Controlled rectifiers (ideal operation)

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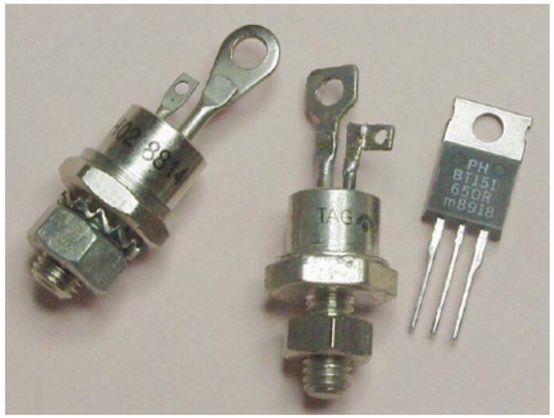
Short Revision on Thyristors

Introduction

- One of the most important type of power semiconductor device.
- Compared to transistors, thyristors have lower on-state conduction losses and higher power handling capability.
- However, they have worse switching performances than transistors.
- Have the highest power handling capability.
- It has a rating of 1200V / 1500A with switching frequencies ranging from 0.1kHz to 20kHz.



Background On Thyristors

- They name Thyristor comes from two similar device names ‘Thyratron’ and ‘Transistor’
- Thyristors are useful due to their ability to handle large current in power applications and fast switching
- The most common thyristor is the SCR which stands for “Silicon Controlled Rectifier”

Ultra-High Power Thyristor

- Quantitatively understand the upper bound these devices can achieve

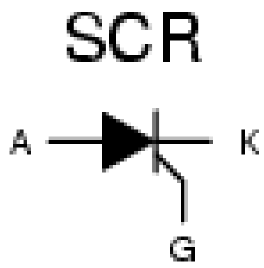
Absolute Maximum Ratings, T_j = 25 °C unless otherwise specified

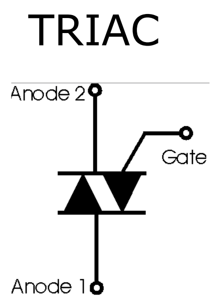
Ratings	Symbol	FT1500AU-240	Units
Repetitive Peak Reverse Voltage	V_{RRM}	12000	Volts
Non-Repetitive Peak Reverse Voltage	V_{RSM}	12000	Volts
DC Reverse Voltage	$V_{R(DC)}$	9600	Volts
Repetitive Peak Off-state Voltage	V_{DRM}	12000	Volts
Non-Repetitive Peak Off-state Voltage	V_{DSM}	12000	Volts
RMS On-state Current	$I_{T(RMS)}$	2360	Amperes
Average On-state Current, f = 60Hz, Sine Wave θ = 180°C, T_f = 88°C	$I_{T(AV)}$	1500	Amperes
Surge (Non-repetitive) On-state Current, One Half Cycle at 60Hz	I_{TSM}	34	kA
Current-squared, Time Integration, One Cycle at 60Hz	I^2t	4.8 x 10[illegible]	A^2s
Critical Rate of Rise of On-state Current, V_D = 1/2 V_{DRM}, I_G = 2.0A, T_j = 125°C	di_T/dt	100	A/µs
Peak Forward Gate Power Dissipation	P_{FGM}	30	Watts
Average Forward Gate Power Dissipation	$P_{FG(AV)}$	8.0	Watts
Peak Forward Gate Voltage	V_{FGM}	20	Volts
Peak Reverse Gate Voltage	V_{RGM}	10	Volts
Peak Forward Gate Current	I_{FGM}	6.0	Amperes
Junction Temperature	T_j	-40 to 125	°C
Storage Temperature	T_{stg}	-40 to 150	°C
Mounting Force Required, Recommended Value 118	–	108 ~ 132	kN
Weight, Standard Value	–	4000	Grams
DC Off-state Voltage	$V_{D(DC)}$	9600	Volts

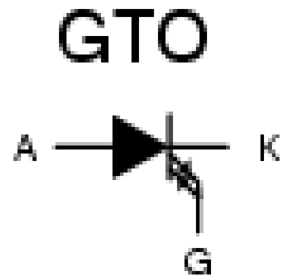
http://www.datasheetarchive.com/dl/Datasheet-020/DSA00357098.pdf

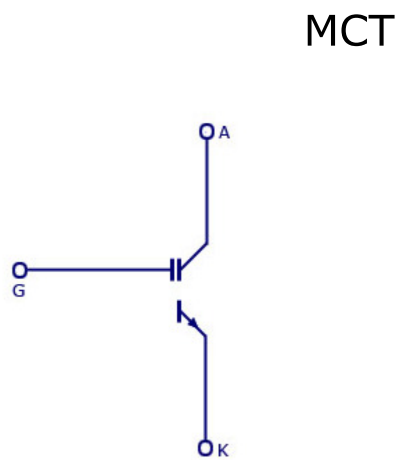
Main types of thyristors

- SCR (Silicon Controlled rectifier)
  - TRIAC
- GTO (Gate Turn Off)
  - IGTO (Integrated Gate Turn Off)
- MCT (MOS-controlled Rectifier)

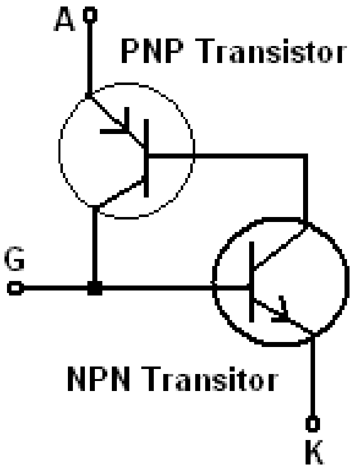




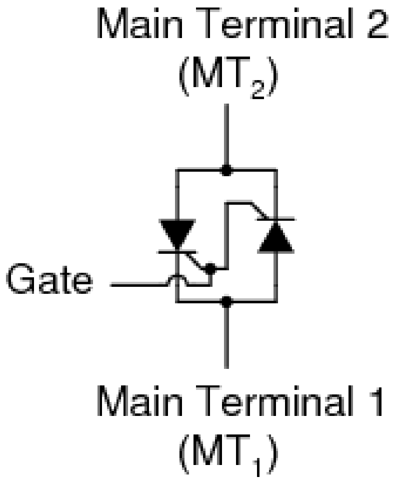




Equivalent circuits

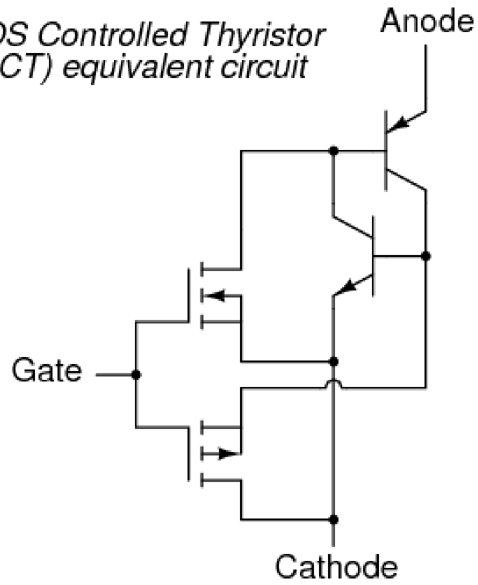


SCR Equivalent Circuit



TRIAC equivalent circuit

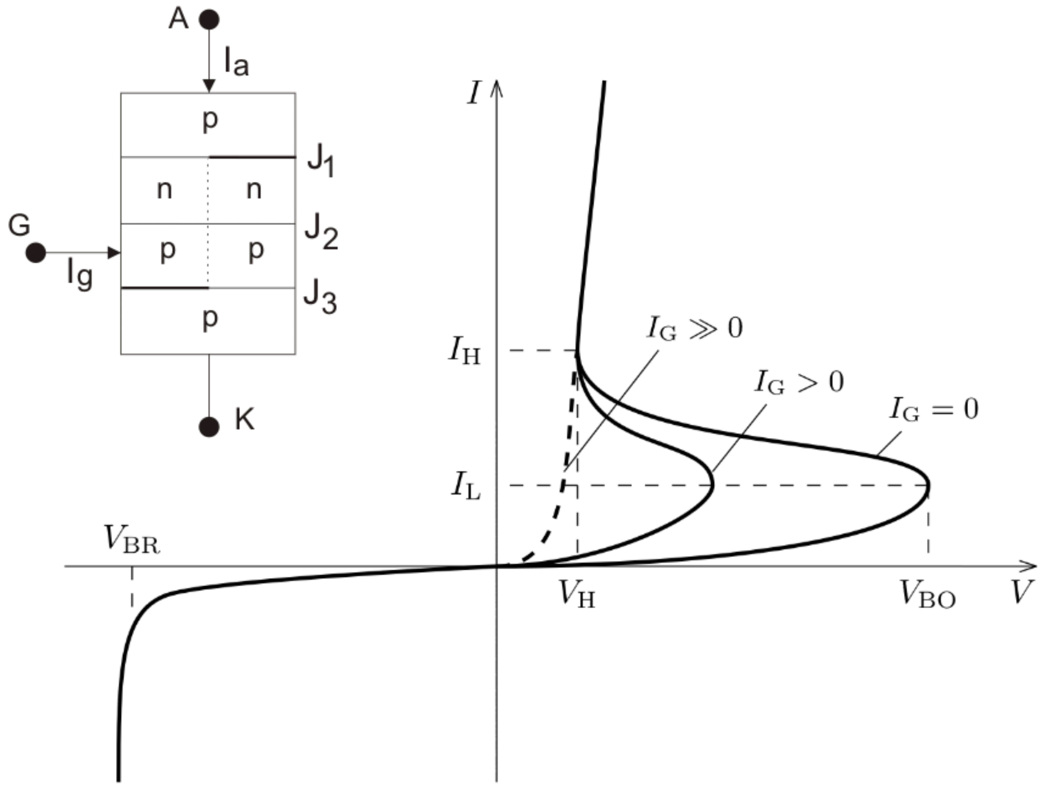
MOS Controlled Thyristor (MCT) equivalent circuit



http://www.allaboutcircuits.com/vol_3/chpt_7/6.html

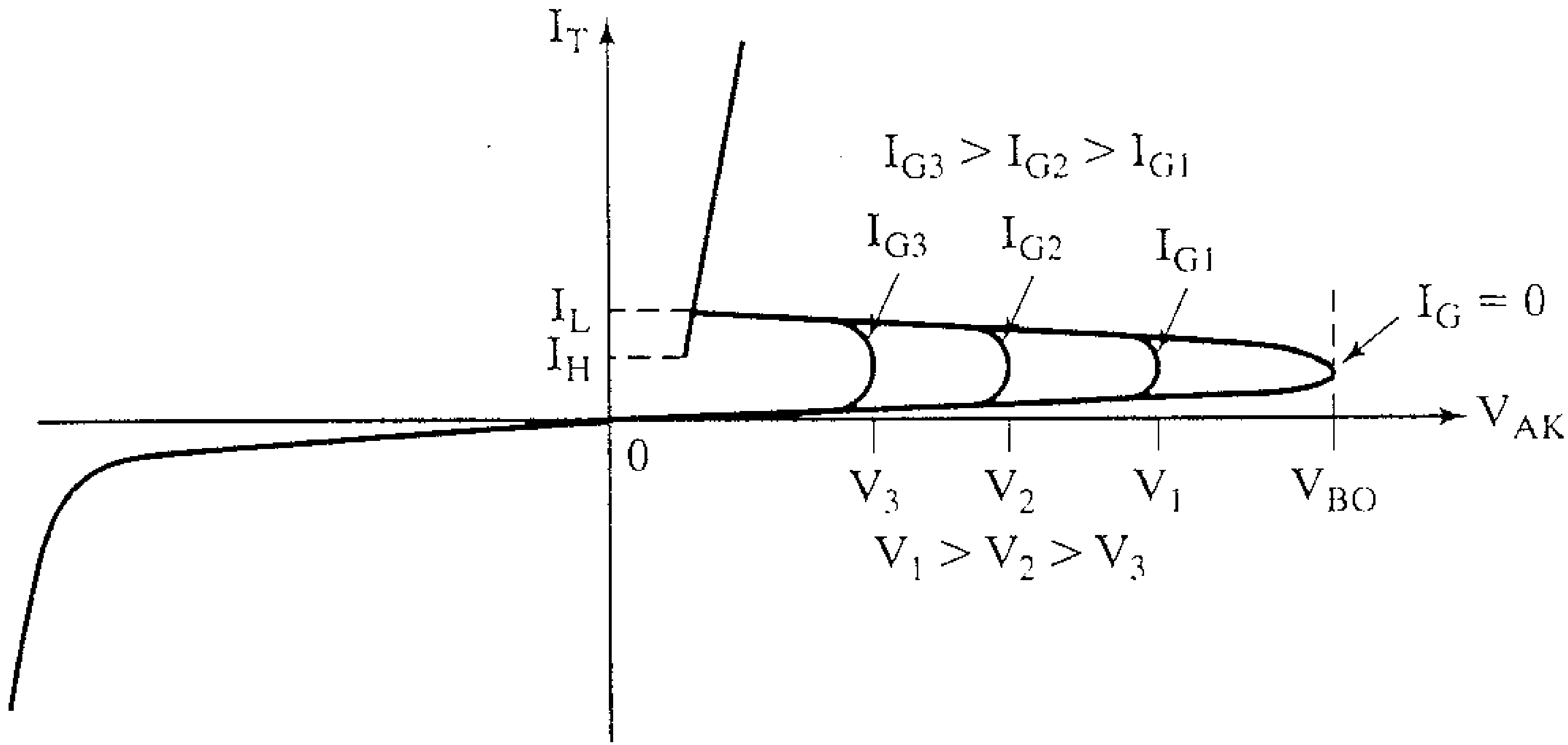
How the SCR operates

- Three modes of operation:
  - Reverse Blocking mode
  - Forward Blocking mode
  - Forward Active conducting mode



http://upload.wikimedia.org/wikipedia/commons/thumb/f/f1/Thyristor_I-V_diagram.svg/1280px-Thyristor_I-V_diagram.svg.png

Effects of Increasing Gate Current



How the TRIAC Operates

- Modes of operation:
  - Forward conducting mode
  - Reverse conducting mode
  - Forward Blocking mode
  - Reverse Blocking mode

