PROSPECTIVE MATHEMATICS TEACHERS' NOTICING OF CLASSROOM PRACTICE THROUGH CRITICAL EVENTS

Despina Potari*, Giorgos Psycharis*, Eirini Kouletsi* and Maria Diamantis**
*University of Athens, Greece, ** Southern Connecticut State University, USA

This paper reports on a study conducted to explore prospective mathematics teachers' reflections on teaching practice at the secondary level through noticing key aspects of classroom interactions. The study used critical incidents taken from everyday classroom situations as a means to make the act of noticing more concrete. The participating prospective teachers were engaged in a number of different activities including observing, designing and teaching. The results indicate a progression of prospective teachers' noticing of classroom practice and development of teaching awareness marked by shifts in analysing and interpreting classroom events.

Key words: reflection, noticing, critical incidents, teaching awareness

INTRODUCTION

In this paper we aim to tackle the theory-practice problem in mathematics teacher education (cf. Mason, 2002, Jaworski, 2006) by exploring prospective teachers' reflection on teaching practice through noticing key aspects of everyday classroom situations. Our approach involves the use of critical incidents (Goodell, 2006) as the means by which noticing -and thus reflection on teaching practice- is facilitated to emerge. Our theoretical position towards reflection is based on Jaworski's (1998) interpretation of Dewey's definition of reflective thinking, i.e. "firstly, a recognition of questions to address, identifying some perplexity, making some aspects of teaching problematic; and, secondly, through some processes of enquiry, to seek solutions, or resolutions to, or new ways of understanding, the problems identified." (ibid., p. 7). In resonance with a number of current research approaches (c.f., Scherer & Steinbring, 2006, Jansen & Spitzer, 2009) we see noticing as an activity involving description, analysis and interpretation of teaching practice, thus creating a framework for reflection. A number of research approaches have indicated a number of difficulties that prospective teachers face while engaged in reflection on classroom interactions (eg. collecting evidence about students' learning as well as developing interpretative analysis of classroom instruction (Morris, 2006)). A recent focus on the prospective teachers' reflections on critical incidents taken from classroom situations (Goodell, 2006) supports the idea that critical incidents can be a powerful tool towards promoting prospective teachers' reflective practices to develop. In our study, we used critical incidents as means to engage prospective teachers in reflecting on teaching practice so that they could learn to attend to their students' thinking, interpret classroom phenomena and start to develop ideas of alternative teaching actions. This paper demonstrates how this approach can facilitate

the progression of prospective teachers' noticing of classroom practice and development of teaching awareness marked by shifts in interpreting classroom events.

THEORETICAL FRAMEWORK

Two main bodies of research informed our study: the first, concerns reflective thinking through *noticing* of classroom interactions in teacher education, and the second, *critical incidents* of classroom practice as a concrete basis for reflection and interpretation. In his elaboration of the idea of noticing in teachers' professional development, Mason (2002) pointed out the importance of teachers' attention on the students' learning processes as well as to the teachers' self-observation practices in the classroom. In this view, *noticing* has been related to systematic reflection on acts or issues, leading to shifts in the structure of attention and, through this, to different levels of awareness both in mathematics and in mathematics teaching. In the research reported by Scherer and Steinbring (2006), noticing of students' learning processes was at the core of the joint reflection of teachers and researchers. The analysis suggests that a critical step towards a positive change of teaching activity consists in moving the dominating focus of mathematical interaction in teaching from the teacher to the learning students.

In the domain of preservice teacher education existing research studies suggest that it may be possible to help prospective teachers engage in reflective thinking through noticing so as to enhance their ability to focus on key aspects of teaching practice. Morris (2006) reported that under the condition to form hypothesis about the sources of students' difficulties in a videotaped mathematics lesson, prospective teachers appeared to be able to develop claims and conjectures about the connection between specific instructional activities and students' mathematical understanding. Spitzer et al. (in press) reported that a rather short classroom intervention (2 lessons) involving joint reflection and discussion on written classroom transcripts provided by the researchers, produced substantial improvement in prospective teachers' ability to identify and analyze evidence of student understanding. Similar findings were also reported by research studies in which prospective teachers were engaged in reflecting on their own teaching. The results revealed that reflective activities served as transition mechanisms that promoted prospective teachers' awareness of the need to monitor student understanding during the lesson (Artzt, 1999) and develop hypothesis when interpreting how their teaching affects their students' learning (Jansen & Spitzer, 2009).

