ΔΗΜΟΚΡΙΤΕΙΟ
ΠΑΝΕΠΙΣΤΗΜΙΟ
ΘΡΑΚΗΣ

Analysis of Power Electronic Converters DC / AC

Third Task

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Task:

For a Voltage Source Inverter (VSI) which supplies symmetrical three-phase loads of star wiring and the conduction interval of each switching element is 180° calculate the harmonic component analysis for both the phase and the polar voltage of this converter.

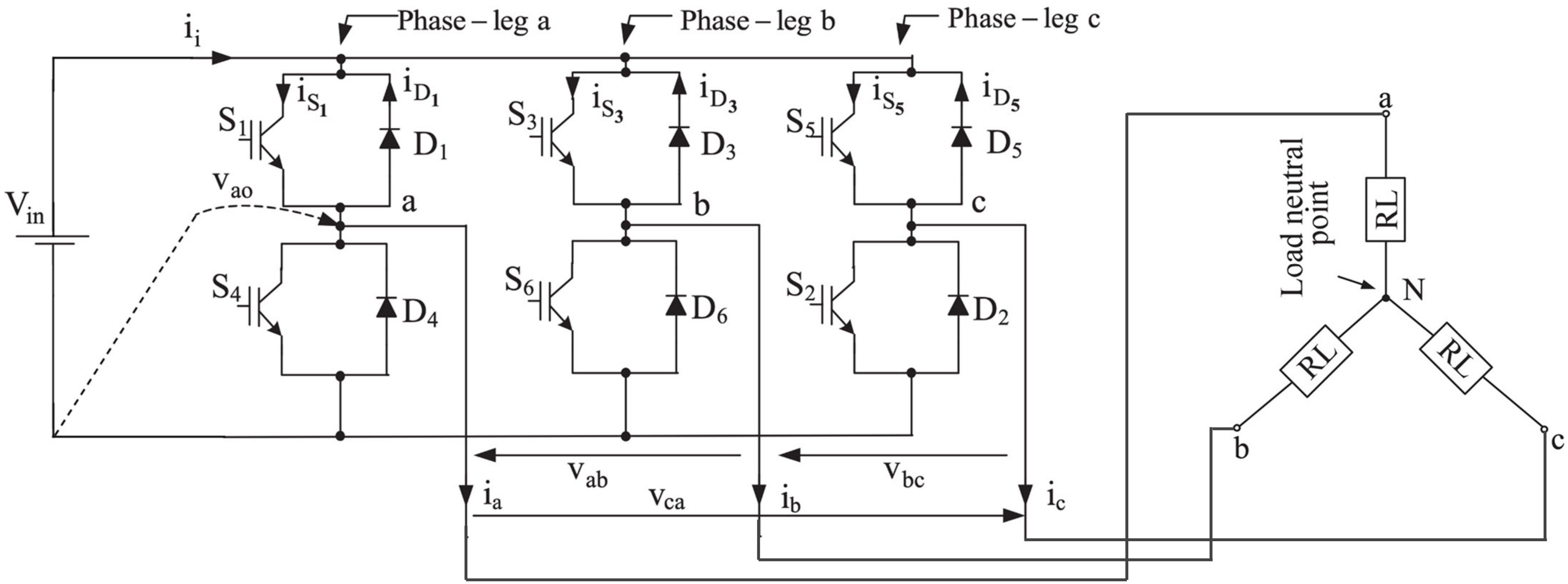


Fig. 1 Three-phase voltage source inverter.

Solution:

In this converter the simplest method of creating a three phase output voltage is the six step one.

In this method we have a new conducting switching element every 60° and every switching element conducts for a time period of 180°.

Through this consecutive conduction of the 6 switching elements a rough sinusoidal waveform is produced at the output.

Because of that the output waveform is enriched with a variety of harmonic components.