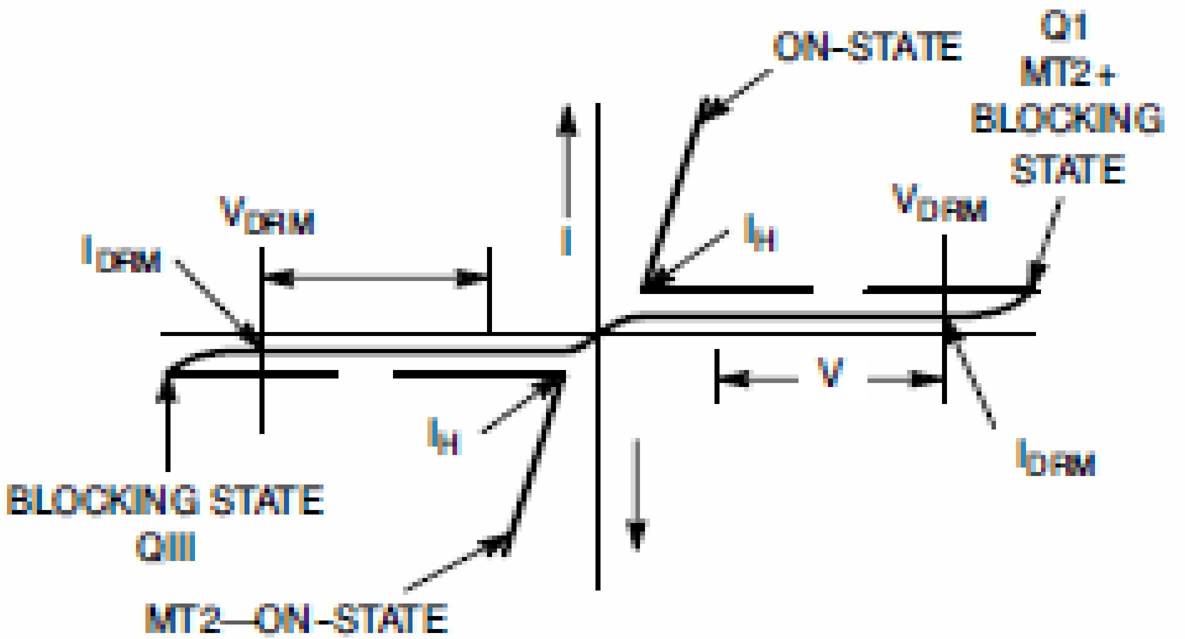


Figure 2.16. Triac Voltage−Current Characteristic

http://www.onsemi.com/pub_link/Collateral/HBD855-D.PDF

Some Important Parameters

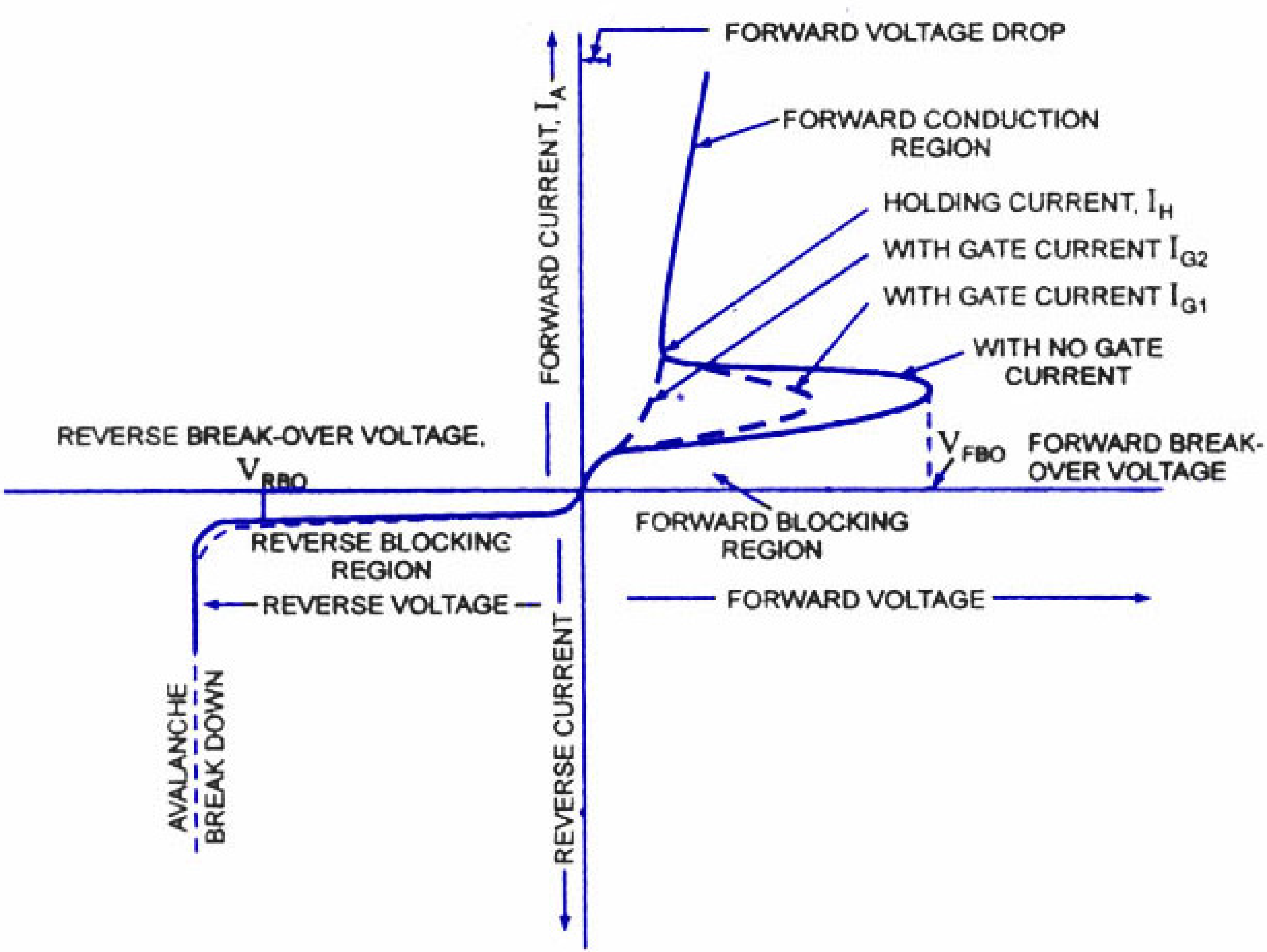
- di/dt dv/dt – Critical Rise of On-State Current/voltage
  - Maximum rise of current/voltage that the device can handle
  - Things to consider: High frequencies and large amounts of current/voltage
- I_{gm} V_{gm} – Forward Peak Gate Current/Voltage
  - Largest amount of current/voltage that can be applied to gate while in conduction mode
- I_H – Holding Current
  - Minimum current flow (from anode to cathode) to keep device on
- I_L – Latching Current
  - Current flow applied to anode in order to turn the device on

More parameters…

- t_{gt} – Gate Turn-On Time
  - Time it takes for a gate pulse to send the SCR into conducting mode
- t_q – Turn-Off Time
  - Time it takes for SCR to start blocking current after external voltage has switched to negative cycle
- V_{DRM} $I_{leakage}$ – Peak Repetitive Off-State Voltage / Forward current
  - Maximum repetitive voltage/current applied to Anode that won't breakdown the SCR or damage it
- V_{RRM} I_{RRM} – Peak Repetitive Off-State Voltage / Reverse Current
  - Maximum repetitive voltage/current applied to Cathode that won't breakdown the SCR or damage it

Even More Parameters

- I_{GT} V_{GT} – Gate Trigger Current/Voltage
  - Minimum value of current/voltage that will trigger the device from off to on
  - Important for considering false triggering
- I^2t – Circuit Fusing Consideration
  - Max non-repetitive over-current capability without damage (typically rated for 60Hz)
- T_j – Junction Temperature
  - Temperature range which this device may operate without damage under load conditions

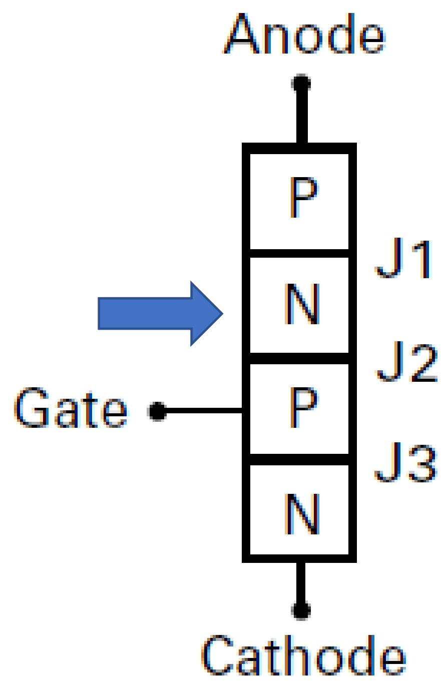


V-I Characteristics of SCR

http://forum.allaboutcircuits.com/attachments/scr-voltage-current-characteristics-jpg.69922/

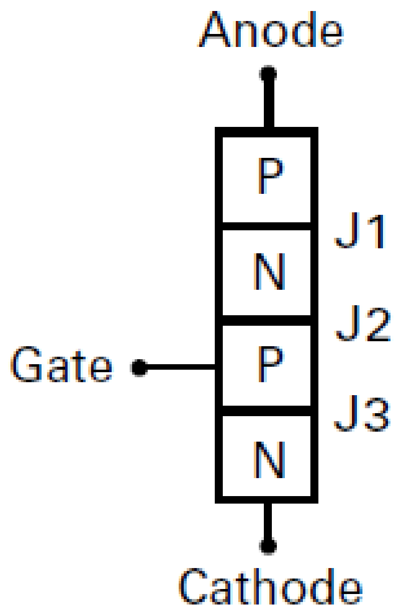
Characteristics of the SCR N- Regions

- SCR’s have a high resistive N-base region which forms a junction J2 as shown
  - This region is typically doped with Phosphorus atoms where N_D has a range of values from 10^{13} to 10^{14} cm^{-3}
  - This region’s thickness generally ranges from 1um to 100um depending on the voltage ratings
- Thicker N-base region increases forward conducting voltage drop
- The Cathode region is only 2um-5 um thick and has N_D range of 10^{16} to 10^{18} cm^{-3}



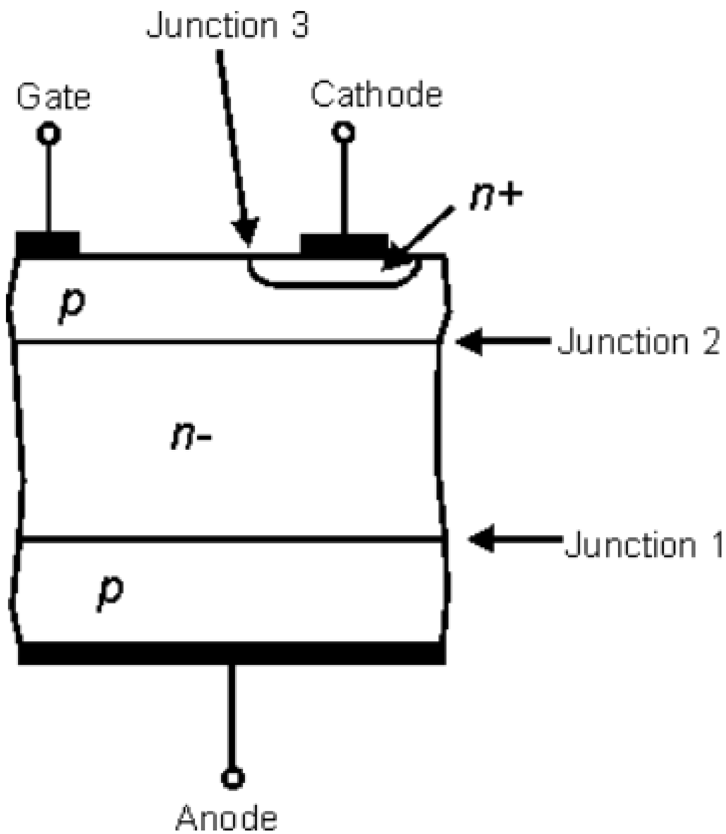
Characteristics of the SCR P-Regions

- High voltage SCRs are generally made by diffusing Al or Ga making it a P-region
- Typical N_A values range from 10^{15} to 10^{17} cm^{-3}
- These P-regions are generally on the order of 10-50 um thick



Comparing Doping Concentrations

- Highest Doping Concentration:
  - Cathode region or n+
- Next Highest level of Concentration:
  - Anode and Gate or p
- Lowest Doping level:
  - Mid N-Base region or n-
  - However, note that this is the thickest



http://www.radio-electronics.com/info/data/semicond/thyristor/structure-fabrication.php

Typical Materials Used In SCR

- Si - Silicon
- SiC – Silicon Carbide
- GaN – Gallium Nitride
- C – Carbon
- P – Phosphorus
- Al – Aluminum
- Au – Gold
- Pl - Platinum

- Used to create charge carrier recombination sites
- This slows the switching time but decreases forward conducting voltage drop

Trade-Offs In Design

- Forward Blocking Voltage vs. Switching time
- Forward Blocking Voltage vs. Forward Voltage Drop during Conduction Mode

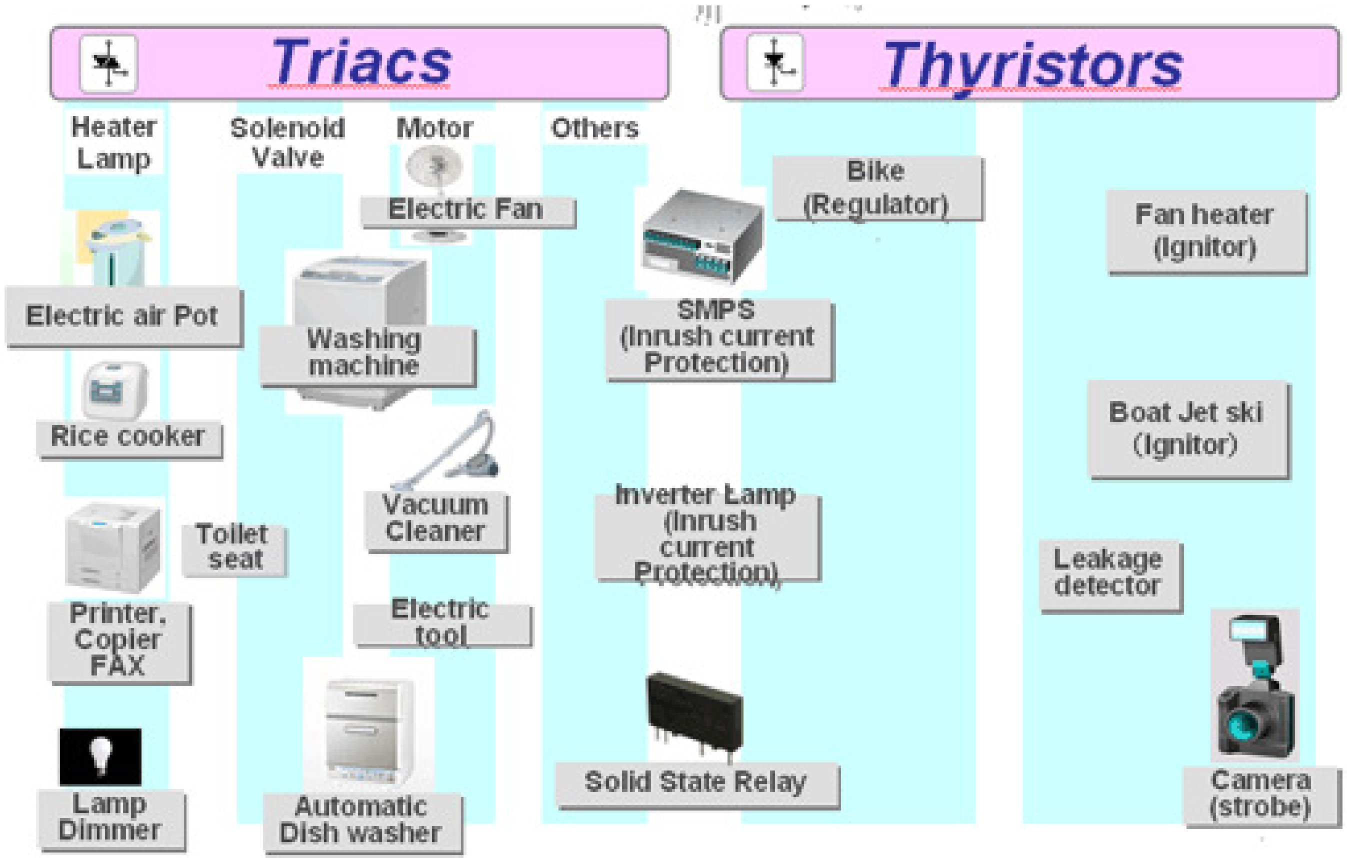
Application Area

- Mainly used where high currents and voltages are involved, and are often used to control alternating currents, where the change of polarity of the current causes the device to switch off automatically; referred to as Zero Cross operation.

- Thyristors can be used as the control elements for phase angle triggered controllers, also known as phase fired controllers.

Applications of Thyristors

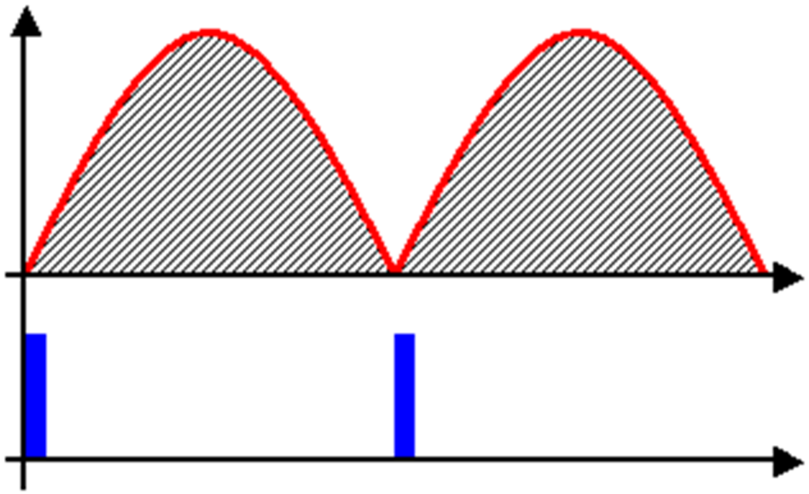
- Rectifiers
- Phase Fired Controllers
- Light Dimmers
- Motor Drive Speed Control
- Strobe Lights



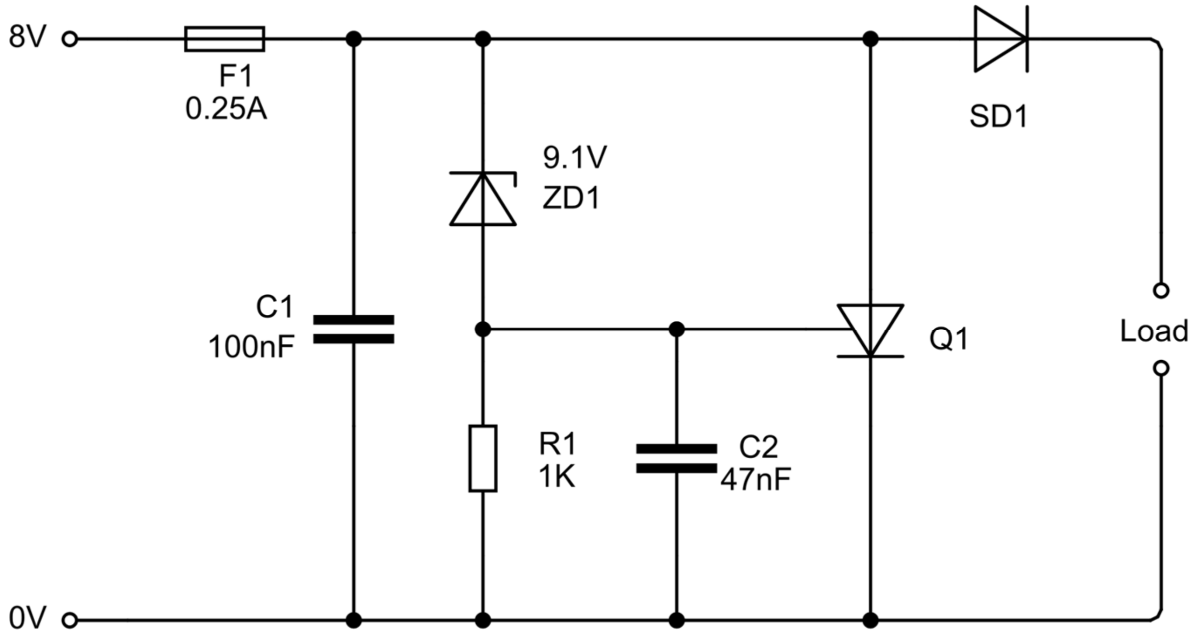
http://www.renesas.eu/products/discrete/thyristor_triac/index.jsp

Phase Control

- In power supplies application, thyristor can be used as a sort of "circuit breaker" or "crowbar" to prevent a failure in the power supply from damaging downstream components, by shorting the power supply output to ground.