Along with a focus on prospective teachers' reflective practices, researchers have been concerned about the introduction of sufficient structures for making the act of inquiry into teaching practice more concrete. A particular example of a structured framework for reflection on classroom episodes are *critical incidents*, i.e. everyday classroom events which are significant for the teachers, make them question their

practice and seem to provide an entry for their better understanding of teaching-learning situations (Hole & McEntee, 1999). Recent research focus on the use of critical incidents in pre-service teacher education (Goodell, 2006) supports the idea that critical incidents can be considered as a means to facilitate prospective teachers' productive engagement in noticing teaching events and critically reflecting on them. In resonance with this approach, in this study we were interested to stimulate prospective teachers' noticing through critical events and see if and how the developing process of selecting and reflecting on critical incidents in different contexts (i.e. observing, designing and teaching) might promote changes in the prospective teachers' stance towards analysing and interpreting classroom events.

METHODOLOGY

Context of the study and participants

The research took place in the context of a 16-week mathematics education undergraduate course (taught in one semester by the first author, mentioned as teacher educator in this paper) at the University of Athens in Greece. The philosophy of the course was to link theory-driven instruction on the teaching and learning of mathematics at the secondary level with realisation of mathematics teaching in real classroom settings. The aim was to engage prospective students in critically consideration of aspects of mathematics teaching as they emerge from the complexity of teaching practice in schools. Every second week (for the entire semester) prospective teachers were asked to participate in a number of field activities such as to observe other teachers' course in cooperating schools, to conduct a didactical intervention in one group of students and to design and implement lessons in the classroom. Each week following the field activities-week included a 3-hour class session taking place at the University. Instructional practice in this session aimed to support prospective teachers' reflective activities on their recent field experience and to link emergent issues with existing mathematics education research. The 22 prospective teachers (9 males, 13 females) who served as participants in this study were divided in pairs and carried out collaboratively the field activities under the supervision of 8 experienced secondary mathematics teachers who served as mentors. Apart from the teacher educator, the research team consisted of two more researchers of mathematics education (the second and fourth author) and an experienced teacher (the third author) who acted as mentor-researcher in the study. Enrolling in the course, prospective teachers had a background of undertaking at least four other mathematics education courses as a part of their teacher education program at the University. Most of them also parallel to their university studies were helping school students on a private base with their mathematical homework.

Research design and data sources

Reflection on teaching practice through noticing of classroom events in the framework of critical incidents was the foundation of our research design. Critical

incidents were considered as a methodological tool for triggering prospective teachers' reflection on teaching practice. In the first class meeting, prospective teachers were introduced to the idea of critical incidents mostly through examples and also by reference to teacher education research. In the next class meeting and in the first half of the third, the groups of prospective teachers completed tasks that required them to identify why some classroom incidents "provided" by the teacher educator (e.g. transcripts of lessons or videotaped teaching episodes) could be considered as critical, discuss the features of those incidents, and finally analyze and interpret them. These incidents were considered as critical by the researchers as they could indicate an important teaching and learning moment (eg. an unexpected student's response, an "effective" classroom interaction etc.) In the second half of the third class meeting and in all the subsequent ones, the groups of prospective teachers were asked to select and present in the next class session a critical incident that represents an unexpected situation that they had experienced during their fieldwork activities (i.e. observation of lessons, didactical interventions, design and implementation of lesson plans). It was expected that these presentations and the subsequent class discussions would provide a fruitful terrain for studying the development and evolution of prospective teachers' noticing through critical incidents in different contexts. All class sessions (8 in total) were video recorded. The data for this study was conducted over the entire semester, and consisted of: (a) prospective teachers' personal portfolios including their written accounts of critical incidents and material related to the design, implementation and presentation of their field activities in the classroom (e.g. worksheets, lesson plans, presentation files); (b) video recordings of all class sessions at the University; (c) audio-recordings of interviews with some of the prospective teachers regarding their field activities, and (d) researchers' field notes. For the analysis verbatim transcriptions of all recordings were made. The analysis presented in this paper is based only on the video transcripts of the class sessions.

