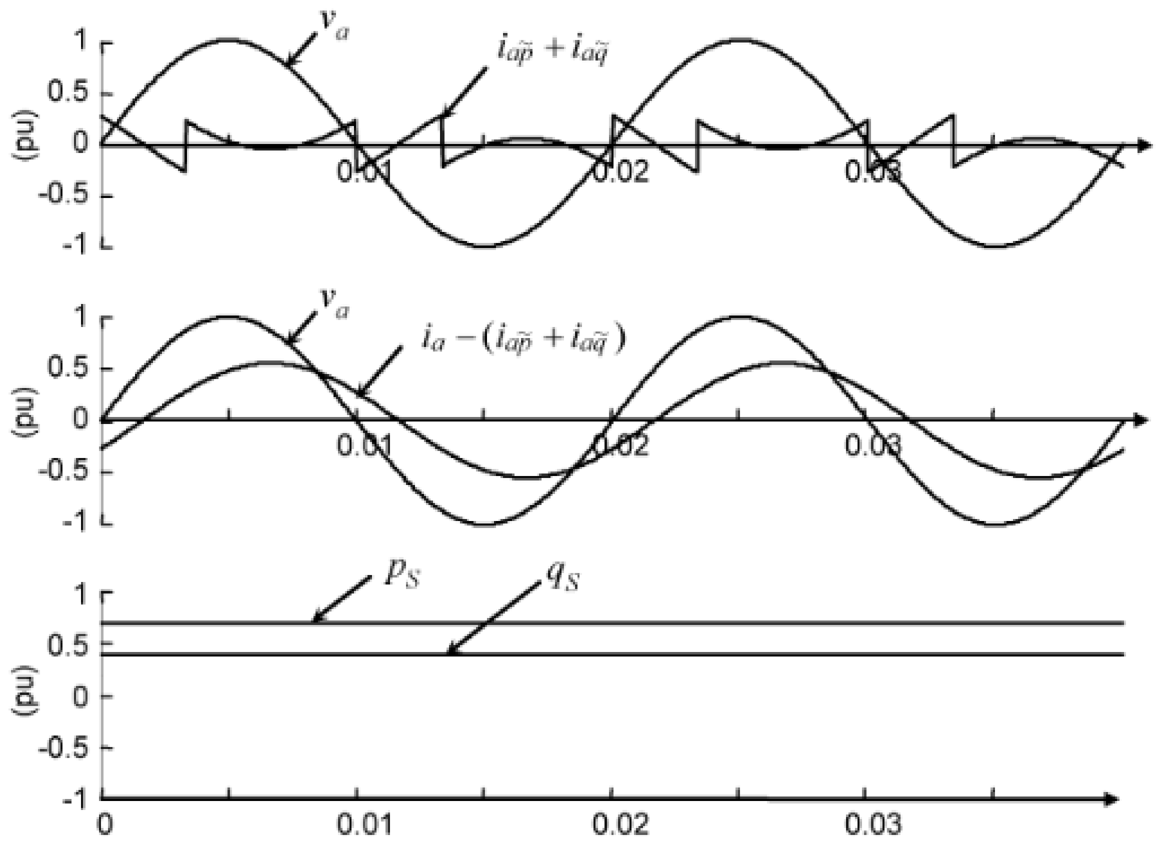


Figure 3-15. Eliminated current $i_{a\tilde{p}} + i_{a\tilde{q}}$, compensated current $i_a - (i_{a\tilde{p}} + i_{a\tilde{q}})$, and the real and imaginary powers produced by the compensated current.

This kind of compensation is applicable when harmonic elimination is the most important issue.