Load voltage regulated by thyristor phase control.
Red trace: load voltage
Blue trace: trigger signal.



https://www.sunpower-uk.com/glossary/crowbar-protection/

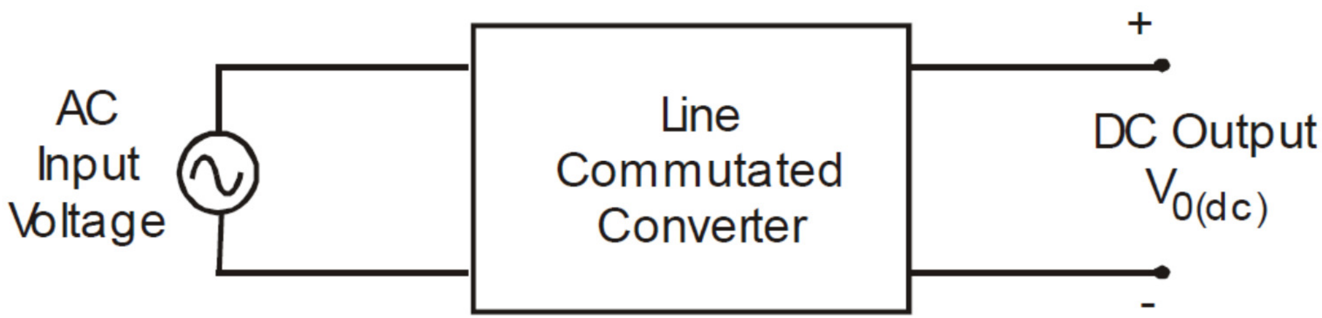
This circuit uses an 8V supply and has its overvoltage protection set at 9.1V; this can be adjusted by changing to the zener diode with the preferred voltage. At 9.1V, the zener diode ZD1 starts to conduct and causes a trigger signal to switch on the thyristor Q1.

The fuse F1 blows up once the current exceeds its rated 250mA. Capacitor C2 is used to take care of the small voltage spikes, noise and other harmless fluctuations which may erroneously trigger the circuit.

CONTROLLED RECTIFIERS
(Line Commutated AC to DC converters)

INTRODUCTION TO CONTROLLED RECTIFIERS

Controlled rectifiers are line commutated ac to dc power converters which are used to convert a fixed voltage, fixed frequency ac power supply into variable dc output voltage.



Type of input: Fixed voltage, fixed frequency ac power supply.
Type of output: Variable dc output voltage

APPLICATIONS OF PHASE CONTROLLED RECTIFIERS

- DC motor control in steel mills, paper and textile mills employing dc motor drives.
- AC fed traction system using dc traction motor.
- Electro-chemical and electro-metallurgical processes.
- Magnet power supplies.
- Reactor controls.
- Portable hand tool drives.
- Variable speed industrial drives.
- Battery charges.
- High voltage DC transmission.
- Uninterruptible power supply systems (UPS).

CLASSIFICATION OF PHASE CONTROLLED RECTIFIERS

The phase controlled rectifiers can be classified based on the type of input power supply as

- Single Phase Controlled Rectifiers which operate from single phase ac input power supply.
- Three Phase Controlled Rectifiers which operate from three phase ac input power supply.

DIFFERENT TYPES OF SINGLE PHASE CONTROLLED RECTIFIERS

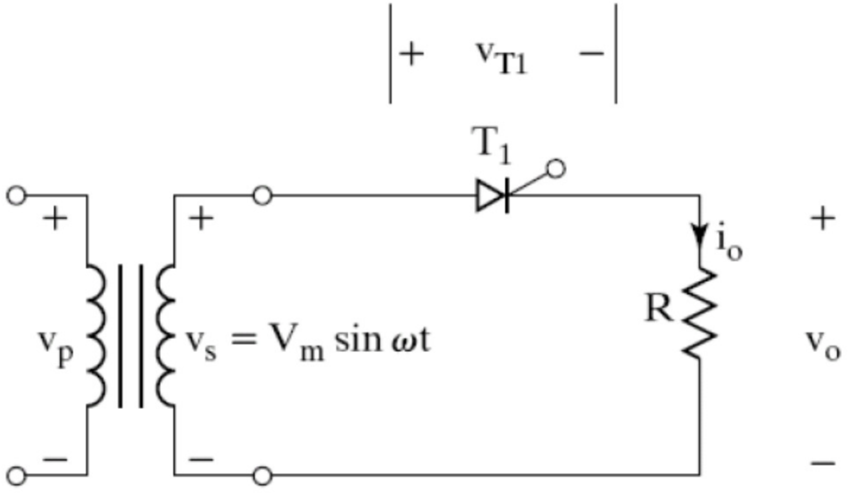
*Single Phase Controlled Rectifier*s are further subdivided into different types

- *Half wave controlled rectifier* which uses a single thyristor device (which provides output control only in one half cycle of input ac supply, and it provides low dc output).

- *Full wave controlled rectifiers* (which provide higher dc output)
    - Full wave controlled rectifier using a center tapped transformer (which requires two thyristors).
    - Full wave bridge controlled rectifiers (which do not require a center tapped transformer)
        - *Single phase semi-converter* (half controlled bridge converter, using two SCR's and two diodes, to provide single quadrant operation).
        - *Single phase full converter* (fully controlled bridge converter which requires four SCR's, to provide two quadrant operation).

*Three Phase Controlled Rectifier*s are of different types

- Three phase half wave controlled rectifiers.
- Three phase full wave controlled rectiriers.
    - Semi converter (half controlled bridge converter).
    - Full converter (fully controlled bridge converter).



$R = R_L =$ Load Resistance

Fig.: Single Phase Half-Wave Thyristor Converter with a Resistive Load

EQUATIONS

$v_s = V_m \sin \omega t =$ the ac supply voltage across the transformer secondary.

$V_m =$ max. (peak) value of input ac supply voltage across transformer secondary.

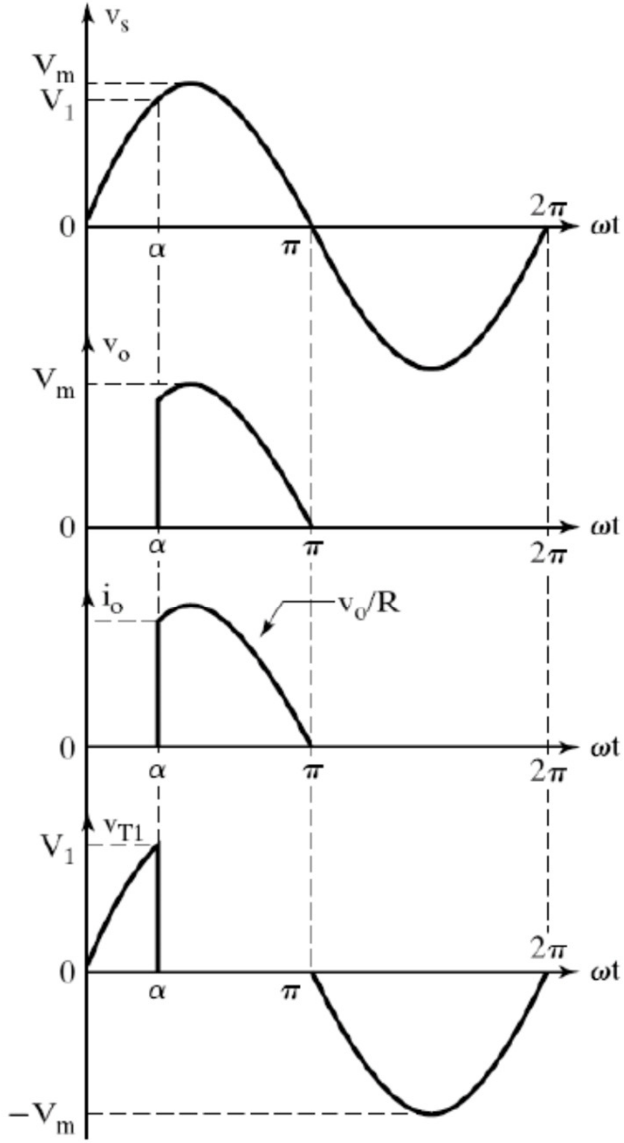
$V_S = \frac{V_m}{\sqrt{2}} =$ RMS value of input ac supply voltage across transformer secondary.

$v_O = v_L =$ the output voltage across the load ; $i_O = i_L =$ output (load) current.

When the thyristor is triggered at $\omega t = \alpha$ (an ideal thyristor behaves as a closed switch) and hence the output voltage follows the input supply voltage.

$v_O = v_L = V_m \sin \omega t$; for $\omega t = \alpha$ to π, when the thyristor is on.

$i_O = i_L = \frac{v_O}{R} =$ Load current for $\omega t = \alpha$ to π, when the thyristor is on.



Waveforms of single phase half-wave controlled rectifier with resistive load

TO DERIVE AN EXPRESSION FOR THE AVERAGE (DC) OUTPUT VOLTAGE ACROSS THE LOAD

If V_m is the peak input supply voltage, the average output voltage V_{dc} can be found from

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi}\int_{\alpha}^{\pi} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin \omega t . d(\omega t)$$

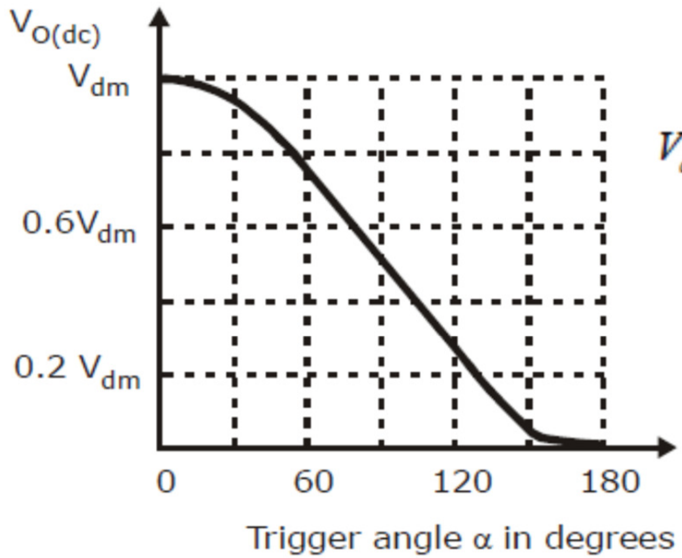
$$V_{O(dc)} = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin \omega t . d(\omega t)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\int_{\alpha}^{\pi} \sin \omega t . d(\omega t)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[-\cos \omega t \Big/_{\alpha}^{\pi}\right]$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[-\cos \pi + \cos \alpha\right] \quad ; \quad \cos \pi = -1$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[1 + \cos \alpha\right] \quad ; \quad V_m = \sqrt{2} V_S$$



$$V_{dcn} = \frac{V_{O(dc)}}{V_{dc(\max)}} = \frac{V_{dc}}{V_{dm}}$$

Fig.: Control characteristic

TO DERIVE AN EXPRESSION FOR THE RMS VALUE OF OUTPUT VOLTAGE OF A SINGLE PHASE HALF WAVE CONTROLLED RECTIFIER WITH RESISTIVE LOAD

The rms output voltage is given by

$$V_{O(RMS)} = \left[\frac{1}{2\pi} \int_0^{2\pi} v_O^2 . d(\omega t) \right]$$

Output voltage $v_O = V_m \sin \omega t$; for $\omega t = \alpha$ to π

$$V_{O(RMS)} = \left[\frac{1}{2\pi} \int_\alpha^\pi V_m^2 \sin^2 \omega t . d(\omega t) \right]^{\frac{1}{2}}$$

By substituting $\sin^2 \omega t = \frac{1 - \cos 2\omega t}{2}$, we get

$$V_{O(RMS)} = \left[\frac{1}{2\pi} \int_\alpha^\pi V_m^2 \frac{(1 - \cos 2\omega t)}{2} . d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{4\pi} \int_\alpha^\pi (1 - \cos 2\omega t) \, d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{4\pi} \left\{ \int_\alpha^\pi d(\omega t) - \int_\alpha^\pi \cos 2\omega t . d(\omega t) \right\} \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2} \left[\frac{1}{\pi} \left\{ (\omega t) \Big/_\alpha^\pi - \left(\frac{\sin 2\omega t}{2} \right) \Big/_\alpha^\pi \right\} \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2} \left[\frac{1}{\pi} \left((\pi - \alpha) - \frac{(\sin 2\pi - \sin 2\alpha)}{2} \right) \right]^{\frac{1}{2}} \text{ ; } \sin 2\pi = 0$$

Hence we get,

$$V_{O(RMS)} = \frac{V_m}{2} \left[\frac{1}{\pi} \left((\pi - \alpha) + \frac{\sin 2\alpha}{2} \right) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}} \left((\pi - \alpha) + \frac{\sin 2\alpha}{2} \right)^{\frac{1}{2}}$$

PERFORMANCE PARAMETERS OF PHASE CONTROLLED RECTIFIERS

Output dc power (average or dc output power delivered to the load)

$$P_{O(dc)} = V_{O(dc)} \times I_{O(dc)} \text{ ; i.e., } P_{dc} = V_{dc} \times I_{dc}$$

Where

$V_{O(dc)} = V_{dc} =$ average or dc value of output (load) voltage.

$I_{O(dc)} = I_{dc} =$ average or dc value of output (load) current.

Output ac power

$$P_{O(ac)} = V_{O(RMS)} \times I_{O(RMS)}$$

Efficiency of Rectification (Rectification Ratio)

$$\text{Efficiency } \eta = \frac{P_{O(dc)}}{P_{O(ac)}} \text{ ;} \qquad \% \text{ Efficiency } \eta = \frac{P_{O(dc)}}{P_{O(ac)}} \times 100$$

The output voltage can be considered as being composed of two components

- The dc component $V_{O(dc)}$ = DC or average value of output voltage.
- The ac component or the ripple component $V_{ac} = V_{r(rms)}$ = RMS value of all the ac ripple components.

The total RMS value of output voltage is given by

$$V_{O(RMS)} = \sqrt{V_{O(dc)}^2 + V_{r(rms)}^2}$$

Therefore

$$V_{ac} = V_{r(rms)} = \sqrt{V_{O(RMS)}^2 - V_{O(dc)}^2}$$

Form Factor (FF) which is a measure of the shape of the output voltage is given by

$$FF = \frac{V_{O(RMS)}}{V_{O(dc)}} = \frac{\text{RMS output (load) voltage}}{\text{DC output (load) voltage}}$$

The Ripple Factor (RF) which is a measure of the ac ripple content in the output voltage waveform. The output voltage ripple factor defined for the output voltage waveform is given by

$$r_v = RF = \frac{V_{r(rms)}}{V_{O(dc)}} = \frac{V_{ac}}{V_{dc}}$$

$$r_v = \frac{\sqrt{V_{O(RMS)}^2 - V_{O(dc)}^2}}{V_{O(dc)}} = \sqrt{\left[\frac{V_{O(RMS)}}{V_{O(dc)}}\right]^2 - 1}$$

Therefore

$$r_v = \sqrt{FF^2 - 1}$$

Current Ripple Factor defined for the output (load) current waveform is given by

$$r_i = \frac{I_{r(rms)}}{I_{O(dc)}} = \frac{I_{ac}}{I_{dc}}$$

Where $I_{r(rms)} = I_{ac} = \sqrt{I_{O(RMS)}^2 - I_{O(dc)}^2}$

Some times the peak to peak output ripple voltage is also considered to express the peak to peak output ripple voltage as

$V_{r(pp)} =$ peak to peak ac ripple output voltage

The peak to peak ac ripple load current is the difference between the maximum and the minimum values of the output load current.

$$I_{r(pp)} = I_{O(\max)} - I_{O(\min)}$$

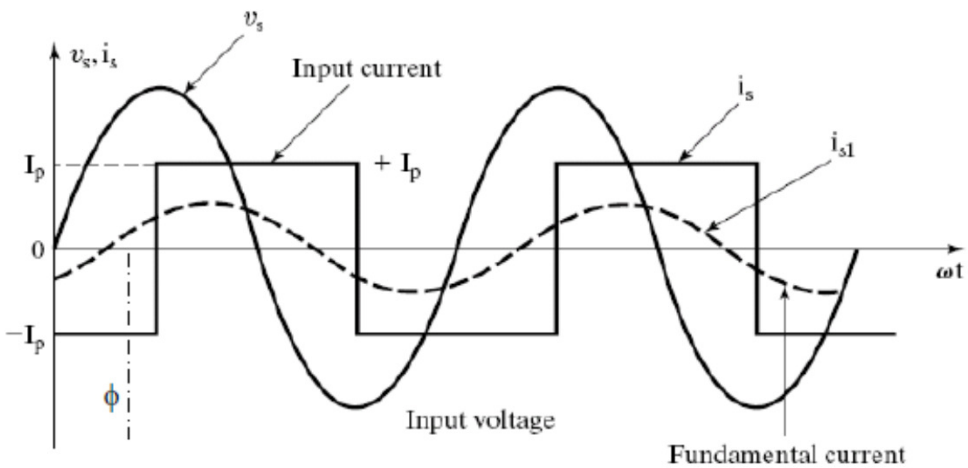
Transformer Utilization Factor (TUF)

$$TUF = \frac{P_{O(dc)}}{V_S \times I_S}$$

Where

$V_S =$ RMS value of transformer secondary output voltage (RMS supply voltage at the secondary)

$I_S =$ RMS value of transformer secondary current (RMS line or supply current).



$v_S =$ Supply voltage at the transformer secondary side.

$i_S =$ Input supply current (transformer secondary winding current).

$i_{S1} =$ Fundamental component of the input supply current.

$I_P =$ Peak value of the input supply current.

$\phi =$ Phase angle difference between (sine wave components) the fundamental components of input supply current and the input supply voltage.