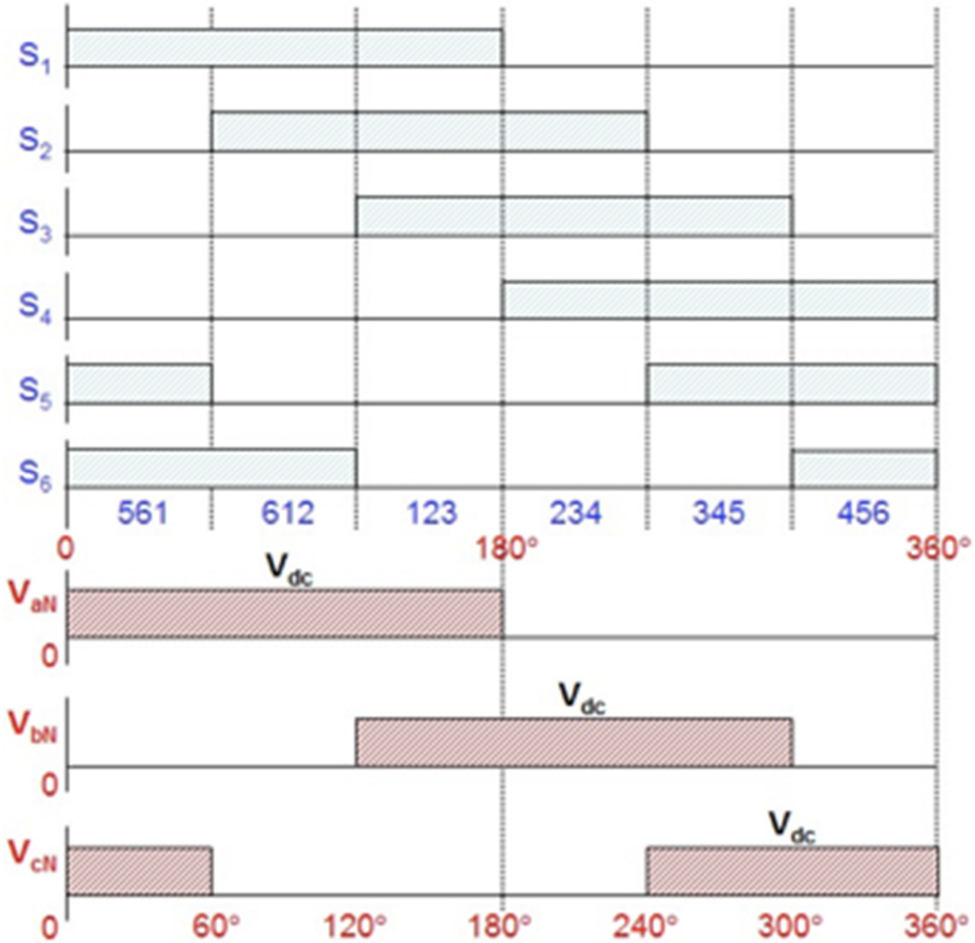


Fig. 2 Waveforms of gating signals, switching sequence, line to negative voltages for six-step voltage source inverter.

Solution:

In this converter there are two types of voltages depending on the reference of them:

- Line to line voltages (Vab, Vbc, Vca) called polar voltage:

  Vab = VaN - VbN

  Vbc = VbN - VcN

  Vca = VcN - VaN

- Line to neutral voltages (Van, Vbn, Vcn) called phase voltage:

  Van = 2/3VaN - 1/3VbN - 1/3VcN

  Vbn = -1/3VaN + 2/3VbN - 1/3VcN

  Vcn = -1/3VaN - 1/3VbN + 2/3VcN

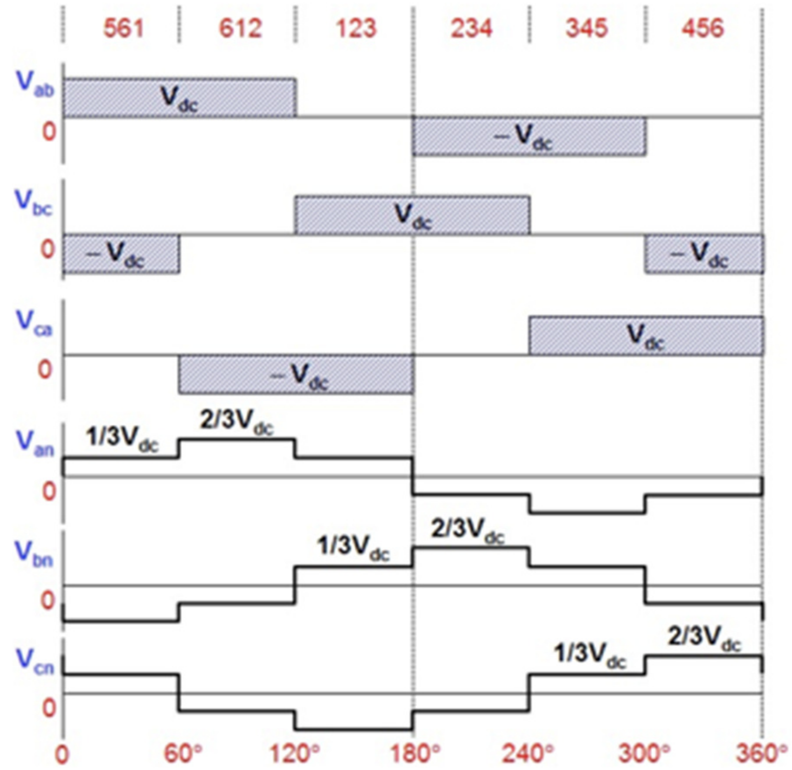


Fig. 3 Waveforms of line to neutral (phase) voltages and line to line voltages for six-step voltage source inverter.

Solution:

For 561 conduction

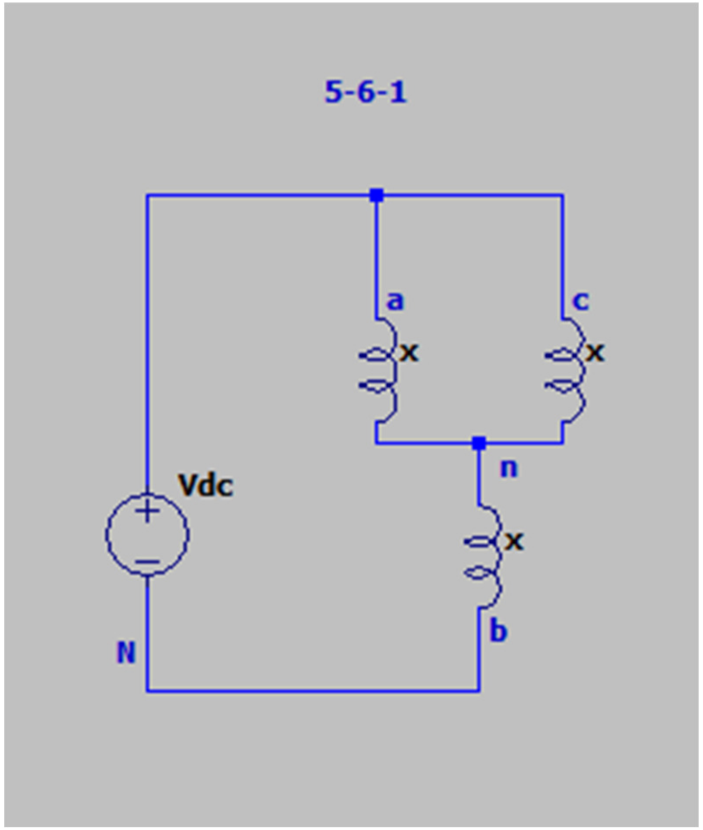
The Phase Voltages are:

$$Van = Vcn = Vdc \cdot \frac{x/2}{3x/2} = 1/3 \cdot Vdc$$

$$Vbn = -\ Van - \ Vcn = -\ 2 \cdot Vdc/3$$

The Polar Voltages are:

$$Vab = Vdc,\ \ Vbc = -\ Vdc,\ \ Vca = 0$$



Solution:

For 612 conduction

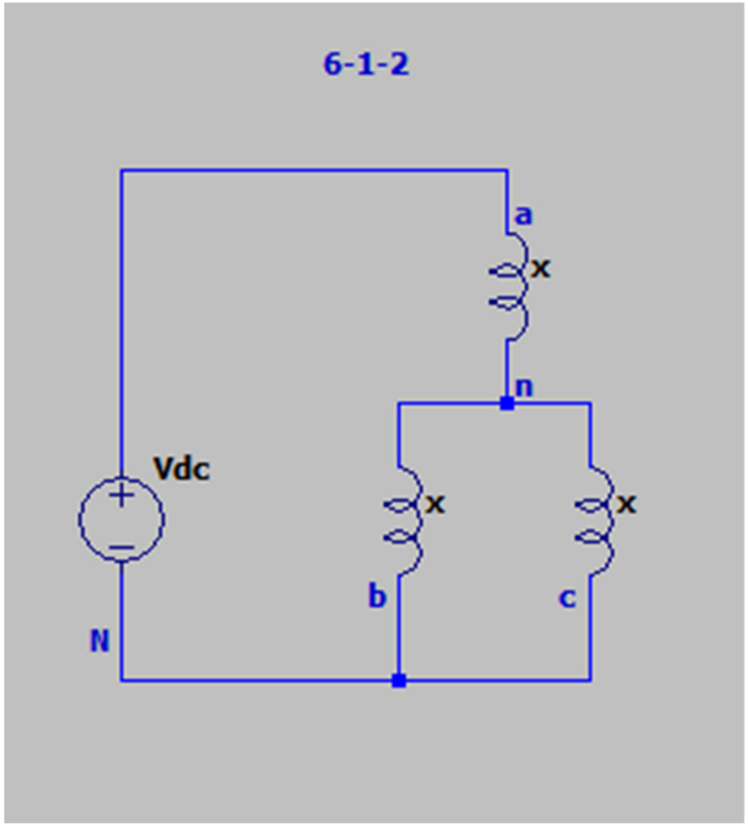
The Phase Voltages are:

$$Vbn = Vcn = -\,Vdc \cdot \frac{x/2}{3x/2} = -\,1/3 \cdot Vdc$$

$$Van = -\,Vbn - Vcn = 2 \cdot Vdc/3$$

The Polar Voltages are:

$$Vab = Vdc,\ \ Vbc = 0,\ \ Vca = -\,Vdc$$



Solution:

For 123 conduction

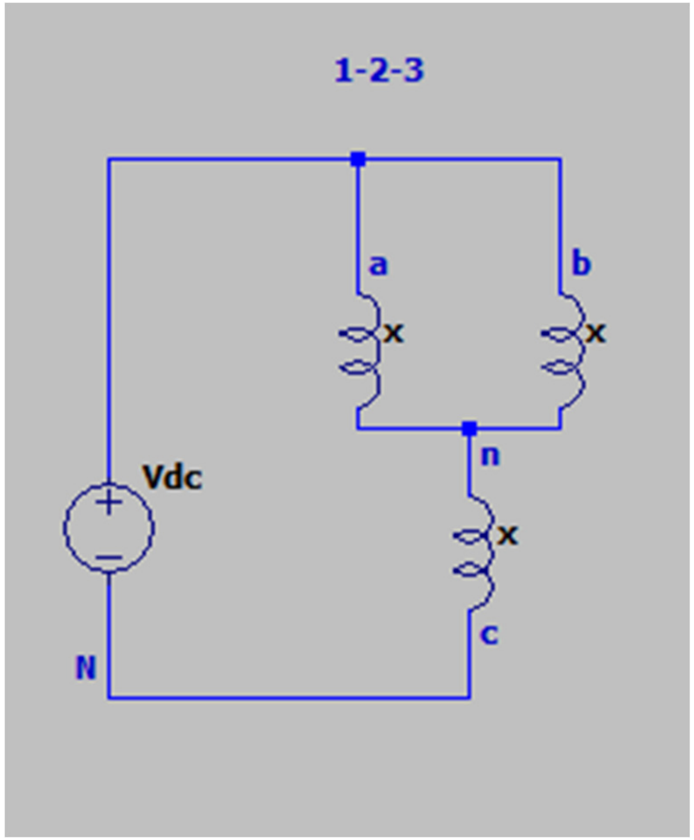
The Phase Voltages are:

$$Van = Vbn = Vdc \cdot \frac{x/2}{3x/2} = 1/3 \cdot Vdc$$

$$Vcn = -Van - Vbn = -2 \cdot Vdc/3$$

The Polar Voltages are:

$$Vab = 0,\ Vbc = Vdc,\ Vca = -Vdc$$



Solution:

For 234 conduction

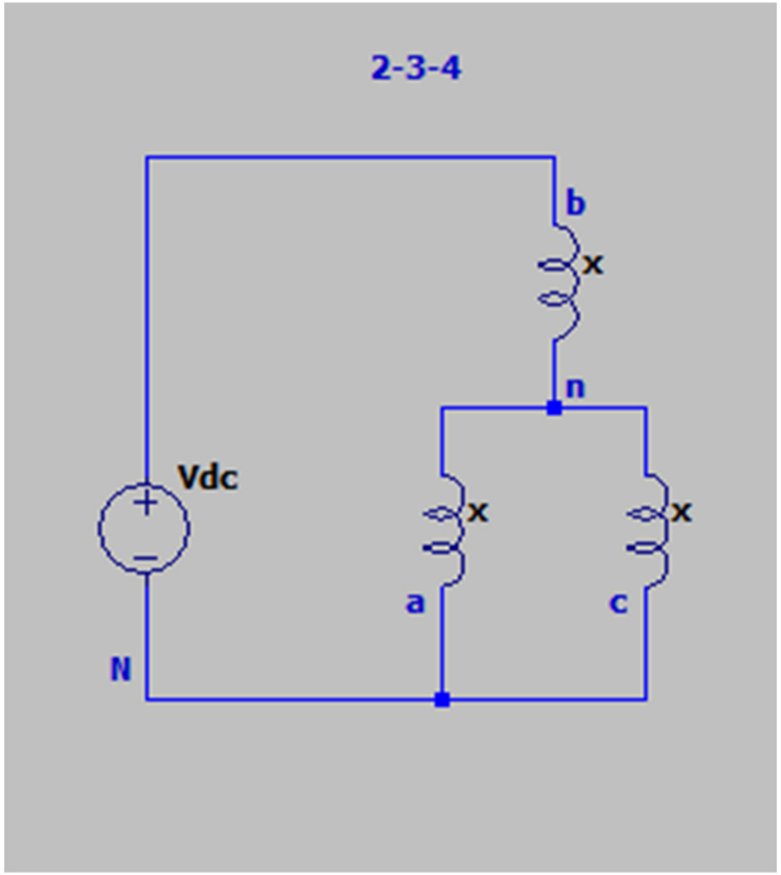
The Phase Voltages are:

$$Van = Vcn = -Vdc \cdot \frac{x/2}{3x/2} = -1/3 \cdot Vdc$$

$$Vbn = -Van - Vcn = 2 \cdot Vdc/3$$

The Polar Voltages are:

$$Vab = -Vdc, Vbc = Vdc, Vca = 0$$



Solution:

For 345 conduction

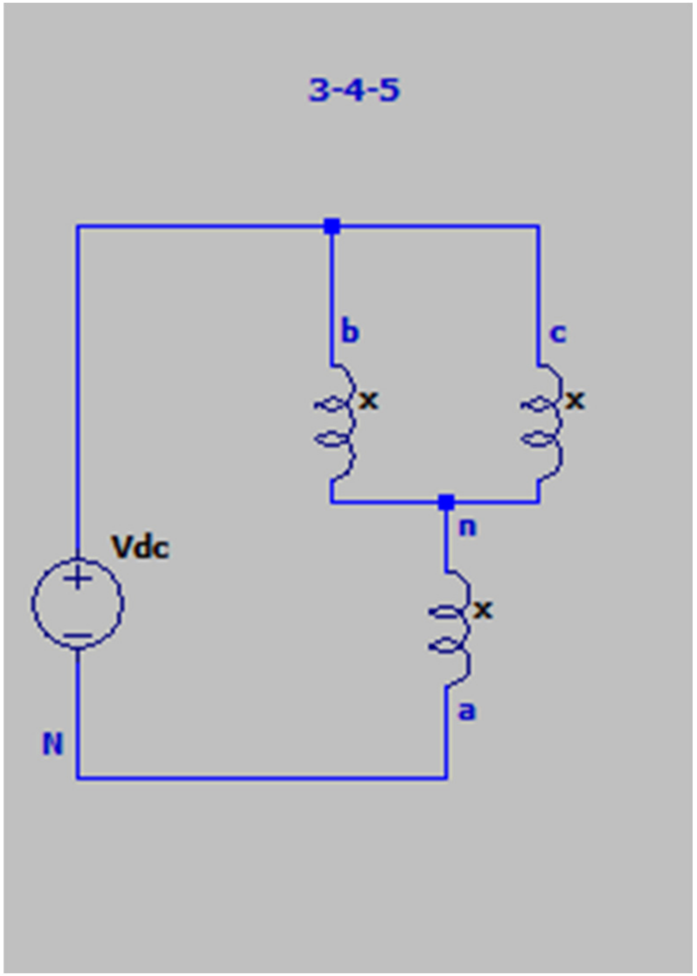
The Phase Voltages are:

$$Vbn = Vcn = Vdc \cdot \frac{x/2}{3x/2} = 1/3 \cdot Vdc$$

$$Van = -Vbn - Vcn = -2 \cdot Vdc/3$$

The Polar Voltages are:

$$Vab = -Vdc, Vbc = 0, Vca = Vdc$$



Solution:

For 456 conduction

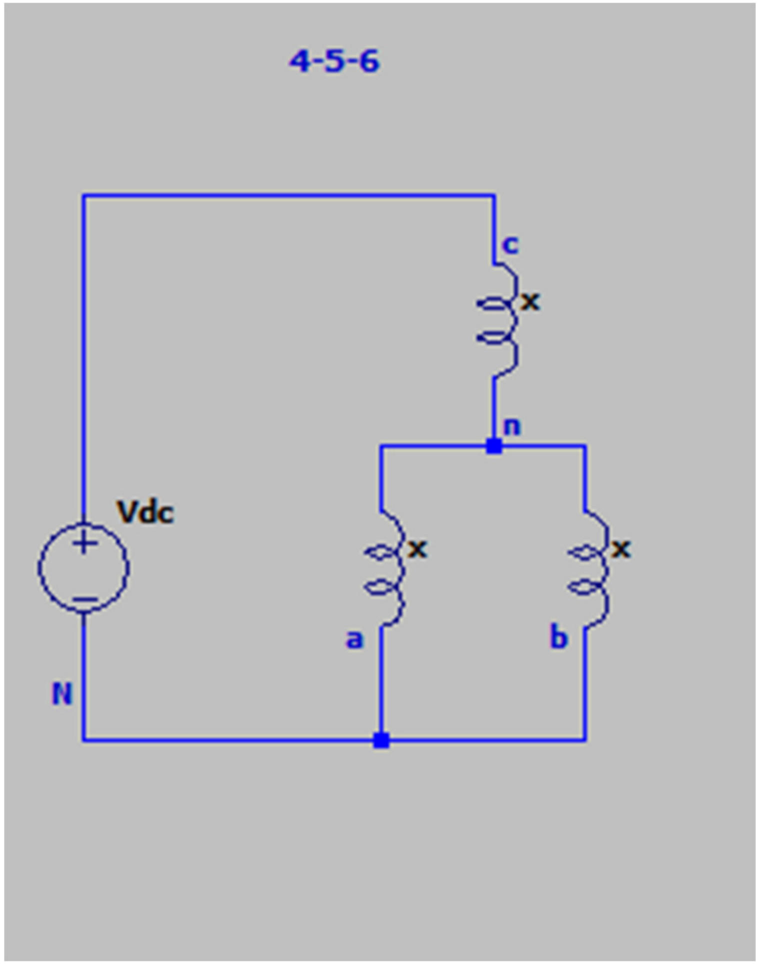
The Phase Voltages are:

$$Van = Vbn = Vdc \cdot \frac{x/2}{3x/2} = -1/3 \cdot Vdc$$

$$Vcn = -Van - Vbn = 2 \cdot Vdc/3$$

The Polar Voltages are:

$$Vab = 0, Vbc = -Vdc, Vca = Vdc$$



Solution:

The Fourier series of line-to-line voltages:

$$v_{ab} = \sum_{n=1,3,5..}^{\infty} \hat{V}_{ab,n} \sin[n(\omega t + 30^o)]$$

Where Vab,n is the amplitude of Vab:

$$\hat{V}_{ab,n} = \frac{2}{T}\int_{-T/2}^{T/2} v_{ab} \sin(n\omega t) d(\omega t) = \frac{1}{\pi}\int_{-\pi}^{\pi} v_{ab} \sin(n\omega t) d(\omega t) =$$

$$= \frac{4}{\pi}\int_{\pi/6}^{\pi/2} v_{ab} \sin(n\omega t) d(\omega t) = \frac{4V_{in}}{n\pi}\cos(\frac{n\pi}{6})$$

Solution:

Finally the line-to-line voltages:

$$v_{ab} = \sum_{n=1,3,5..}^{\infty} \frac{4V_{in}}{n\pi}\cos(\frac{n\pi}{6})\sin[n(\omega t + 30^{o})]$$

$$v_{bc} = \sum_{n=1,3,5..}^{\infty} \frac{4V_{in}}{n\pi}\cos(\frac{n\pi}{6})\sin[n(\omega t - 90^{o})]$$

$$v_{ca} = \sum_{n=1,3,5..}^{\infty} \frac{4V_{in}}{n\pi}\cos(\frac{n\pi}{6})\sin[n(\omega t - 210^{o})]$$

Solution:

The output linear voltage amplitude for the first 21 harmonics:

- n=1 $V_{ab,1}$= 623.75V
- n=5 $V_{ab,5}$= -124.75V
- n=7 $V_{ab,7}$= -89.1V
- n=11 $V_{ab,11}$= 56.7V
- n=13 $V_{ab,13}$= 48V
- n=17 $V_{ab,17}$= -36.7V
- n=19 $V_{ab,19}$= -32.8V

For the rest of them between 1 and 21 the voltage amplitude is 0V.

The amplitudes were calculated with $Vdc = 400\sqrt{2} = 565.68V$ which is the output voltage of a 3-phase rectifier.

The amplitudes for Vab, Vbc and Vca are the same. The phases of them are simply 120° apart between of waveforms.

Solution:

The load phase voltages can be replaced with the Fourier series:

$$v_{aN} = \sum_{n=1,2,3\ldots}^{\infty} \hat{V}_{aN} \sin(n\omega t)$$

Where VaN is the amplitude of vaN:

$$\hat{V}_{aN} = \frac{2}{T}\int_{-T/2}^{T/2} v_{aN} \sin(n\omega t) d(\omega t) = \frac{1}{\pi}\int_{-\pi}^{\pi} v_{aN} \sin(n\omega t) d(\omega t) =$$

$$= \frac{4}{\pi}\int_{0}^{\pi/2} v_{aN} \sin(n\omega t) d(\omega t) = \frac{4}{\pi}\int_{0}^{\pi/3} \frac{V_{in}}{3} \sin(n\omega t) d(\omega t) + \frac{4}{\pi}\int_{\pi/3}^{\pi/2} \frac{2V_{in}}{3} \sin(n\omega t) d(\omega t) =$$

$$= \frac{4V_{in}}{3n\pi}(1 + \cos\frac{n\pi}{3})$$

Solution:

Finally the load phase voltages are:

$$v_{aN} = \sum_{n=1,5,7,11..}^{\infty} \frac{4V_{in}}{3n\pi}(1+\cos\frac{n\pi}{3})\sin(n\omega t)$$