Compensation of the load imaginary power q and the oscillating real power p

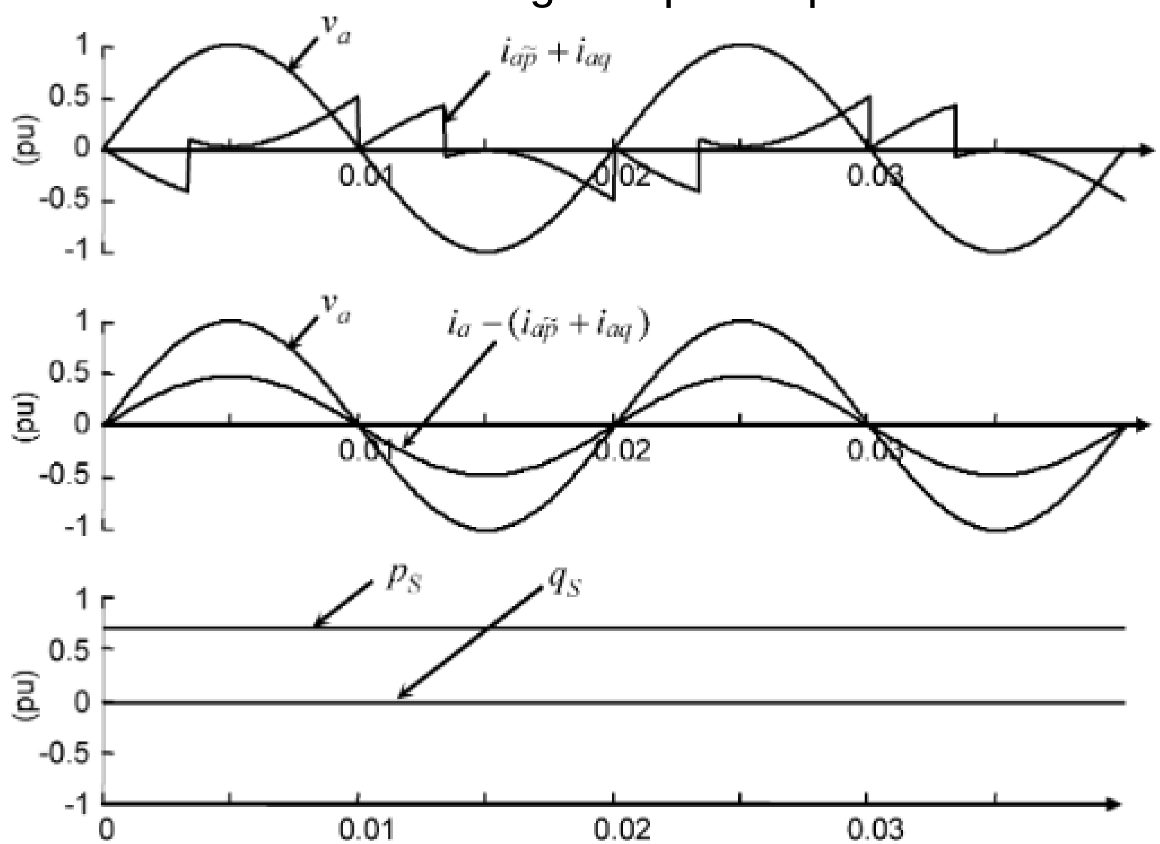


Figure 3-16. Eliminated current $i_{a\tilde{p}} + i_{aq}$, compensated current $i_a - (i_{a\tilde{p}} + i_{aq})$, and the real and imaginary powers produced by the compensated current.

This is the best compensation that can be made from the power-flow point of view, because it smoothes the power drawn from the generator system. Besides, it eliminates all the harmonic currents.

The nonlinear load and the compensator form an ideal, linear, purely resistive load.

Presence of the Fifth Harmonic in Load Current

- Besides its fundamental component, the nonlinear load current generally contains a large harmonic spectrum.

$$\begin{cases} i_{a5}(t) = \sqrt{2} I_{-5} \sin(5\omega t + \delta_{-5}) \\ i_{b5}(t) = \sqrt{2} I_{-5} \sin\left(5\omega t + \delta_{-5} + \dfrac{2\pi}{3}\right) \\ i_{c5}(t) = \sqrt{2} I_{-5} \sin\left(5\omega t + \delta_{-5} - \dfrac{2\pi}{3}\right) \end{cases} \Rightarrow \begin{cases} i_{\alpha 5} = \sqrt{3} I_{-5} \sin(5\omega t + \delta_{-5}) \\ i_{\beta 5} = \sqrt{3} I_{-5} \cos(5\omega t + \delta_{-5}) \end{cases}$$

$$\tilde{p} = -3 V_{+1} I_{-5} \cos(6\omega t + \delta_{-5})$$

$$\tilde{q} = -3 V_{+1} I_{-5} \sin(6\omega t + \delta_{-5})$$

$$\Rightarrow$$

$$i_{\alpha p5} = \frac{v_\alpha}{v_\alpha^2 + v_\beta^2} \tilde{p} = \frac{\sqrt{3}}{2} I_{-5} \sin(5\omega t + \delta_{-5}) - \frac{\sqrt{3}}{2} I_{-5} \sin(7\omega t + \delta_{-5}) \quad (3.47)$$

$$i_{\alpha q5} = \frac{v_\beta}{v_\alpha^2 + v_\beta^2} \tilde{q} = \frac{\sqrt{3}}{2} I_{-5} \sin(5\omega t + \delta_{-5}) + \frac{\sqrt{3}}{2} I_{-5} \sin(7\omega t + \delta_{-5}) \quad (3.48)$$

$$i_{\beta p5} = \frac{v_\beta}{v_\alpha^2 + v_\beta^2} \tilde{p} = \frac{\sqrt{3}}{2} I_{-5} \cos(5\omega t + \delta_{-5}) + \frac{\sqrt{3}}{2} I_{-5} \cos(7\omega t + \delta_{-5}) \quad (3.49)$$