$\phi =$ Displacement angle (phase angle)

For an RL load $\phi =$ Displacement angle = Load impedance angle

$$\therefore \quad \phi = \tan^{-1}\left(\frac{\omega L}{R}\right) \text{ for an RL load}$$

Displacement Factor (DF) or Fundamental Power Factor

$$DF = Cos\phi$$

Harmonic Factor (HF) or Total Harmonic Distortion Factor (THD)

The harmonic factor is a measure of the distortion in the output waveform and is also referred to as the total harmonic distortion (THD)

$$HF = \left[\frac{I_S^2 - I_{S1}^2}{I_{S1}^2}\right]^{\frac{1}{2}} = \left[\left(\frac{I_S}{I_{S1}}\right)^2 - 1\right]^{\frac{1}{2}}$$

Where

$I_S =$ RMS value of input supply current.

$I_{S1} =$ RMS value of fundamental component of the input supply current.

Input Power Factor (PF)

$$PF = \frac{V_S I_{S1}}{V_S I_S}\cos\phi = \frac{I_{S1}}{I_S}\cos\phi$$

The Crest Factor (CF)

$$CF = \frac{I_{S(peak)}}{I_S} = \frac{\text{Peak input supply current}}{\text{RMS input supply current}}$$

For an Ideal Controlled Rectifier

$FF = 1$; which means that $V_{O(RMS)} = V_{O(dc)}$.

Efficiency $\eta = 100\%$; which means that $P_{O(dc)} = P_{O(ac)}$.

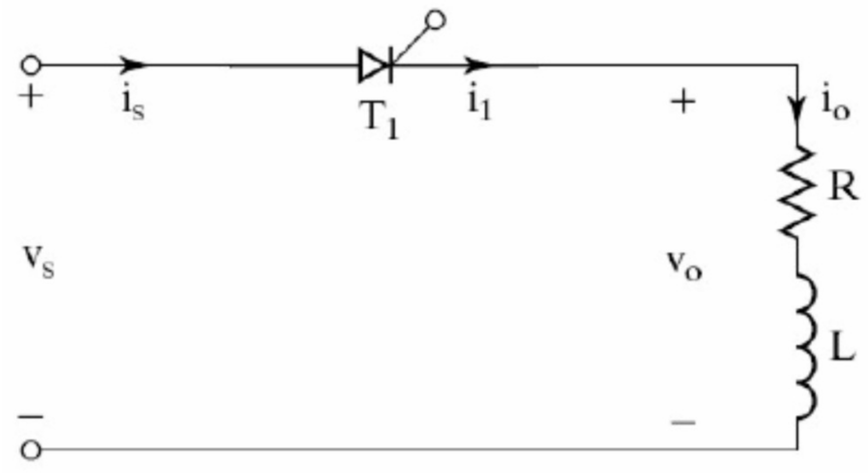
$V_{ac} = V_{r(rms)} = 0$; so that $RF = r_v = 0$; Ripple factor = 0 (ripple free converter).

$TUF = 1$; which means that $P_{O(dc)} = V_S \times I_S$

$HF = THD = 0$; which means that $I_S = I_{S1}$

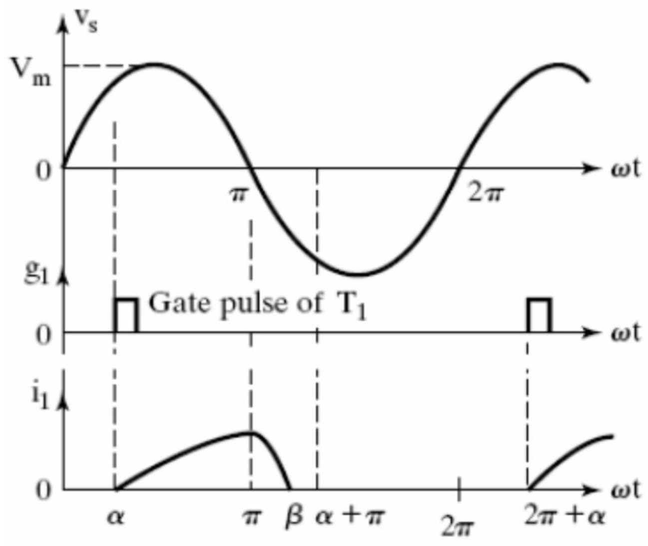
$PF = DPF = 1$; which means that $\phi = 0$

SINGLE PHASE HALF WAVE CONTROLLED RECTIFIER WITH AN RL LOAD



The thyristor T_1 will continue to conduct the load current until all the inductive energy stored in the load inductor L is completely utilized and the load current through T_1 falls to zero at $\omega t = \beta$, where β is referred to as the Extinction angle, (the value of ωt) at which the load current falls to zero. The extinction angle β is measured from the point of the beginning of the positive half cycle of input supply to the point where the load current falls to zero.

The thyristor T_1 conducts from $\omega t = \alpha$ to β. The conduction angle of T_1 is $\delta = (\beta - \alpha)$, which depends on the delay angle α and the load impedance angle ϕ.



$$i_1 = i_O = i_S$$

Fig.: Input supply voltage & Thyristor current waveforms

β is the extinction angle which depends upon the load inductance value.

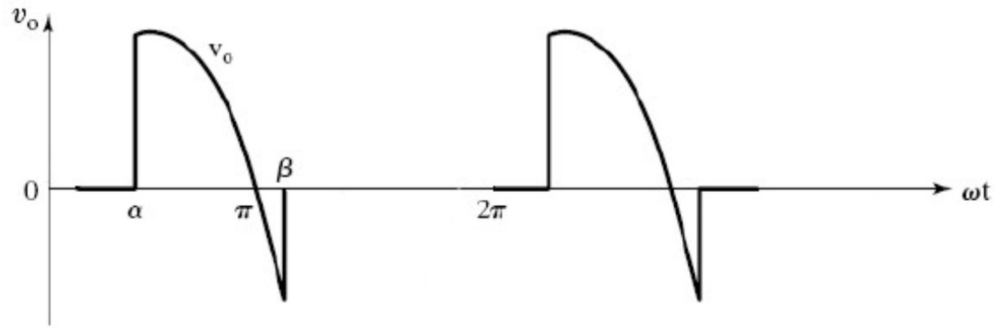


Fig.: Output (load) voltage waveform of a single phase half wave controlled rectifier with RL load

TO DERIVE AN EXPRESSION FOR THE OUTPUT (INDUCTIVE LOAD) CURRENT, DURING $\omega t = \alpha$ to β WHEN THYRISTOR T_1 CONDUCTS

$v_S = V_m \sin \omega t$ = instantaneous value of the input supply voltage.

Let us assume that the thyristor T_1 is triggered by applying the gating signal to T_1 at $\omega t = \alpha$. The load current which flows through the thyristor T_1 during $\omega t = \alpha$ to β can be found from the equation

$$L\left(\frac{di_O}{dt}\right) + Ri_O = V_m \sin \omega t \quad ;$$

The solution of the above differential equation gives the general expression for the output load current which is of the form

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) + A_1 e^{\frac{-t}{\tau}} \quad ;$$

Where $V_m = \sqrt{2}V_S$ = maximum or peak value of input supply voltage.

$Z = \sqrt{R^2 + (\omega L)^2}$ = Load impedance.

$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$ = Load impedance angle (power factor angle of load).

$\tau = \frac{L}{R}$ = Load circuit time constant.

Therefore the general expression for the output load current is given by the equation

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) + A_1 e^{\frac{-R}{L}t} \quad ;$$

The value of the constant A_1 can be determined from the initial condition. i.e. initial value of load current $i_O = 0$, at $\omega t = \alpha$. Hence from the equation for i_O equating i_O to zero and substituting $\omega t = \alpha$, we get

$$i_O = 0 = \frac{V_m}{Z}\sin(\alpha - \phi) + A_1 e^{\frac{-R}{L}t}$$

Therefore $\quad A_1 e^{\frac{-R}{L}t} = \frac{-V_m}{Z}\sin(\alpha - \phi)$

$$A_1 = \frac{1}{e^{\frac{-R}{L}t}}\left[\frac{-V_m}{Z}\sin(\alpha - \phi)\right]$$

$$A_1 = e^{\frac{+R}{L}t}\left[\frac{-V_m}{Z}\sin(\alpha - \phi)\right]$$

Therefore we obtain the final expression for the inductive load current of a single phase half wave controlled rectifier with RL load as

$$i_O = \frac{V_m}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right] \quad ; \quad \text{Where } \alpha \le \omega t \le \beta .$$

Equations

$v_s = V_m \sin\omega t =$ Input supply voltage

$v_O = v_L = V_m \sin\omega t =$ Output load voltage for $\omega t = \alpha$ to β,

when the thyristor T_1 conducts (T_1 is on).

Expression for the load current (thyristor current): for $\omega t = \alpha$ to β

$$i_O = \frac{V_m}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right] \; ; \quad \text{Where } \alpha \le \omega t \le \beta.$$

Extinction angle β can be calculated using the equation

$$\sin(\beta - \phi) = \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\beta - \alpha)}$$

TO CALCULATE EXTINCTION ANGLE β

The extinction angle β, which is the value of ωt at which the load current i_O falls to zero and T_1 is turned off can be estimated by using the condition that $i_O = 0$, at $\omega t = \beta$

By using the above expression for the output load current, we can write

$$i_O = 0 = \frac{V_m}{Z}\left[\sin(\beta - \phi) - \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\beta - \alpha)}\right]$$

As $\frac{V_m}{Z} \neq 0$, we can write

$$\left[\sin(\beta - \phi) - \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\beta - \alpha)}\right] = 0$$

Therefore we obtain the expression

$$\sin(\beta - \phi) = \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\beta - \alpha)}$$

The extinction angle β can be determined from this transcendental equation by using the iterative method of solution (trial and error method). After β is calculated, we can determine the thyristor conduction angle $\delta = (\beta - \alpha)$.

β is the extinction angle which depends upon the load inductance value. Conduction angle δ increases as α is decreased for a specific value of β.

Conduction angle $\delta = (\beta - \alpha)$; for a purely resistive load or for an RL load when the load inductance L is negligible the extinction angle $\beta = \pi$ and the conduction angle $\delta = (\pi - \alpha)$

TO DERIVE AN EXPRESSION FOR AVERAGE (DC) LOAD VOLTAGE

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\int_0^{2\pi} v_O.d(\omega t)$$

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_0^{\alpha} v_O.d(\omega t) + \int_{\alpha}^{\beta} v_O.d(\omega t) + \int_{\beta}^{2\pi} v_O.d(\omega t)\right];$$

$v_O = 0$ for $\omega t = 0$ to α & for $\omega t = \beta$ to 2π ;

$$\therefore \quad V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} v_O.d(\omega t)\right]; v_O = V_m \sin\omega t \text{ for } \omega t = \alpha \text{ to } \beta$$

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} V_m \sin\omega t.d(\omega t)\right]$$

$$V_{O(dc)} = V_L = \frac{V_m}{2\pi}\left[-\cos\omega t \Big/_{\alpha}^{\beta}\right] = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta)$$

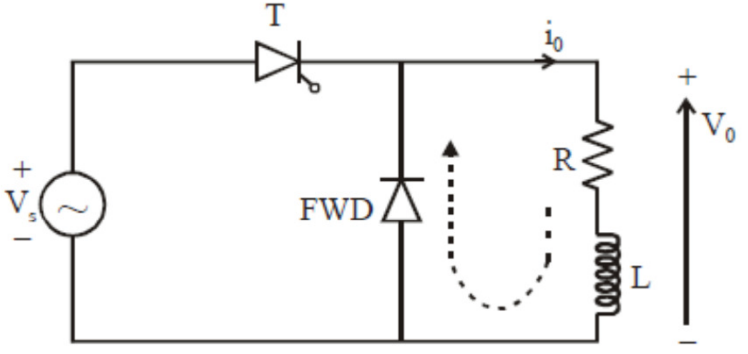
$$\therefore \quad V_{O(dc)} = V_L = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta)$$

Note: During the period $\omega t = \pi$ to β, we can see from the output load voltage waveform that the instantaneous output voltage is negative and this reduces the average or the dc output voltage when compared to a purely resistive load.

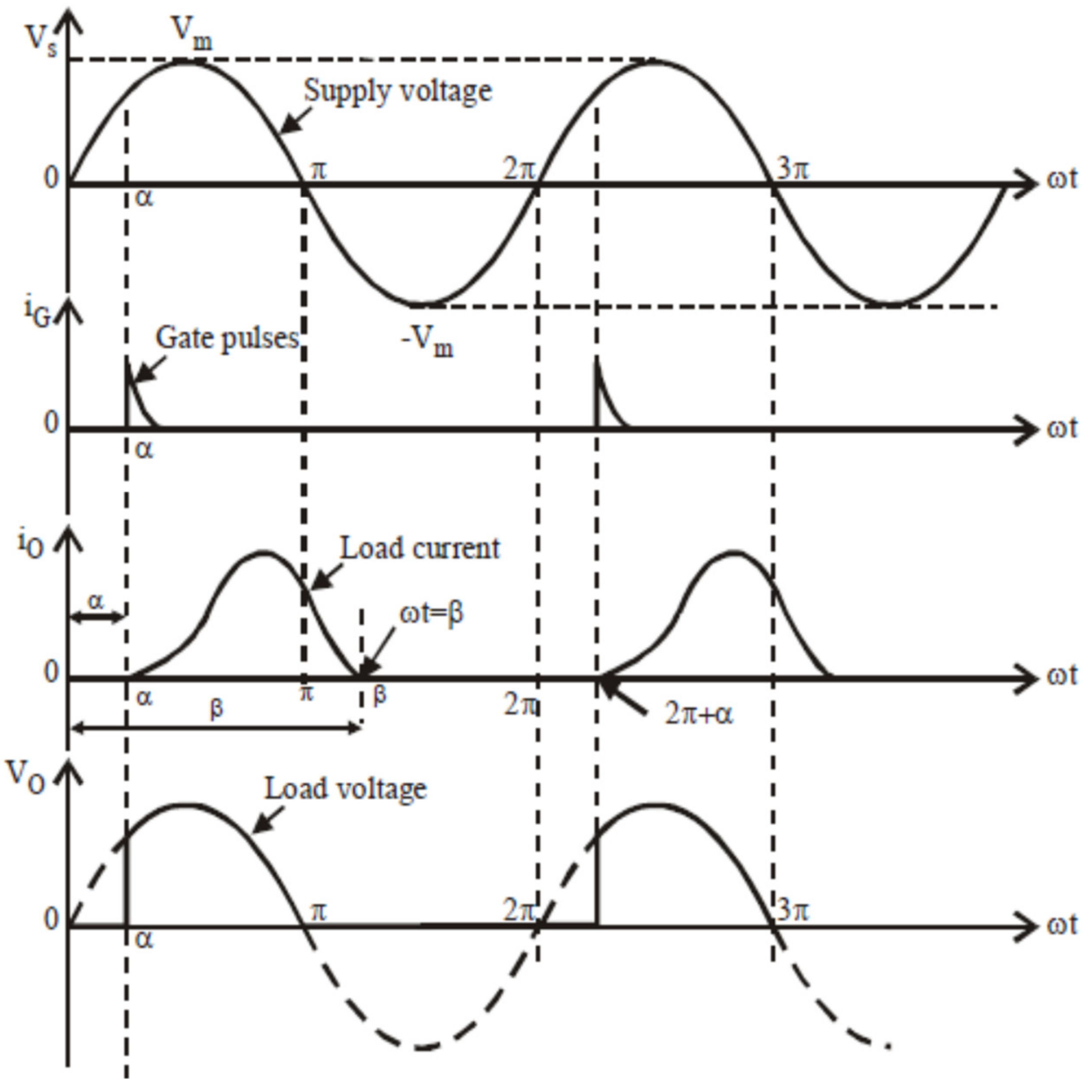
Average DC Load Current

$$I_{O(dc)} = I_{L(Avg)} = \frac{V_{O(dc)}}{R_L} = \frac{V_m}{2\pi R_L}(\cos\alpha - \cos\beta)$$

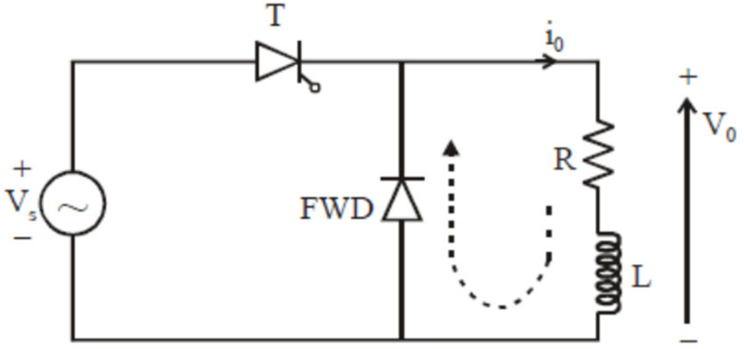
SINGLE PHASE HALF WAVE CONTROLLED RECTIFIER WITH RL LOAD AND FREE WHEELING DIODE