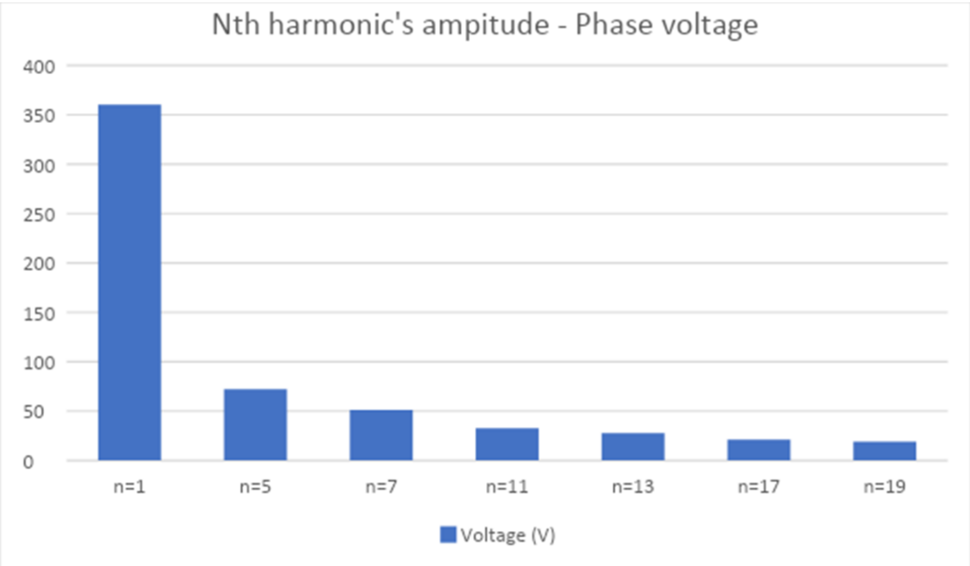
$$v_{bN} = \sum_{n=1,5,7,11..}^{\infty} \frac{4V_{in}}{3n\pi}(1+\cos\frac{n\pi}{3})\sin(n\omega t-120^{o})$$

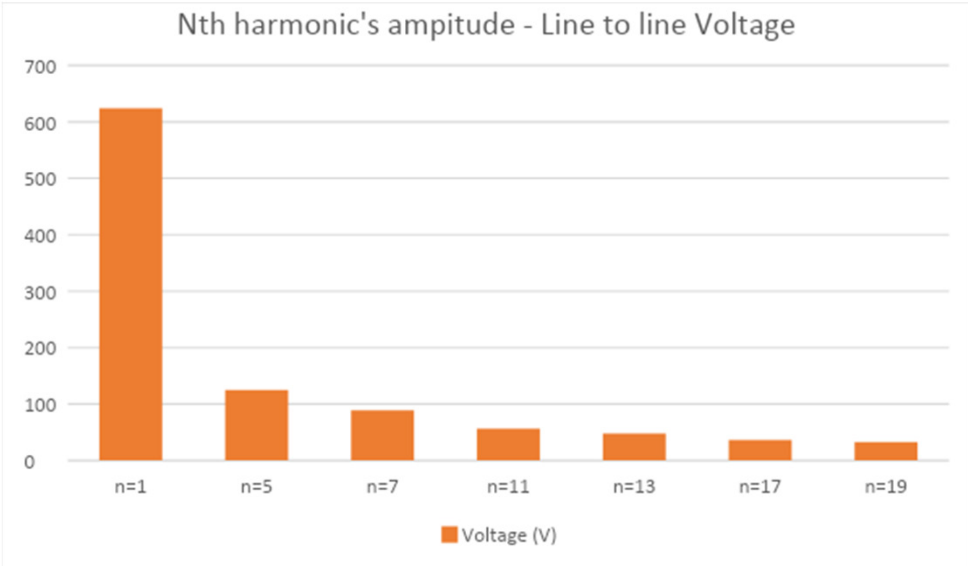
$$v_{cN} = \sum_{n=1,5,7,11..}^{\infty} \frac{4V_{in}}{3n\pi}(1+\cos\frac{n\pi}{3})\sin(n\omega t-240^{o})$$

Solution:

It’s worth to mention that due to the lack of the neutral connection the third and the multiples of the third harmonic are not existing.

Also due to the symmetry across the 90° there is also lack of the even harmonics in the output voltage of this converter.





Thank you for your attention!