$$i_{\beta q5} = \frac{-v_\alpha}{v_\alpha^2 + v_\beta^2} \tilde{q} = \frac{\sqrt{3}}{2} I_{-5} \cos(5\omega t + \delta_{-5}) - \frac{\sqrt{3}}{2} I_{-5} \cos(7\omega t + \delta_{-5}) \quad (3.50)$$

$$\begin{cases} i_{\alpha 5} = i_{\alpha p5} + i_{\alpha q5} \\ i_{\beta 5} = i_{\beta p5} + i_{\beta q5} \end{cases}$$

If p-q Theory is used to compensate for the currents that are dependent on ~p and ~q, it is possible to define compensating currents using gains k_p and k_q.

$$\begin{cases} i_{C\alpha 5} = k_p \cdot i_{\alpha p5} + k_q \cdot i_{\alpha q5} \\ i_{C\beta 5} = k_p \cdot i_{\beta p5} + k_q \cdot i_{\beta q5} \end{cases}$$

In this case, the source current would be

$$\begin{cases} i_{S\alpha 5} = i_{\alpha 5} - i_{C\alpha 5} \\ i_{S\beta 5} = i_{\beta 5} - i_{C\beta 5} \end{cases}$$

If $k_p = k_q$, the seventh-harmonic component is totally eliminated in the source current. However, if $k_p \neq k_q$, the seventh-harmonic component in i_{ap5} does not cancel the seventh harmonic component in i_{aq5}.

The Dual p-qTheory

- Instead of using shunt currents compensators we use voltage compensators in series
- There is a three phase current source

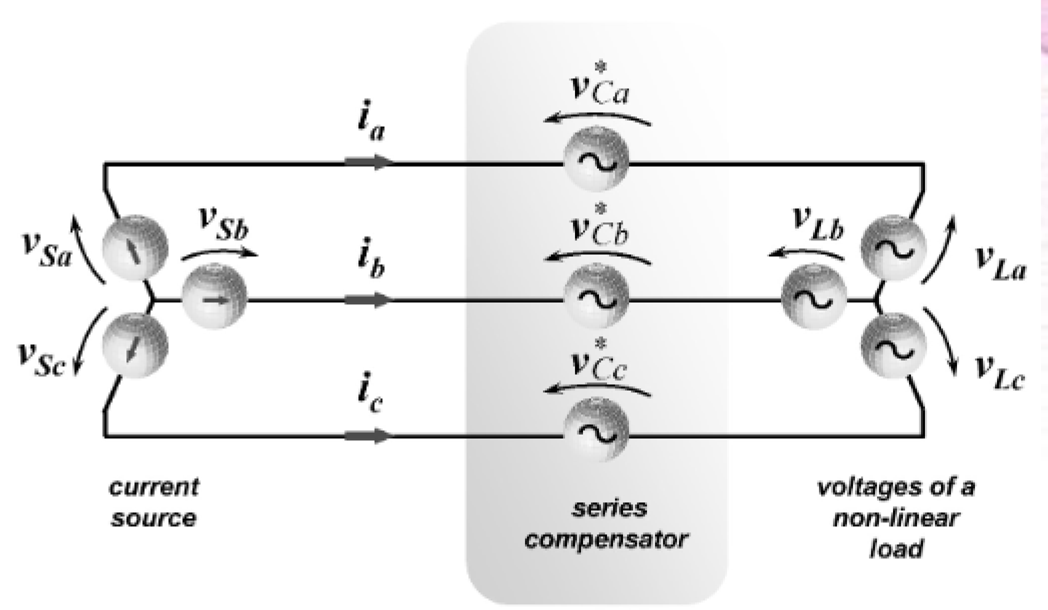


Figure 3-17. Basic principle of series voltage compensation.

Mathematical analysis

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} i_\alpha & i_\beta \\ -i_\beta & i_\alpha \end{bmatrix} \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix}$$

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \frac{1}{i_\alpha^2 + i_\beta^2} \begin{bmatrix} i_\alpha & -i_\beta \\ i_\beta & i_\alpha \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix}$$

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \frac{1}{i_\alpha^2 + i_\beta^2} \begin{bmatrix} i_\alpha & -i_\beta \\ i_\beta & i_\alpha \end{bmatrix} \begin{bmatrix} p \\ 0 \end{bmatrix} + \frac{1}{i_\alpha^2 + i_\beta^2} \begin{bmatrix} i_\alpha & -i_\beta \\ i_\beta & i_\alpha \end{bmatrix} \begin{bmatrix} 0 \\ q \end{bmatrix}$$