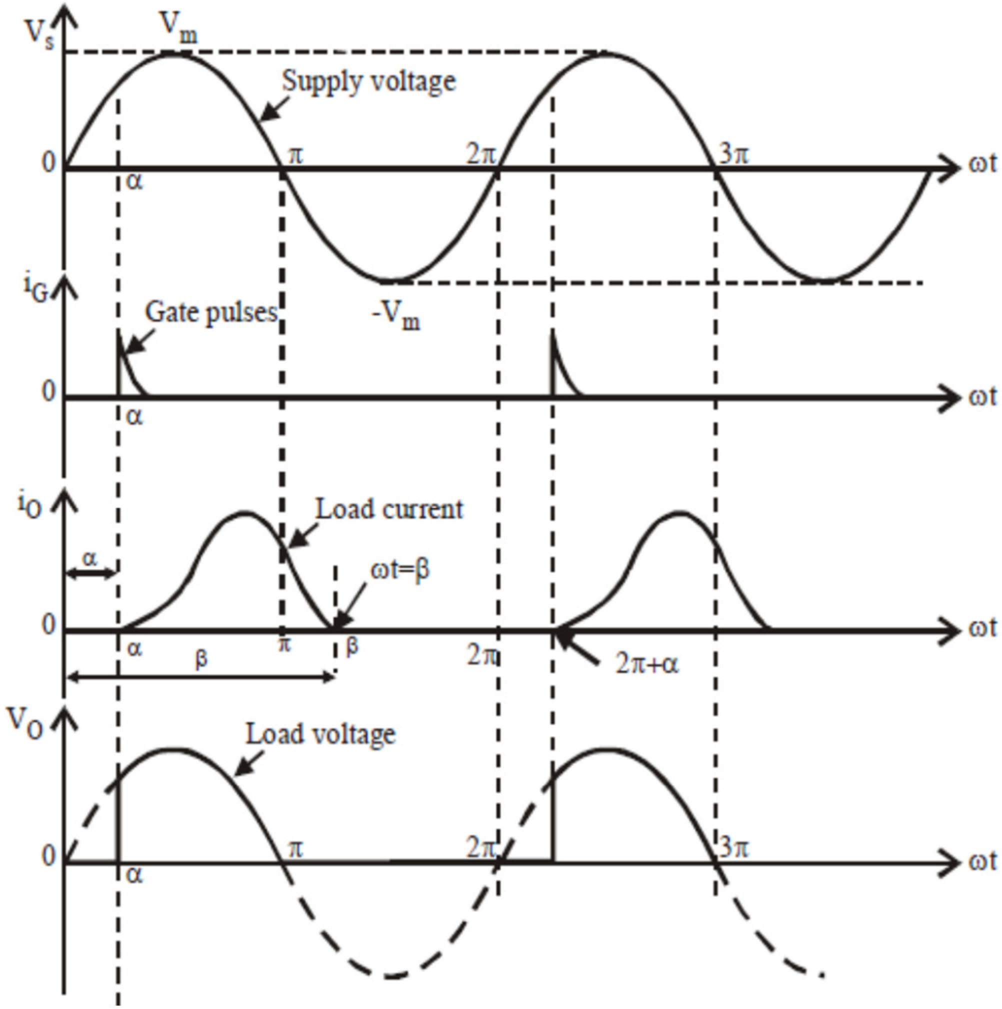


At $\omega t = \pi$, the source voltage v_S falls to zero and as v_S becomes negative, the free wheeling diode is forward biased. The stored energy in the inductance maintains the load current flow through R, L, and the FWD. Also, as soon as the FWD is forward biased, at $\omega t = \pi$, the SCR becomes reverse biased, the current through it becomes zero and the SCR turns off. During the period $\omega t = \pi$ to β, the load current flows through FWD (free wheeling load current) and decreases exponentially towards zero at $\omega t = \beta$.



SINGLE PHASE HALF WAVE CONTROLLED RECTIFIER WITH RL LOAD AND FREE WHEELING DIODE



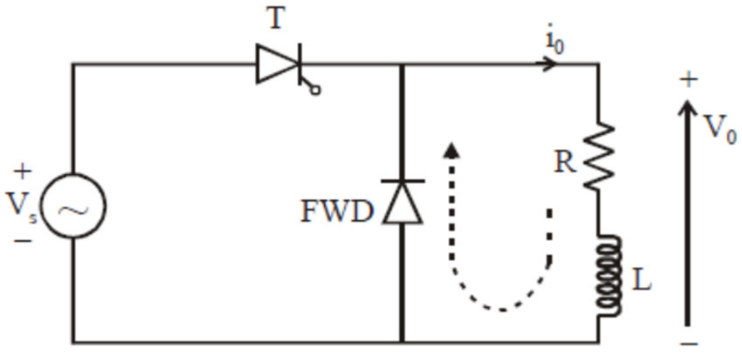


The average output voltage $V_{dc} = \frac{V_m}{2\pi}\left[1+\cos\alpha\right]$, which is the same as that of a purely resistive load. The output voltage across the load appears similar to the output voltage of a purely resistive load.

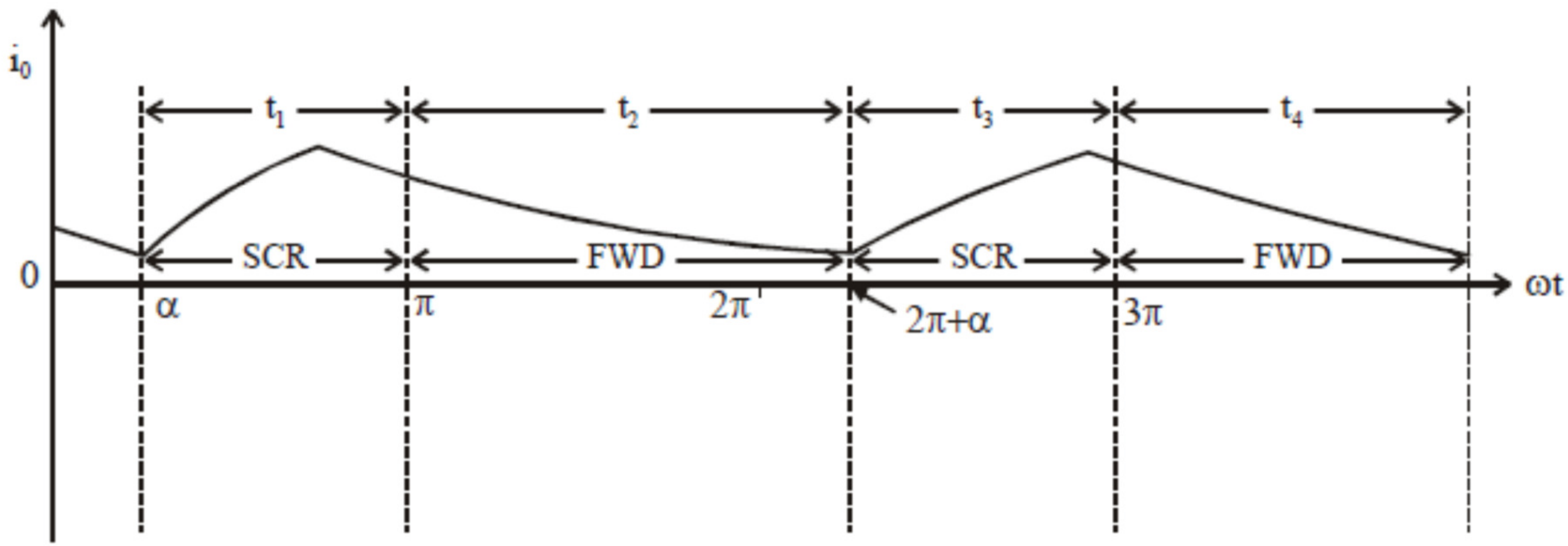
The following points are to be noted.

- If the inductance value is not very large, the energy stored in the inductance is able to maintain the load current only upto $\omega t = \beta$, where $\pi < \beta < 2\pi$, well before the next gate pulse and the load current tends to become discontinuous.
- During the conduction period α to π, the load current is carried by the SCR and during the free wheeling period π to β, the load current is carried by the free wheeling diode.
- The value of β depends on the value of R and L and the forward resistance of the FWD. Generally $\pi < \beta < 2\pi$.

SINGLE PHASE HALF WAVE CONTROLLED RECTIFIER WITH RL LOAD AND FREE WHEELING DIODE

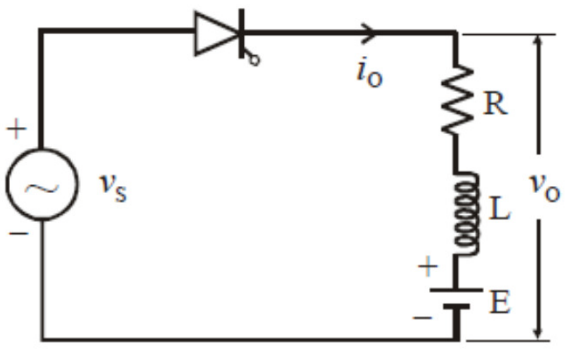


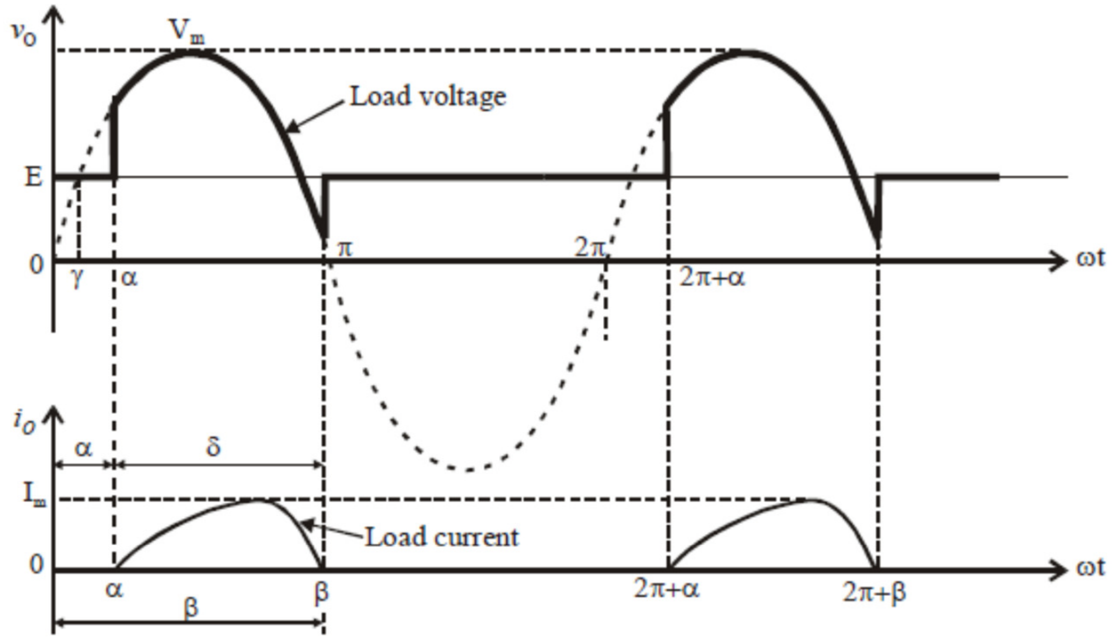
If the value of the inductance is very large, the load current does not decrease to zero during the free wheeling time interval and the load current waveform appears as shown in the figure.



SINGLE PHASE HALF WAVE CONTROLLED RECTIFIER WITH A GENERAL LOAD

A general load consists of R, L and a DC source 'E' in the load circuit





When the supply voltage is less than the dc voltage 'E' in the circuit the thyristor is reverse biased and hence the thyristor cannot conduct for supply voltage less than the load circuit dc voltage.

The value of ωt at which the supply voltage increases and becomes equal to the load circuit dc voltage can be calculated by using the equation $V_m \sin \omega t = E$. If we assume the value of ωt is equal to γ then we can write $V_m \sin \gamma = E$. Therefore γ is calculated as $\gamma = \sin^{-1}\left(\frac{E}{V_m}\right)$.

For trigger angle $\alpha < \gamma$, the thyristor conducts only from $\omega t = \gamma$ to β.

For trigger angle $\alpha > \gamma$, the thyristor conducts from $\omega t = \alpha$ to β.

Equations

$v_S = V_m \sin \omega t =$ Input supply voltage.

$v_O = V_m \sin \omega t =$ Output load voltage for $\omega t = \alpha$ to β

$v_O = E$ for $\omega t = 0$ to α & for $\omega t = \beta$ to 2π

Expression for the Load Current

When the thyristor is triggered at a delay angle of α, the equation for the circuit can be written as

$$V_m \sin \omega t = i_O \times R + L\left(\frac{di_O}{dt}\right) + E \ ; \ \alpha \le \omega t \le \beta$$

The general expression for the output load current can be written as

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) - \frac{E}{R} + Ae^{\frac{-t}{\tau}}$$

Where

$Z = \sqrt{R^2 + (\omega L)^2}$ = Load Impedance

$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$ = Load impedance angle

$\tau = \frac{L}{R}$ = Load circuit time constant

The general expression for the output load current can be written as

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) - \frac{E}{R} + Ae^{\frac{-R}{L}t}$$

To find the value of the constant 'A' apply the initial condition at $\omega t = \alpha$, load current $i_O = 0$. Equating the general expression for the load current to zero at $\omega t = \alpha$, we get

$$i_O = 0 = \frac{V_m}{Z}\sin(\alpha - \phi) - \frac{E}{R} + Ae^{\frac{-R}{L}\times\frac{\alpha}{\omega}}$$

We obtain the value of constant 'A' as

$$A = \left[\frac{E}{R} - \frac{V_m}{Z}\sin(\alpha - \phi)\right]e^{\frac{R}{\omega L}\alpha}$$

Substituting the value of the constant 'A' in the expression for the load current, we get the complete expression for the output load current as

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) - \frac{E}{R} + \left[\frac{E}{R} - \frac{V_m}{Z}\sin(\alpha - \phi)\right]e^{\frac{-R}{\omega L}(\omega t - \alpha)}$$

The Extinction angle β can be calculated from the final condition that the output current $i_O = 0$ at $\omega t = \beta$. By using the above expression we get,

$$i_O = 0 = \frac{V_m}{Z}\sin(\beta - \phi) - \frac{E}{R} + \left[\frac{E}{R} - \frac{V_m}{Z}\sin(\alpha - \phi)\right]e^{\frac{-R}{\omega L}(\beta - \alpha)}$$

To derive an expression for the average or dc load voltage

$$V_{O(dc)} = \frac{1}{2\pi}\int_0^{2\pi} v_O.d(\omega t)$$

$$V_{O(dc)} = \frac{1}{2\pi}\left[\int_0^{\alpha} v_O.d(\omega t) + \int_{\alpha}^{\beta} v_O.d(\omega t) + \int_{\beta}^{2\pi} v_O.d(\omega t)\right]$$

$v_O = V_m \sin\omega t =$ Output load voltage for $\omega t = \alpha$ to β

$v_O = E$ for $\omega t = 0$ to α & for $\omega t = \beta$ to 2π

$$V_{O(dc)} = \frac{1}{2\pi}\left[\int_0^{\alpha} E.d(\omega t) + \int_{\alpha}^{\beta} V_m \sin\omega t + \int_{\beta}^{2\pi} E.d(\omega t)\right]$$

$$V_{O(dc)} = \frac{1}{2\pi}\left[E(\omega t)\Big/_{0}^{\alpha} + V_m(-\cos\omega t)\Big/_{\alpha}^{\beta} + E(\omega t)\Big/_{\beta}^{2\pi}\right]$$

$$V_{O(dc)} = \frac{1}{2\pi}\left[E(\alpha - 0) - V_m(\cos\beta - \cos\alpha) + E(2\pi - \beta)\right]$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[(\cos\alpha - \cos\beta)\right] + \frac{E}{2\pi}(2\pi - \beta + \alpha)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta) + \left[\frac{2\pi - (\beta - \alpha)}{2\pi}\right]E$$

Conduction angle of thyristor $\delta = (\beta - \alpha)$

RMS Output Voltage can be calculated by using the expression

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_0^{2\pi} v_O^2.d(\omega t)\right]}$$

DISADVANTAGES OF SINGLE PHASE HALF WAVE CONTROLLED RECTIFIERS

Single phase half wave controlled rectifier gives

- Low dc output voltage.
- Low dc output power and lower efficiency.
- Higher ripple voltage & ripple current.
- Higher ripple factor.
- Low transformer utilization factor.
- The input supply current waveform has a dc component which can result in dc saturation of the transformer core.

Single phase half wave controlled rectifiers are rarely used in practice as they give low dc output and low dc output power. They are only of theoretical interest.

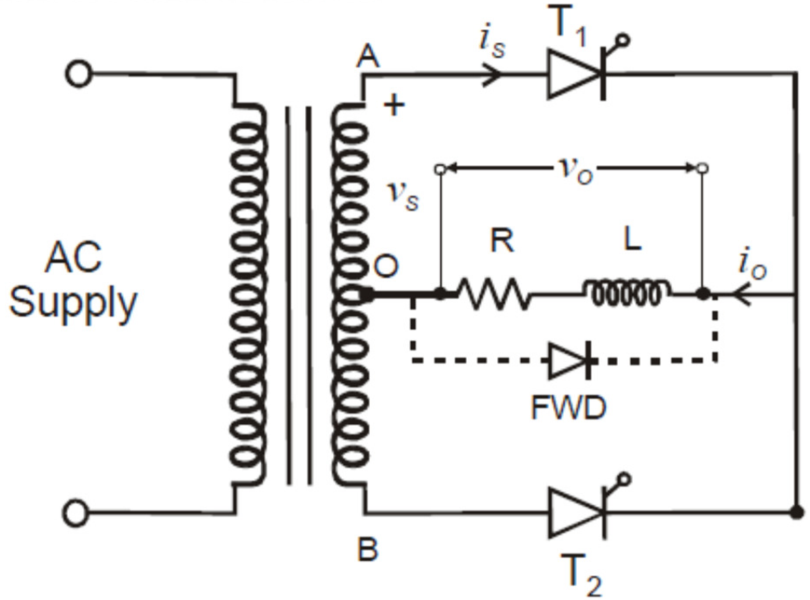
The above disadvantages of a single phase half wave controlled rectifier can be over come by using a full wave controlled rectifier circuit. Most of the practical converter circuits use full wave controlled rectifiers.