Data analysis

In this study we worked broadly through a grounded theory approach (Strauss & Corbin, 1998) as our goal was rather exploratory. The unit of analysis was the episode, defined as an extract of interactions performed in a continuous period of time around a particular issue. The episodes which are the main means of presenting and discussing the data were selected (a) to involve prospective teachers' interactions on student learning according to an unexpected teaching event and (b) to represent indications of emerging shifts in prospective teachers' noticing of classroom interactions involved in the episode.

RESULTS

The prospective teachers identified a variety of critical incidents throughout the activities of the course. The issues that were addressed by them concerned mainly

students' difficulties; classroom management; curriculum and textbooks, overall educational and social context. In this paper, we report findings indicating the shifts in analysis and interpretations of students' learning that occurred during the university class meetings.

Noticing students' learning in teaching examples provided by the researchers

In the initial class sessions the prospective teachers' comments and interpretations of critical incidents were mostly related to students' misconceptions and to ineffective teacher's strategies. They appeared to attribute these problems either exclusively to students' responsibility or to wider social factors that framed teaching and learning. Moreover, these factors seemed to have been considered in a fragmented way despite of the teacher educator's attempts to encourage prospective teachers to see teaching and learning at its complexity. We provide below some evidence of some of the issues described above by referring to an extract from the second classroom meeting. The teacher educator presented a task related to students' proofs for the statement: "The sum of two consecutive odd numbers is divisible by four". (taken from Boero and Guala, 2008). One 14- year- old student provided the following response:

"By making some trials like for instance, 3+5, 15+17, 31+ 33 I realise that I always get sums made by the first odd number and by the same odd number increased by two, thus I get the double of an odd number plus two. This result is divisible by four because the sum of two equal odd numbers would be (alone) an even number divisible only by two, but if I add two I get the consecutive even number, which is divisible by four because even numbers follow each other with the rule that if one is divisible only by two, the following one is divisible by four (like: 2, 4; 6, 8; 22, 24; etc) because the multiples of four are four units far from each other" (ibid, p.238)

Initially, prospective teachers considered student's reasoning empirical:

"The student makes an attempt to generalise but constructs some rules that hold for small numbers but then he concludes arbitrary that this is true for all" (Lefteris, 2nd class meeting)

Later on in their attempts to develop a better understanding of the student's thinking they started to consider much deeper issues such as the symbols' use in a mathematical proof, what constitutes a mathematical proof, and the distance between curriculum demands and students' mathematical understanding. The following example indicates prospective teachers' initially rather narrow perspective about the nature of mathematical proof:

"It is like another example we had seen in the previous lesson where the student could not use the symbols. Although this student seems to understand what the answer is and how more or less to get it, this is not a mathematical proof... it does not have operations and relations" (Ioanna, 2nd class meeting)

Although in this part of the discussion the teacher educator's intervention was minimal, the prospective teachers started to express different opinions from their

initial ones indicating their appreciation of student's reasoning and their efforts to provide an adequate justification (2nd meeting):

Spirithoula: We need to remember that this student is only in the 9th grade and he does not have yet the experience to write the even number in the form of 2k and the odd in the form of 2k+1, so that to construct an accurate mathematical proof. I think that for a student of that age the whole thinking was very good

Adriana: I would also agree, let's not forget that it took ages to develop the formalism ...

Oliver: Diofantus also did not use algebraic symbols...

Adriana: How can a student of that age construct such a proof?

Although the class seemed to come to a consensus, one prospective teacher reminded the audience that the mathematics curriculum suggested a formal approach to problems of that kind. This creates some tensions again:

"I completely disagree. It is very good that a student does something different from what he has been taught. Following a problem solving method mechanically is not good for his future mathematical development. It is more important to encourage him to make explorations." (Oliver, 2nd meeting)

Through the above comment, Oliver brought to the foreground the critical role of exploration in students' mathematical development in the long term.

Prospective teachers' attention to students' thinking in this context seemed to have made a number of shifts in the ways they analysed students' understanding. In particular, they started to recognise students' reasoning beyond the formality of the symbols and to view it as an integral part of students' mathematical future development.