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \begin{bmatrix} v_{\alpha p} \\ v_{\beta p} \end{bmatrix} + \begin{bmatrix} v_{\alpha q} \\ v_{\beta q} \end{bmatrix}$$

- Instantaneous active voltage on the α axis $v_{\alpha p}$

$$v_{\alpha p} = \frac{i_\alpha}{i_\alpha^2 + i_\beta^2} p$$

- Instantaneous reactive voltage on the α axis $v_{\alpha q}$

$$v_{\alpha q} = \frac{-i_\beta}{i_\alpha^2 + i_\beta^2} q$$

- Instantaneous active voltage on the β axis $v_{\beta p}$

$$v_{\beta p} = \frac{i_\beta}{i_\alpha^2 + i_\beta^2} p$$

- Instantaneous reactive voltage on the β axis $v_{\beta q}$

$$v_{\beta q} = \frac{i_\alpha}{i_\alpha^2 + i_\beta^2} q$$

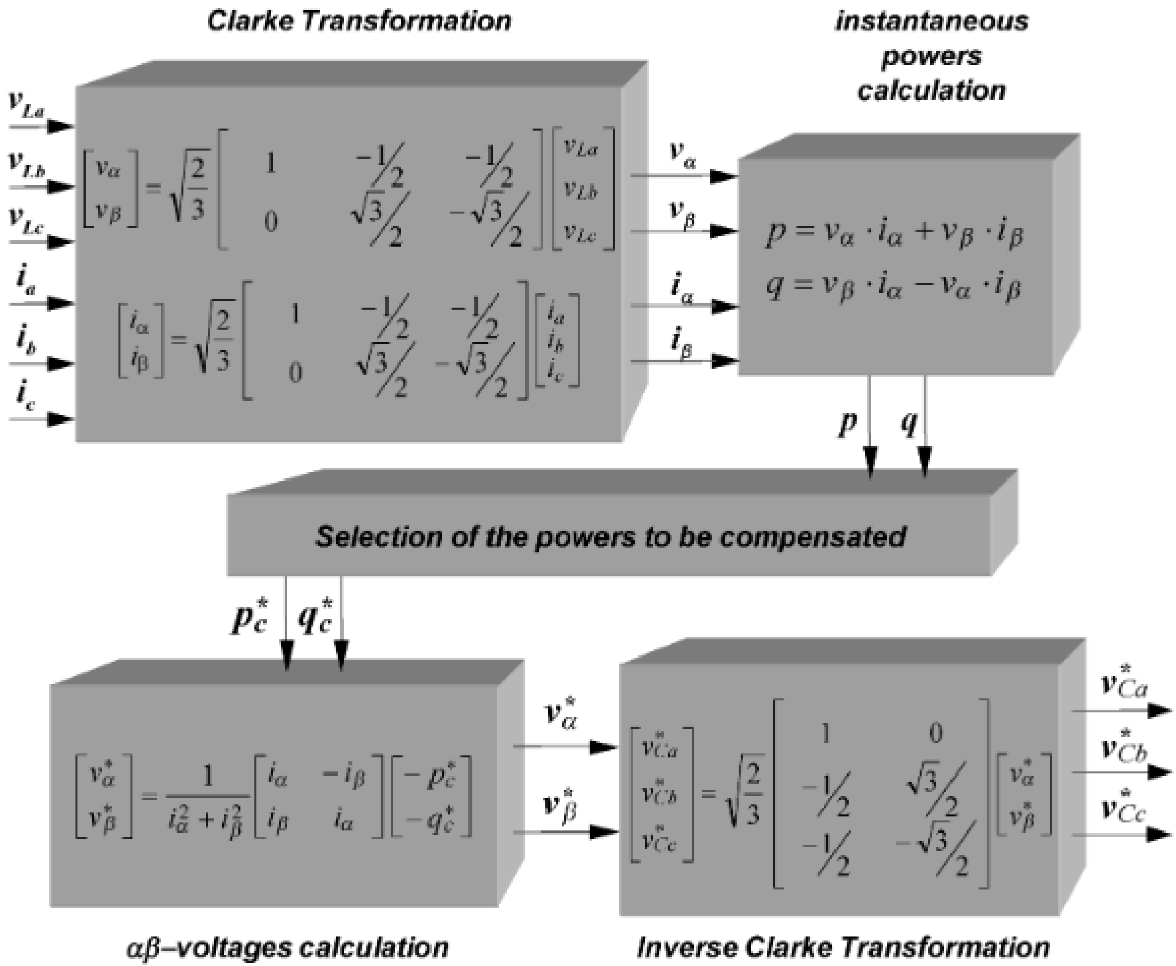


Figure 3-18. Control method for series voltage compensation based on the *p-q* Theory.

Thank you for your attention!

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