SINGLE PHASE FULL WAVE CONTROLLED RECTIFIERS

Single phase full wave controlled rectifiers are of various types

- Single phase full wave controlled rectifier using a center tapped transformer (two pulse converter with mid point configuration).
- Single phase full wave bridge controlled rectifier
  - Half controlled bridge converter (semi converter).
  - Fully controlled bridge converter (full converter).

SINGLE PHASE FULL WAVE CONTROLLED RECTIFIER USING A CENTER TAPPED TRANSFORMER



The upper half of the secondary winding and the thyristor T_1 along with the load act as a half wave controlled rectifier, the lower half of the secondary winding and the thyristor T_2 with the common load act as the second half wave controlled rectifier so as to produce a full wave load voltage waveform.

There are two types of operations possible.

- Discontinuous load current operation, which occurs for a purely resistive load or an RL load with low inductance value.
- Continuous load current operation which occurs for an RL type of load with large load inductance.

v_S = Supply Voltage across the upper half of the transformer secondary winding

$$v_S = v_{AO} = V_m \sin \omega t$$

$v_{BO} = -v_{AO} = -V_m \sin \omega t =$ supply voltage across the lower half of the transformer secondary winding.

Discontinuous Load Current Operation (for low value of load inductance)

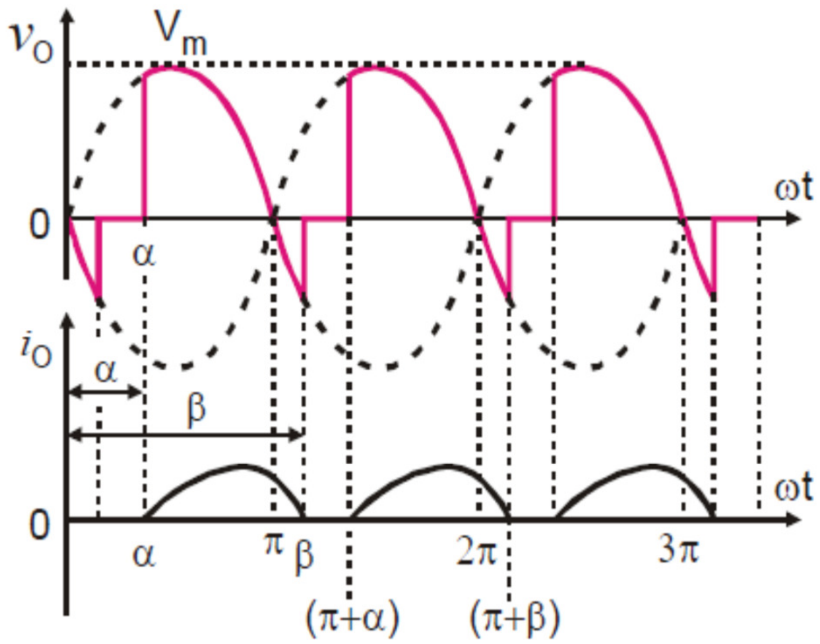


Fig.: Waveform for Discontinuous Load Current Operation without FWD

Positive half cycle of the input supply:

The output voltage across the load follows the input supply voltage that appears across the upper part of the secondary winding from $\omega t = \alpha$ to β. The load current through the thyristor T_1 decreases and drops to zero at $\omega t = \beta$, where $\beta > \pi$ for RL type of load and the thyristor T_1 naturally turns off at $\omega t = \beta$.

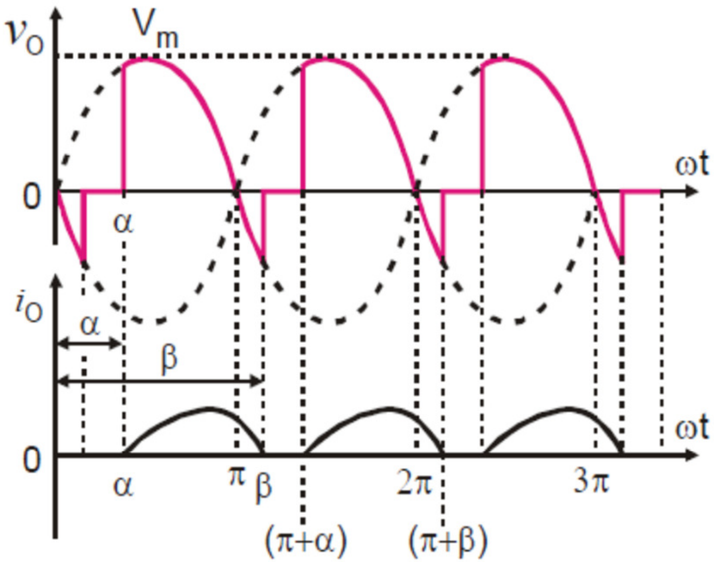
During the negative half cycle of the input supply the voltage at the supply line 'A' becomes negative whereas the voltage at line 'B' (at the lower side of the secondary winding) becomes positive with respect to the center point 'O'. The thyristor T_2 is forward biased during the negative half cycle and it is triggered at a delay angle of $(\pi + \alpha)$. The current flows through the thyristor T_2, through the load, and through the lower part of the secondary winding when T_2 conducts during the negative half cycle the load is connected to the lower half of the secondary winding when T_2 conducts.

For purely resistive loads when L = 0, the extinction angle $\beta = \pi$

For low values of load inductance the load current would be discontinuous and the extinction angle $\beta > \pi$ but $\beta < (\pi + \alpha)$.

For large values of load inductance the load current would be continuous and does not fall to zero. The thyristor T_1 conducts from α to $(\pi + \alpha)$, until the next thyristor T_2 is triggered. When T_2 is triggered at $\omega t = (\pi + \alpha)$, the thyristor T_1 will be reverse biased and hence T_1 turns off.

TO DERIVE AN EXPRESSION FOR THE DC OUTPUT VOLTAGE OF A SINGLE PHASE FULL WAVE CONTROLLED RECTIFIER WITH RL LOAD (WITHOUT FREE WHEELING DIODE (FWD))



$$V_{O(dc)} = V_{dc} = \frac{2}{2\pi}\int_{\omega t=\alpha}^{\beta} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi}\int_{\omega t=\alpha}^{\beta} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi}\left[\int_{\alpha}^{\beta} V_m \sin \omega t . d(\omega t)\right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[-\cos \omega t \Big/_{\alpha}^{\beta}\right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}(\cos\alpha - \cos\beta)$$

Therefore $V_{O(dc)} = \frac{V_m}{\pi}(\cos\alpha - \cos\beta)$, **for discontinuous load current operation,** $\pi < \beta < (\pi + \alpha)$.

When the load inductance is small and negligible that is $L \approx 0$, the extinction angle $\beta = \pi$ radians. Hence the average or dc output voltage for resistive load is obtained as

$$V_{O(dc)} = \frac{V_m}{\pi}(\cos\alpha - \cos\pi) \quad ; \cos\pi = -1$$

$$V_{O(dc)} = \frac{V_m}{\pi}(\cos\alpha - (-1))$$

$$V_{O(dc)} = \frac{V_m}{\pi}(1 + \cos\alpha) \; ; \text{ for resistive load, when } L \approx 0$$

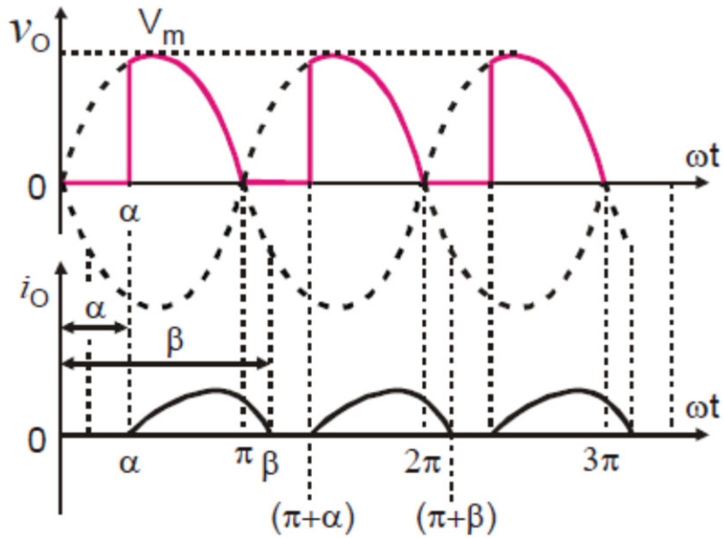
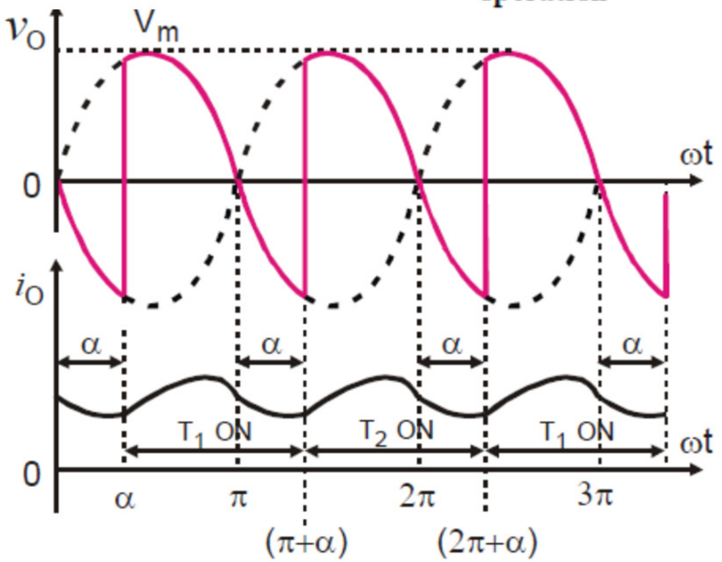


Fig.: Waveform for Discontinuous Load Current Operation with FWD

CONTINUOUS LOAD CURRENT OPERATION (WITHOUT FWD)

Fig.: Load voltage and load current waveform of a single phase full wave controlled rectifier with RL load & without FWD for continuous load current operation



In the case of continuous current operation the thyristor T_1 which is triggered at a delay angle of α, conducts from $\omega t = \alpha$ to $(\pi + \alpha)$. Output voltage follows the input supply voltage across the upper half of the transformer secondary winding $v_O = v_{AO} = V_m \sin \omega t$.

The next thyristor T_2 is triggered at $\omega t = (\pi + \alpha)$, during the negative half cycle input supply. As soon as T_2 is triggered at $\omega t = (\pi + \alpha)$, the thyristor T_1 will be reverse biased and T_1 turns off due to natural commutation (ac line commutation). The load current flows through the thyristor T_2 from $\omega t = (\pi + \alpha)$ to $(2\pi + \alpha)$.

Each thyristor conducts for π radians (180°) in the case of continuous current operation.

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t = \alpha}^{(\pi+\alpha)} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \left[\int_{\alpha}^{(\pi+\alpha)} V_m \sin \omega t . d(\omega t) \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{(\pi+\alpha)} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[\cos \alpha - \cos(\pi + \alpha) \right] \quad ; \quad \cos(\pi + \alpha) = -\cos \alpha$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[\cos \alpha + \cos \alpha \right]$$

$$V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi} \cos \alpha$$

TO DERIVE AN EXPRESSION FOR RMS OUTPUT VOLTAGE
The rms value of the output voltage is calculated by using the equation

$$V_{O(RMS)}=\left[\frac{2}{2\pi}\int_{\alpha}^{(\pi+\alpha)}v_O^2.d(\omega t)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)}=\left[\frac{1}{\pi}\int_{\alpha}^{(\pi+\alpha)}V_m^2\sin^2\omega t.d(\omega t)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)}=\left[\frac{V_m^2}{\pi}\int_{\alpha}^{(\pi+\alpha)}\sin^2\omega t.d(\omega t)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)}=\left[\frac{V_m^2}{\pi}\int_{\alpha}^{(\pi+\alpha)}\frac{(1-\cos 2\omega t)}{2}.d(\omega t)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)}=V_m\left[\frac{1}{2\pi}\left\{\int_{\alpha}^{(\pi+\alpha)}d(\omega t)-\int_{\alpha}^{(\pi+\alpha)}\cos 2\omega t.d(\omega t)\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)}=V_m\left[\frac{1}{2\pi}\left\{(\omega t)\Big/_{\alpha}^{(\pi+\alpha)}-\left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{(\pi+\alpha)}\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)}=V_m\left[\frac{1}{2\pi}\left\{(\pi+\alpha-\alpha)-\left(\frac{\sin 2(\pi+\alpha)-\sin 2\alpha}{2}\right)\right\}\right]^{\frac{1}{2}}$$

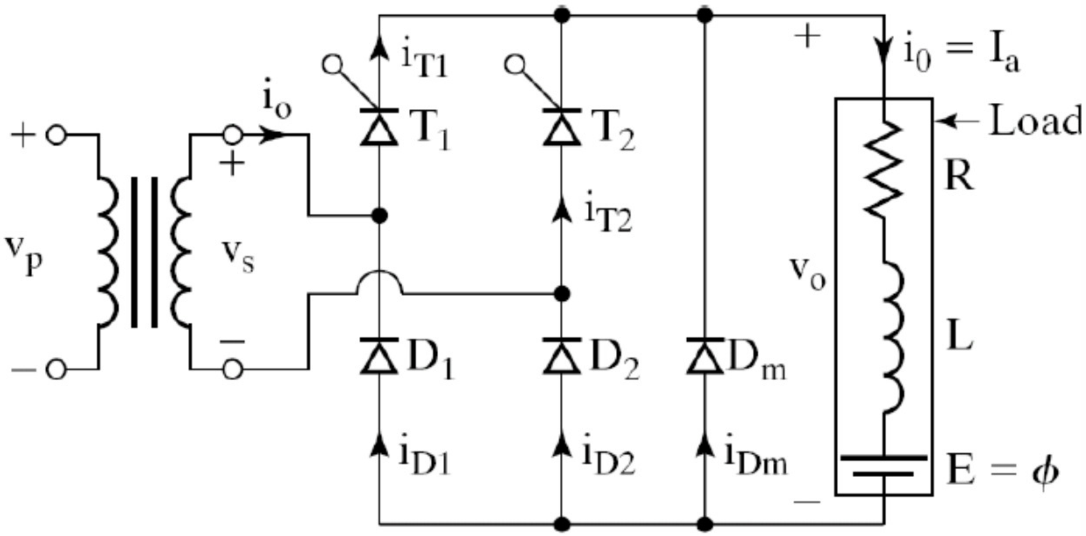
$$V_{O(RMS)}=V_m\left[\frac{1}{2\pi}\left\{(\pi)-\left(\frac{\sin 2\pi\times\cos 2\alpha+\cos 2\pi\times\sin 2\alpha-\sin 2\alpha}{2}\right)\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)}=V_m\left[\frac{1}{2\pi}\left\{(\pi)-\left(\frac{0+\sin 2\alpha-\sin 2\alpha}{2}\right)\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)}=V_m\left[\frac{1}{2\pi}(\pi)\right]^{\frac{1}{2}}=\frac{V_m}{\sqrt{2}}$$

$V_{O(RMS)}=\dfrac{V_m}{\sqrt{2}}$; The rms output voltage is same as the input rms supply voltage.

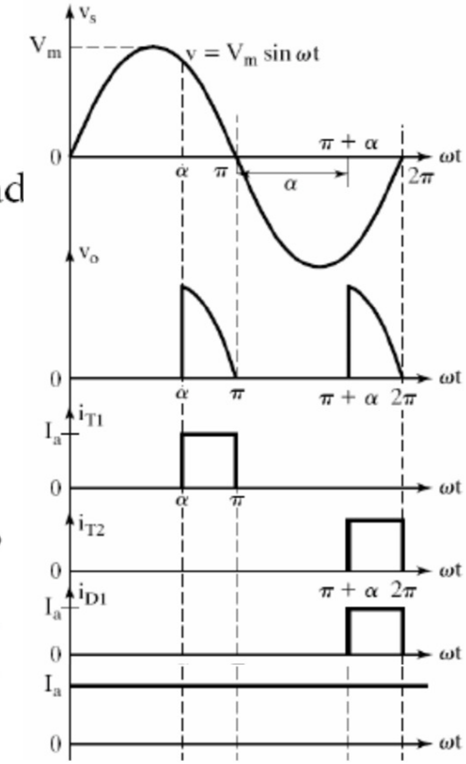
SINGLE PHASE SEMICONVERTERS

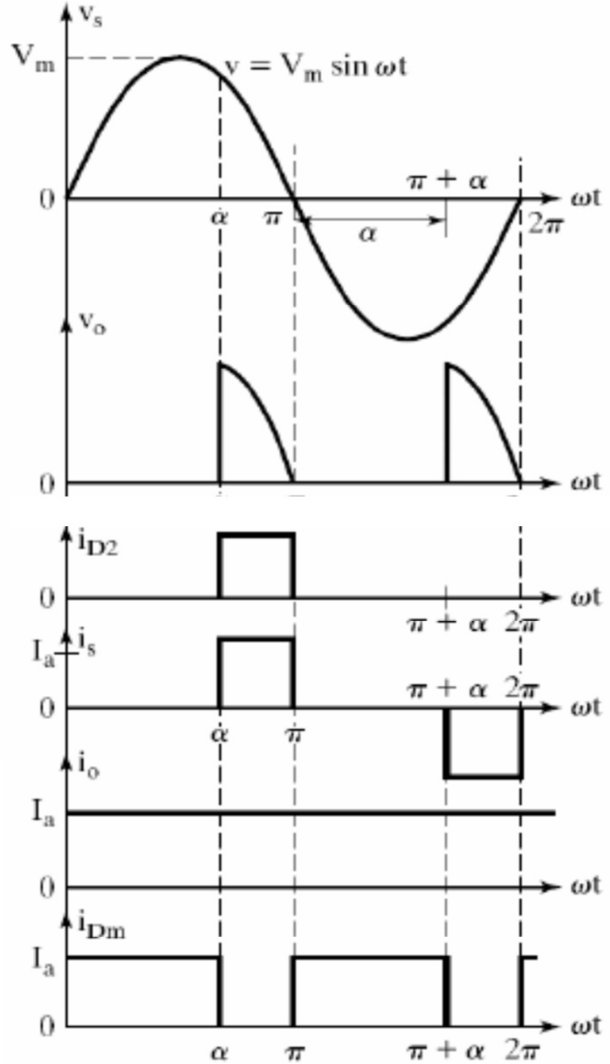


During the positive half cycle of input ac supply voltage, when the transformer secondary output line 'A' is positive with respect to the line 'B' the thyristor T_1 and the diode D_1 are both forward biased. The thyristor T_1 is triggered at $\omega t = \alpha$; $(0 \leq \alpha \leq \pi)$ by applying an appropriate gate trigger signal to the gate of T_1. The current in the circuit flows through the secondary line 'A', through T_1 , through the load in the downward direction, through diode D_1 back to the secondary line 'B'.