Noticing students' learning in classroom observations

In the third, fourth and fifth meeting the prospective teachers commented on critical incidents which they had noticed in the classroom observations. A variety of issues emerged in the discussions such as conceptual and procedural learning, students' difficulties to make connections between different representations, the relation between the nature of teacher's questions and students' answers, curriculum and wider social issues and their impact on learning and teaching, the role of students' prior knowledge in learning and the effectiveness of specific teaching examples and tasks (e.g. the use of paradoxes, the connection among different content areas). In the discussions a central issue was the construction of mathematical meaning. Initially the focus was on students' mistakes that were due to lack of understanding. However, during the discussion the prospective teachers started to link this phenomenon to teachers' choices (examples, tasks, questions) and to research findings. We will try to indicate these shifts by using examples from our data.

A number of critical incidents that the prospective teachers presented were related to the fact that the students often apply a method to solve a mathematical task without understanding the underlying properties. Some examples were the transformation of a fraction to an equivalent one, the solution of a first degree equation, arithmetic or algebraic computations. For example, one critical incident reported by a prospective teacher was about a classroom interaction between two students concerning the transformation of the fraction 7/5 to its equivalent with 30 as denominator. The first student completed the transformation by multiplying both terms of the fraction by 6. Then the second student wondered why he did not use a faster common technique based on the use of the appropriate factor that is "kept" in a place over the nominator. The prospective teacher interpreted the phenomenon by considering this technique as a "picture" in the student's mind which might provide a barrier to conceptual understanding:

"The second student seems to have clear in his mind a picture without knowing why this method works, the essence of the method" (Kostas, 3rd class meeting).

In a subsequent stage of the discussion, the teacher educator attempted to move the class attention on how to deal effectively with the situation in order the make the meaning of this specific technique transparent to the student. Kostas stated that he would ask him to reflect on his actions "What are you actually doing?" "In what way your approach is different from your peer's?". Another prospective teacher recalled from his fieldwork observation how another teacher managed a similar situation. Instead of stressing the rule "change side, change sign", commonly used, in solving algebraic equations, he emphasized the properties involved in the solution process. The prospective teacher found this approach original as it was beyond his own experiences:

"There was not the method of moving it to the other side and change its sign but the teacher was emphasizing that we do the inverse operations. I find this approach very different, more advanced "(Lefteris, 3rd class meeting)

In a subsequent class meeting, the prospective teachers themselves started to build connections between learning, teaching and research. They had been asked to find and read a research paper that would help them to develop adequate explanations of the fact that students often do things at an operational level without deeper understanding of the underlying properties. In terms of students' learning the they managed to give deeper interpretations by realising the meaning of the variable, the double meaning of the equal sign and the transition that the students needed to make from arithmetic to algebra. In terms of teaching they identified tasks such as a tree diagram that could help students to understand the priority of operations and use it for solving equations or they talked about the emphasis needed to be given on algebraic structures in arithmetic. Some typical comments were:

"The students need to understand that a variable is an element of a set, something like this." (Ioanna, 5th class meeting)

"We read about a tree diagram that helps students to read and see how the algebraic relation is structured and it also uses a computer program to represent it." (Lefteris, 5th class meeting)

By summarising, in this context the prospective teachers extended their own examples about mathematics teaching and learning and started to reconsider and evaluate the effectiveness of some of the teaching approaches they experienced as school students. We also noticed deeper interpretations of students' mathematical contributions by relating them to the research findings. Finally, they started to focus on the role of teaching practices to the development of learning and identify fine elements of teaching.

Noticing students' learning in prospective teachers' teaching

In the last three class meetings the prospective teachers presented critical incidents from their own teaching. The critical incidents were related to students' difficulties or unexpected responses, to the appropriateness of the designed tasks, to epistemological aspects and to classroom management. Almost all the prospective teachers participated in the discussion by presenting and justifying their critical incidents as well as by challenging their peers' interpretations and claims. In this phase, the main part of the class discussions was based on the prospective teachers' interactions. Some of the main issues that emerged were: a teacher's difficulty to notice students' learning; the problem of time; the connection among different representations; the difference between procedural and conceptual understanding; the management of students' different mathematical backgrounds and interests; the difficulty to design a mathematically challenging task consistent with students' cognitive and affective needs; and the epistemological characteristics of geometry.