T_1 and D_1 conduct together from $\omega t = \alpha$ to π and the load is connected to the input ac supply. The output load voltage follows the input supply voltage (the secondary output voltage of the transformer) during the period $\omega t = \alpha$ to π .

At $\omega t = \pi$, the input supply voltage decreases to zero and becomes negative during the period $\omega t = \pi$ to $(\pi + \alpha)$. The free wheeling diode D_m across the load becomes forward biased and conducts during the period $\omega t = \pi$ to $(\pi + \alpha)$.





The load current is transferred from T_1 and D_1 to the FWD D_m. T_1 and D_1 are turned off. The load current continues to flow through the FWD D_m. The load current free wheels (flows continuously) through the FWD during the free wheeling time period π to $(\pi + \alpha)$.

TO DERIVE AN EXPRESSION FOR THE AVERAGE OR DC OUTPUT VOLTAGE OF A SINGLE PHASE SEMI-CONVERTER

The average output voltage can be found from

$$V_{dc} = \frac{2}{2\pi}\int_{\alpha}^{\pi} V_m \sin\omega t.d(\omega t)$$

$$V_{dc} = \frac{2V_m}{2\pi}\left[-\cos\omega t\right]_{\alpha}^{\pi}$$

$$V_{dc} = \frac{V_m}{\pi}\left[-\cos\pi + \cos\alpha\right] \; ; \; \cos\pi = -1$$

Therefore $$V_{dc} = \frac{V_m}{\pi}\left[1+\cos\alpha\right]$$

V_{dc} can be varied from $\frac{2V_m}{\pi}$ to 0 by varying α from 0 to π.

The maximum average output voltage is

$$V_{dc(\max)} = V_{dm} = \frac{2V_m}{\pi}$$

Normalizing the average output voltage with respect to its maximum value

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = 0.5\left(1+\cos\alpha\right)$$

TO DERIVE AN EXPRESSION FOR THE RMS OUTPUT VOLTAGE OF A SINGLE PHASE SEMI-CONVERTER

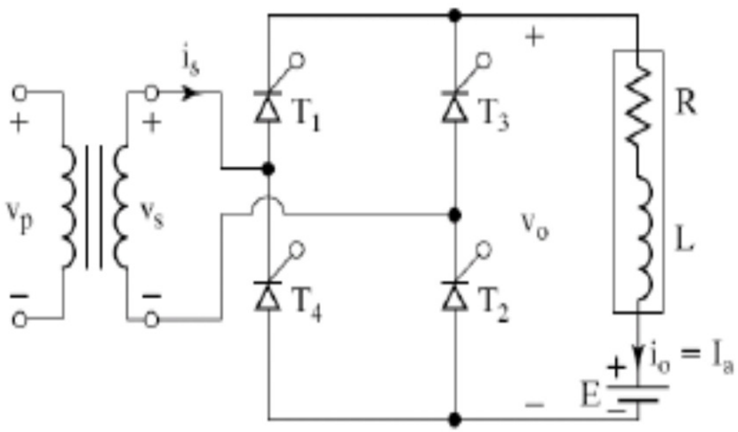
The rms output voltage is found from

$$V_{O(RMS)} = \left[\frac{2}{2\pi}\int_{\alpha}^{\pi} V_m^2 \sin^2\omega t.d(\omega t)\right]^{\frac{1}{2}}$$

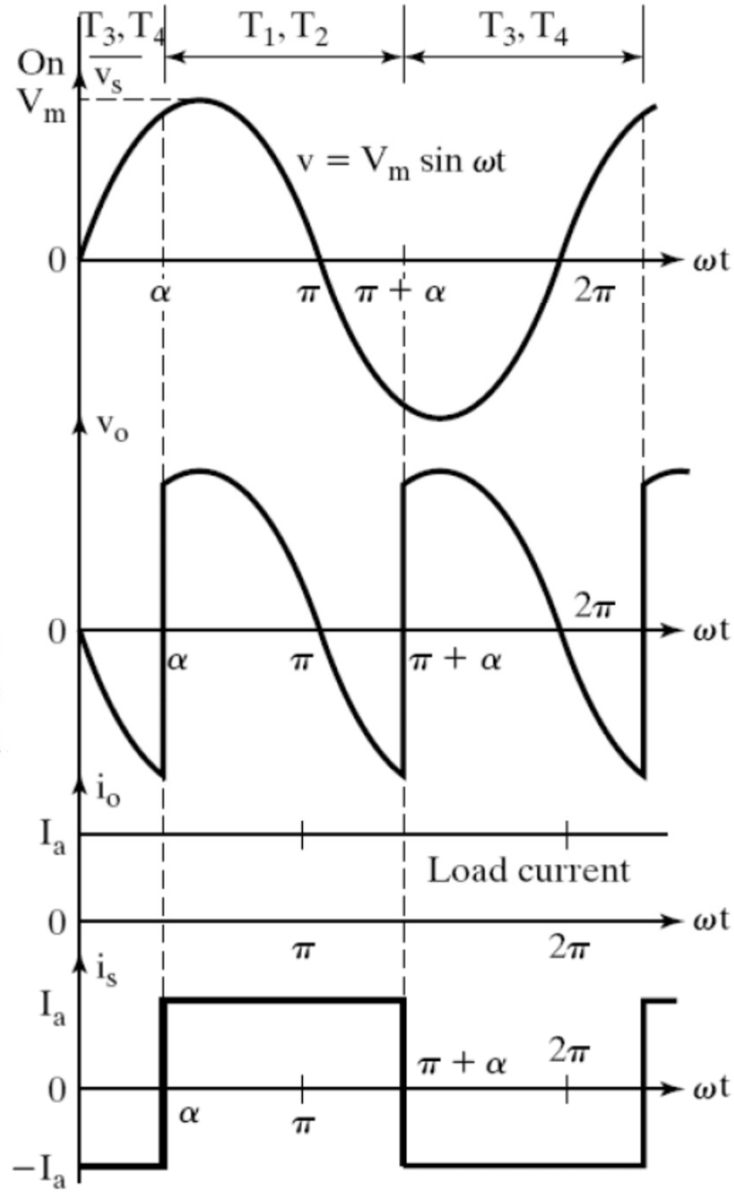
$$V_{O(RMS)} = \left[\frac{V_m^2}{2\pi}\int_{\alpha}^{\pi}(1-\cos 2\omega t).d(\omega t)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}}\left[\frac{1}{\pi}\left(\pi-\alpha+\frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$

SINGLE PHASE FULL CONVERTER (FULLY CONTROLLED BRIDGE CONVERTER)



The single phase full converter is extensively used in industrial applications up to about 15kW of output power. Depending on the value of trigger angle α, the average output voltage may be either positive or negative and two quadrant operation is possible.



TO DERIVE AN EXPRESSION FOR THE AVERAGE (DC) OUTPUT VOLTAGE

The average (dc) output voltage can be determined by using the expression

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi}\left[\int_0^{2\pi} v_O . d(\omega t)\right] \ ;$$

$$V_{O(dc)} = V_{dc} = \frac{2}{2\pi}\left[\int_{\alpha}^{\pi+\alpha} V_m \sin\omega t . d(\omega t)\right]$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi}\left[\int_{\alpha}^{\pi+\alpha} V_m \sin\omega t . d(\omega t)\right]$$

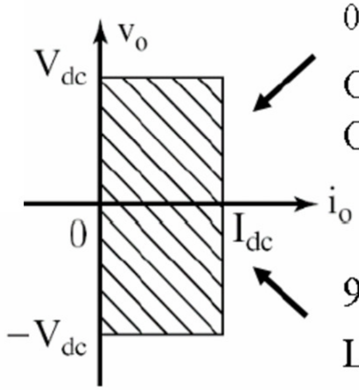
$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[\int_{\alpha}^{\pi+\alpha} \sin\omega t . d(\omega t)\right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[-\cos\omega t\right]_{\alpha}^{\pi+\alpha}$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[-\cos(\pi+\alpha) + \cos\alpha\right] \ ;$$

$$\cos(\pi+\alpha) = -\cos\alpha$$

Therefore $$V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi}\cos\alpha$$

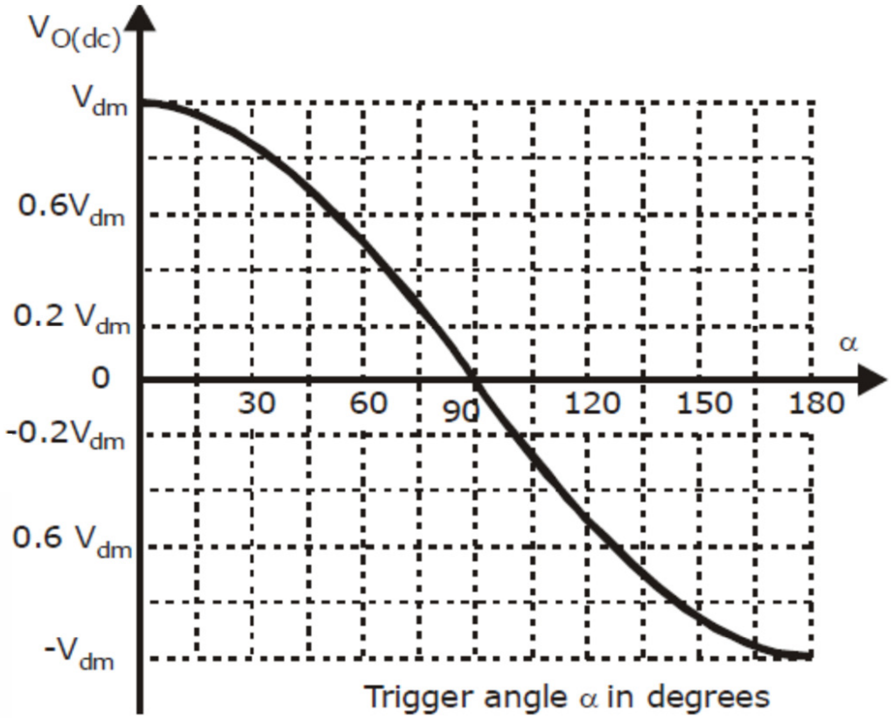


$0<\alpha<90^0$

Controlled Rectifier Operation

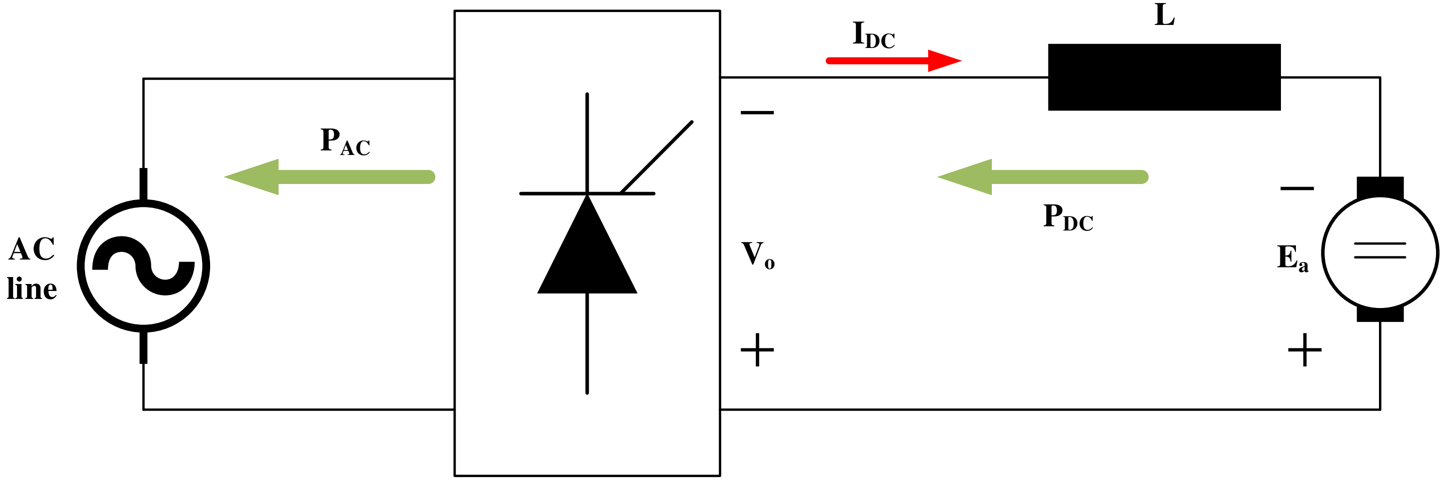
$90^0<\alpha<180^0$

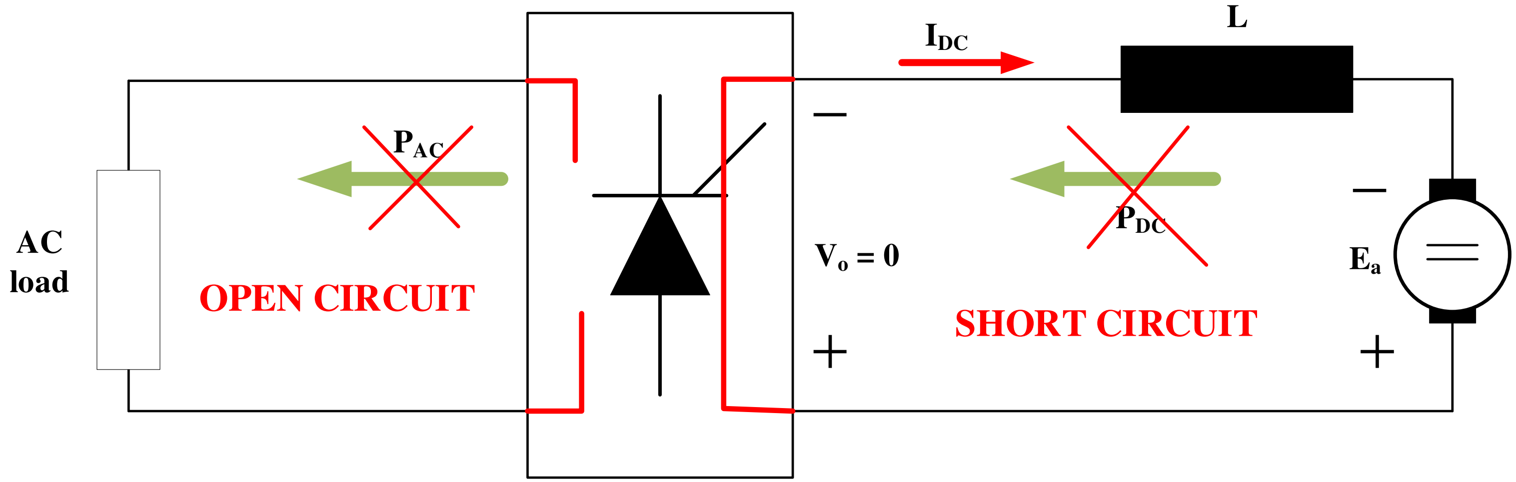
Line Commutated Inverter Operation



$V_{dm} = \left(\frac{2V_m}{\pi}\right)$	Maximum dc output voltage $V_{dc(\max)} = V_{dm} = \left(\frac{2V_m}{\pi}\right)$

LINE COMMUTATED INVERTER: No AC voltage production; power is transferred from DC → AC under the presence of AC line (line commutation)





TO DERIVE AN EXPRESSION FOR THE RMS VALUE OF THE OUTPUT VOLTAGE

The rms value of the output voltage is calculated as

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_0^{2\pi} v_O^2 . d(\omega t)\right]}$$

The single phase full converter gives two output voltage pulses during the input supply time period and hence the single phase full converter is referred to as a two pulse converter. The rms output voltage can be calculated as

$$V_{O(RMS)} = \sqrt{\frac{2}{2\pi}\left[\int_\alpha^{\pi+\alpha} v_O^2 . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{1}{\pi}\left[\int_\alpha^{\pi+\alpha} V_m^2 \sin^2 \omega t . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\pi}\left[\int_\alpha^{\pi+\alpha} \sin^2 \omega t . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\pi}\left[\int_\alpha^{\pi+\alpha} \frac{(1-\cos 2\omega t)}{2} . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[\int_\alpha^{\pi+\alpha} d(\omega t) - \int_\alpha^{\pi+\alpha} \cos 2\omega t . d(\omega t)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[(\omega t)\Big/_{\alpha}^{\pi+\alpha} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{\pi+\alpha}\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[(\pi+\alpha-\alpha) - \left(\frac{\sin 2(\pi+\alpha) - \sin 2\alpha}{2}\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[(\pi) - \left(\frac{\sin(2\pi+2\alpha) - \sin 2\alpha}{2}\right)\right]} \quad ; \quad \sin(2\pi+2\alpha) = \sin 2\alpha$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[(\pi) - \left(\frac{\sin 2\alpha - \sin 2\alpha}{2}\right)\right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}(\pi) - 0} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}}$$

Therefore $\quad V_{O(RMS)} = \dfrac{V_m}{\sqrt{2}} = V_S$

The rms thyristor current can be calculated as

Each thyristor conducts for π radians or 180^0 in a single phase full converter operating at continuous and constant load current.

Therefore rms value of the thyristor current is calculated as

$$I_{T(RMS)} = I_{O(RMS)}\sqrt{\frac{\pi}{2\pi}} = I_{O(RMS)}\sqrt{\frac{1}{2}}$$

$$I_{T(RMS)} = \frac{I_{O(RMS)}}{\sqrt{2}}$$

The average thyristor current can be calculated as

$$I_{T(Avg)} = I_{O(dc)} \times \left(\frac{\pi}{2\pi}\right) = I_{O(dc)} \times \left(\frac{1}{2}\right)$$

$$I_{T(Avg)} = \frac{I_{O(dc)}}{2}$$

THREE PHASE CONTROLLED RECTIFIERS

INTRODUCTION TO 3-PHASE CONTROLLED RECTIFIERS

Single phase half controlled bridge converters & fully controlled bridge converters are used extensively in industrial applications up to about 15kW of output power. The single phase controlled rectifiers provide a maximum dc output of $V_{dc(\max)} = \frac{2V_m}{\pi}$.

The output ripple frequency is equal to the twice the ac supply frequency. The single phase full wave controlled rectifiers provide two output pulses during every input supply cycle and hence are referred to as two pulse converters.

Three phase converters are 3-phase controlled rectifiers which are used to convert ac input power supply into dc output power across the load.

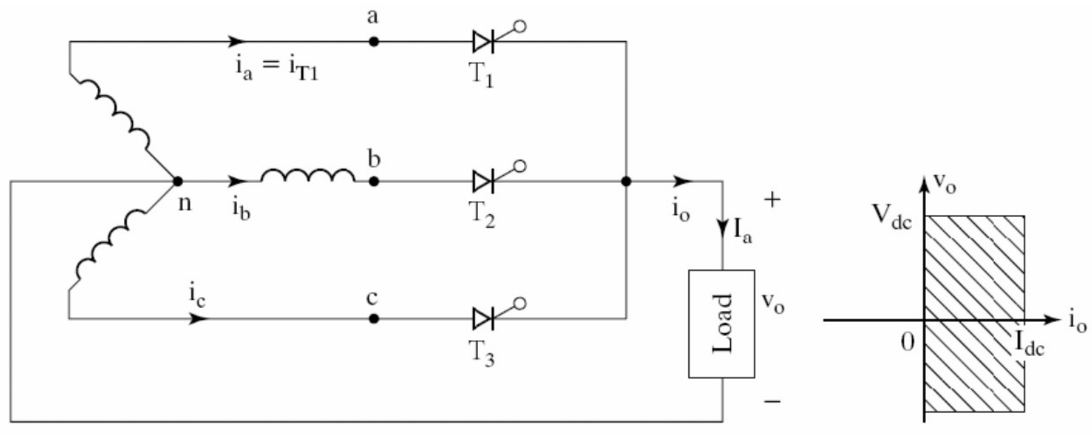
Features of 3-phase controlled rectifiers are

- Operate from 3 phase ac supply voltage.
- They provide higher dc output voltage and higher dc output power.
- Higher output voltage ripple frequency.
- Filtering requirements are simplified for smoothing out load voltage and load current

Three phase controlled rectifiers are extensively used in high power variable speed industrial dc drives.

3-PHASE HALF WAVE CONVERTER

Three single phase half-wave converters are connected together to form a three phase half-wave converter as shown in the figure.



(a) Circuit (b) Quadrant

THEE PHASE SUPPLY VOLTAGE EQUATIONS

We define three line neutral voltages (3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t; \quad V_m = \text{ Max. Phase Voltage}$$

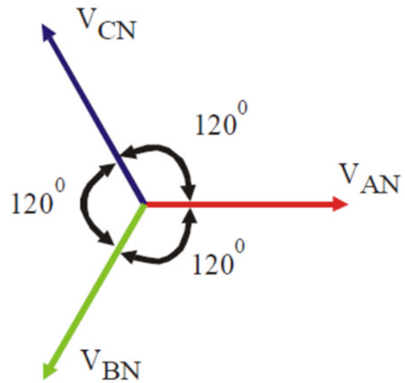
$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - 120^0\right)$$

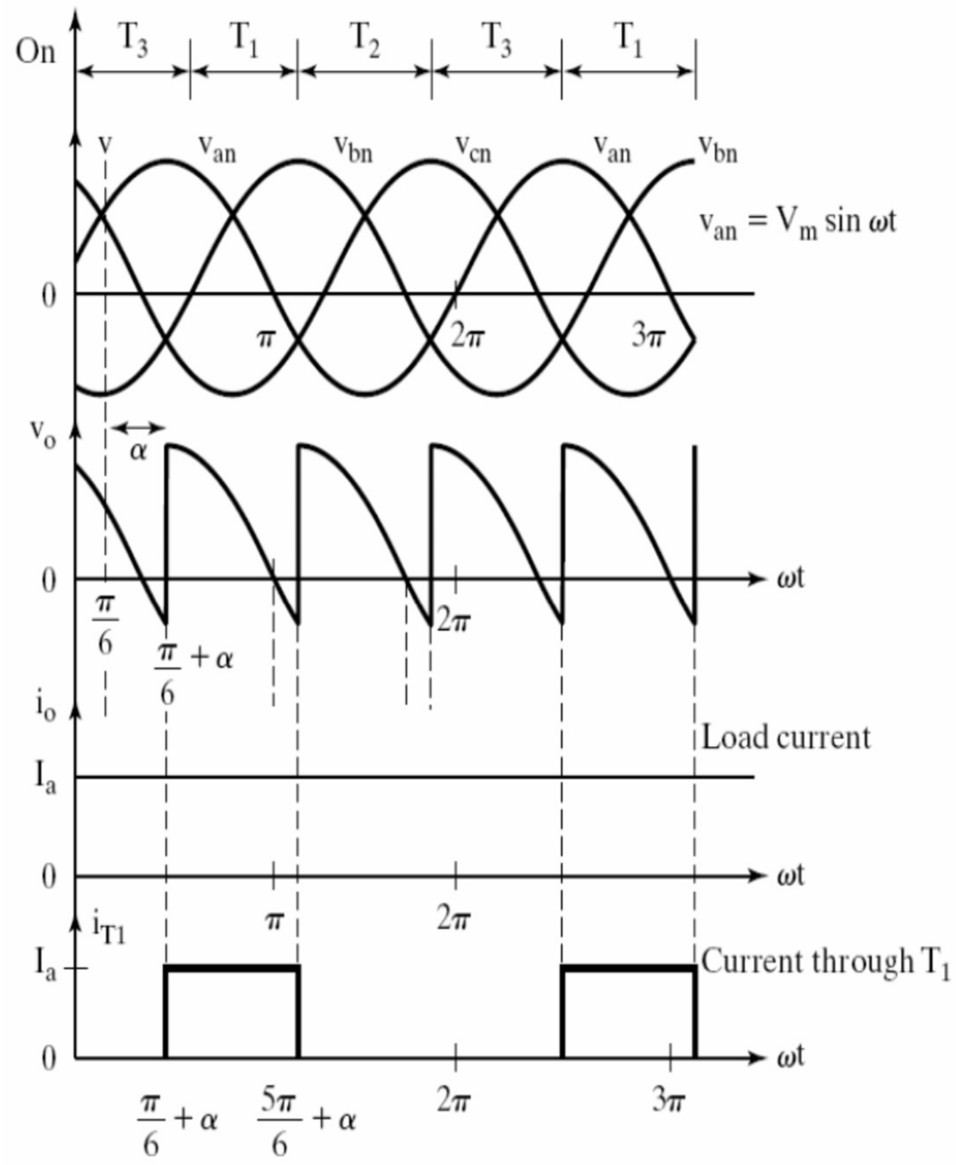
$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + 120^0\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t - 240^0\right)$$



Vector diagram of 3-phase supply voltages



(c) For inductive load

TO DERIVE AN EXPRESSION FOR THE AVERAGE OUTPUT VOLTAGE OF A 3-PHASE HALF WAVE CONVERTER FOR CONTINUOUS LOAD CURRENT

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} v_O.d(\omega t)\right]$$

Output voltage $\quad v_O = v_{an} = V_m \sin\omega t$ for $\omega t = (30^0 + \alpha)$ to $(150^0 + \alpha)$

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin\omega t.d(\omega t)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} \sin\omega t.d(\omega t)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[(-\cos\omega t)\Big/_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha}\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos\left(\frac{5\pi}{6}+\alpha\right)+\cos\left(\frac{\pi}{6}+\alpha\right)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\sqrt{3}\cos(\alpha)\right] = \frac{3\sqrt{3}V_m}{2\pi}\cos(\alpha)$$

As the output load voltage waveform has three output pulses during the input cycle of 2π radians

$$V_{dc} = \frac{3V_{Lm}}{2\pi}\cos(\alpha)$$

Where

$V_{Lm} = \sqrt{3}V_m =$ Max. line to line supply voltage for a 3-phase star connected transformer.

The maximum average or dc output voltage is obtained at a delay angle $\alpha = 0$ and is given by

$$V_{dc(\max)} = V_{dm} = \frac{3\sqrt{3}\ V_m}{2\pi}$$

V_m is the peak phase voltage.

And the normalized average output voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = \cos\alpha$$

TO DERIVE AN EXPRESSION FOR THE RMS VALUE OF THE OUTPUT VOLTAGE OF A 3-PHASE HALF WAVE CONVERTER FOR CONTINUOUS LOAD CURRENT

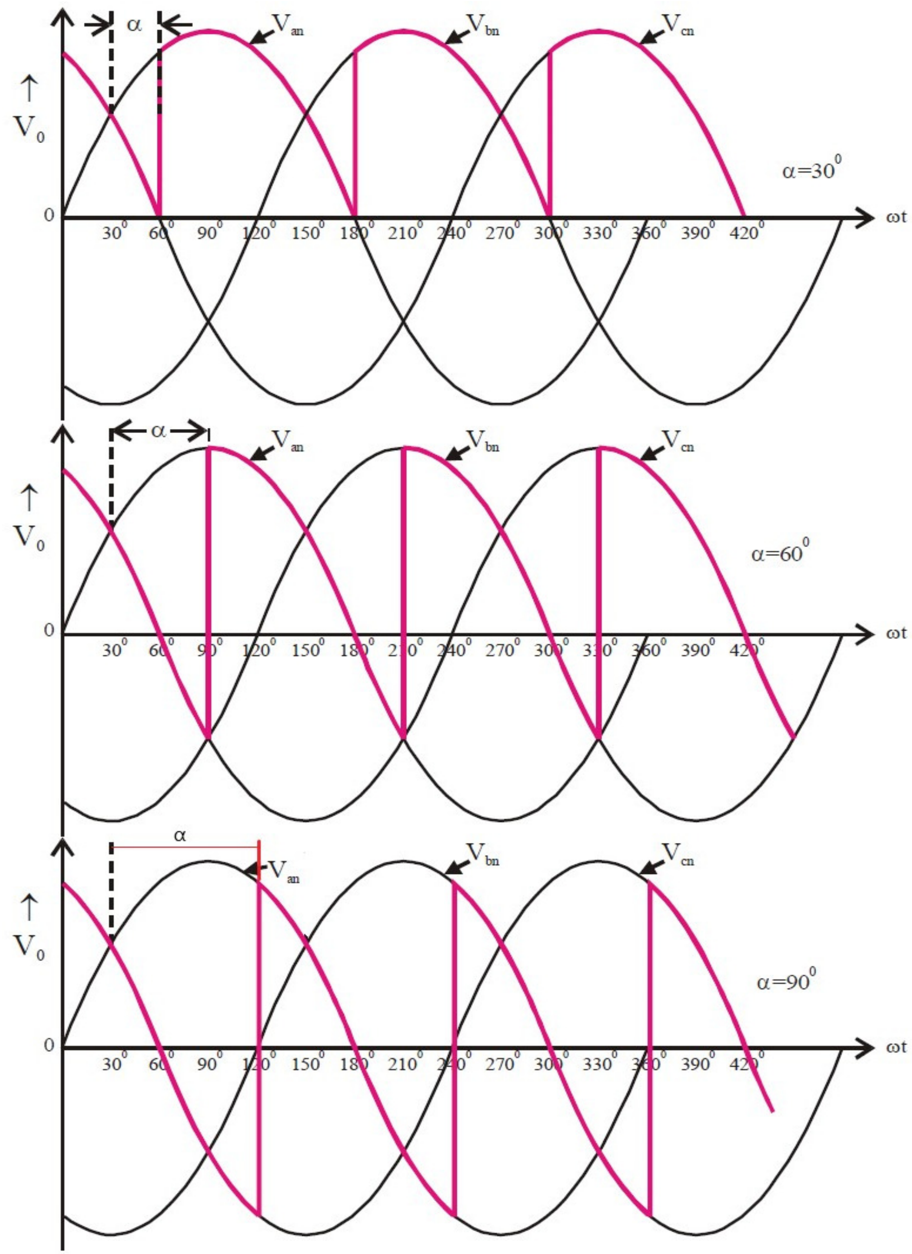
The rms value of output voltage is found by using the equation

$$V_{O(RMS)} = \left[\frac{3}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m^2 \sin^2 \omega t . d\left(\omega t\right) \right]^{\frac{1}{2}}$$

and we obtain

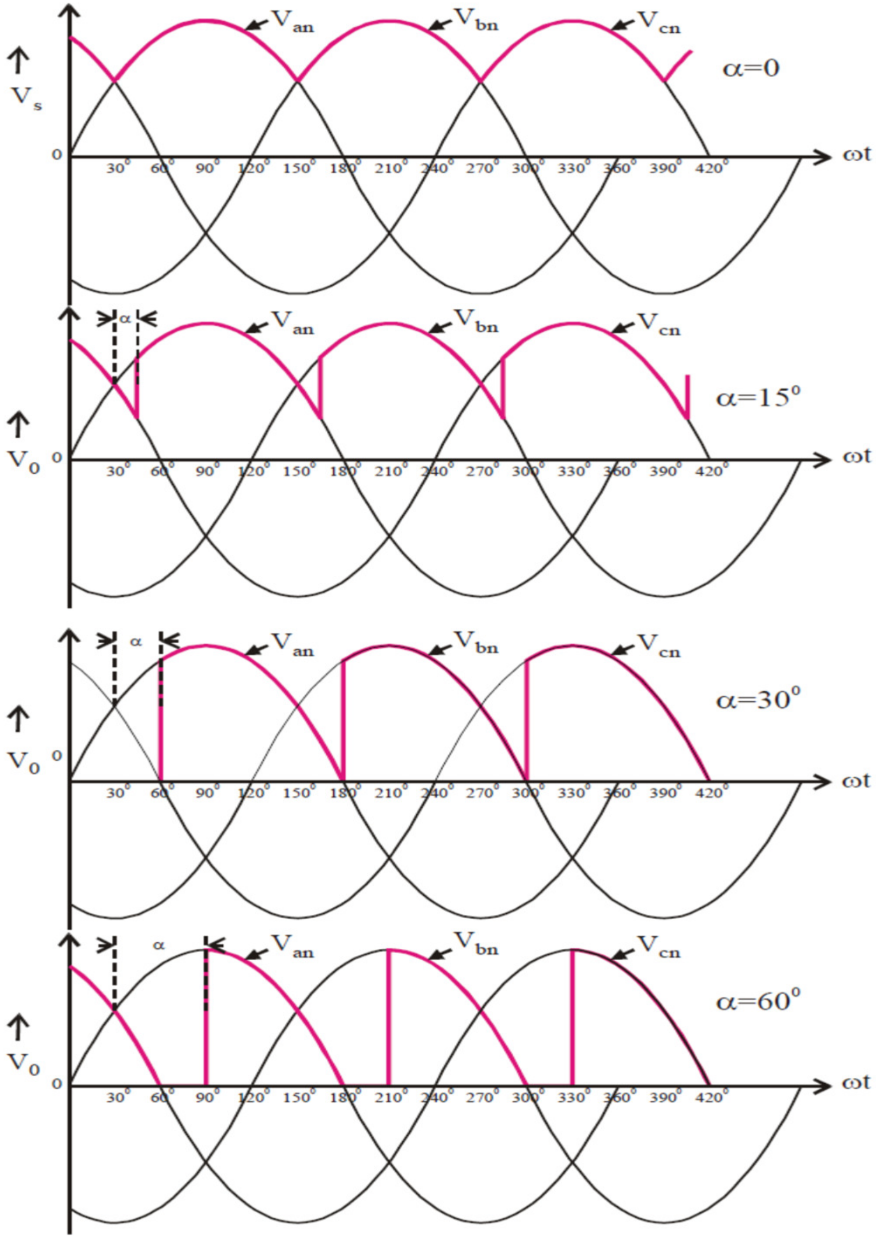
$$V_{O(RMS)} = \sqrt{3} V_m \left[\frac{1}{6} + \frac{\sqrt{3}}{8\pi} \cos 2\alpha \right]^{\frac{1}{2}}$$

3 PHASE HALF WAVE CONTROLLED RECTIFIER OUTPUT VOLTAGE WAVEFORMS FOR DIFFERENT TRIGGER ANGLES WITH RL LOAD



3 PHASE HALF WAVE CONTROLLED RECTIFIER OUTPUT VOLTAGE WAVEFORMS FOR DIFFERENT TRIGGER ANGLES WITH R LOAD

$$V_{dc} = \frac{3V_m}{2\pi}\left[1+\cos\left(\alpha+30^0\right)\right]$$



The same voltage waveforms and dc-value formula stands for the case of half controlled (semiconverter) 3-phase half wave rectifier, regardless load nature (R or RL).