In terms of students' learning the prospective teachers' interpretations focused more on the students' strategies and thinking processes rather than on their difficulties and errors. Moreover, they often seemed to overtly recognise the critical role of tasks in challenging students' mathematical thinking. We are giving below some examples from prospective teachers' reflections.

In the 7th class meeting, Katerina talked about what she learned from her 8th grade students while working on a task she had designed for comparing the areas of three irregular polygonal areas:

"We wanted to see how the students were thinking while they were dividing the areas to regular shapes. We let the students to work on their own. I had expected them to develop three or four different strategies but when I analysed them afterwards I discovered that they were twelve!...What I have understood is that when you let the students to work on themselves they have a lot of different ideas. We can also see how they are thinking... All the students in the class had done something (Katerina, 7th class meeting)

Katerina recognised that students' thinking can be very powerful through the analysis of their strategies. Moreover, she acknowledged the importance to provide space to all students to think mathematically during the teaching. In the discussion that followed the other prospective teachers also commented on the added pedagogical value of students' multiple solutions of a mathematical task:

"I think that it has to do with the nature of the tasks. A very specific task does not allow for multiple solutions and answers. So, I do not have to ask questions that have as answer "yes" or "no". We need to ask why. (Spirithoula, 7th meeting)

In the last class meeting Aggeliki and Maria presented a critical incident from their teaching in a 9th grade class. Their teaching goal was for the students to make sense of the algebraic formula $(a+b)^2 = a^2+2ab+b^2$ through a geometrical task they had developed. One student who was engaged in calculating areas in the geometrical context he recalled the formula without connecting it to the problem. The two prospective teachers did not expect this response and they interpreted that the student did not make any connection to the problem but he only recalled the relation without understanding:

"I expected to hear that the area of the total land was the sum of the four rooms and he gave me the algebraic formula." (Aggeliki, 8th class meeting)

"I used to do this when I was at school. The teacher was telling me something and when I did not know it I was giving him the formula I knew" (Maria, 8th class meeting)

In their attempt to interpret student's approach the prospective teachers were trying to go more deeply to student's thinking process. For example Lefteris mentioned the fact that the student worked at the operational level and could not see the relation structurally:

"The student says that the solution of this relation is... he does not see the equivalence of the two quantities, he only sees that he will expand the $(a+b)^2$ and he will find the result. He has acted only procedurally." (Lefteris, 8th class meeting)

Overall, by reflecting on their actual teaching the prospective teachers seemed to focus on key aspects of student's learning and to relate it to features of the tasks (e.g. openness, kind of representations).

CONCLUDING REMARKS

Our purpose in this paper was to illustrate a particular approach to encourage and study prospective teachers' reflection on teaching practice by noticing key aspects of classroom interactions through critical incidents. The results indicate a progression of prospective teachers' noticing of classroom practice marked by shifts in the analysis and interpretation of critical incidents. An initial analysis of students' thinking at a surface level has gradually been moving towards considering salient features of the learning process. Towards the last class sessions prospective teachers seemed to be able to make connections between students' learning with particular

aspects of teaching practice. Finally, this process seemed to be carried out through the integration of different sources of knowledge such as prospective teachers' tacit knowledge about teaching from their experiences as students and private tutors and the academic knowledge they were developing at the University course.

REFERENCES

Artzt, A. F. (1999). A structure to enable preservice teachers of mathematics to reflect on their teaching. *Journal of Mathematics Teacher Education*, 2, 143–166.

Boero, P. & Guala, E. (2008). Development of mathematical knowledge and beliefs of teachers. In P. Sullivan and T. Wood (eds.), *Knowledge and Beliefs in Mathematics Teaching and Teaching Development* (pp. 223-244). Rotterdam: Sense Publishers

Jaworski B. (2006). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education*, 9, 187-211.

Goodell, J. E. (2006). Using critical incident reflections: A self-study as a mathematics teacher educator. *Journal of Mathematics Teacher Education*, 9, 221-248.

Hole, S., & McEntee, G. H. (1999). Reflection is at the heart of practice. *Educational Leadership*, 56, 34–37.

Jansen, A. & Spitzer, S. M. (2009). Prospective middle school mathematics teachers' reflective thinking skills: descriptions of their students' thinking and interpretations of their teaching. *Journal of Mathematics Teacher Education*, 12, 133–151.

Jaworski, B. (1998). Mathematics teacher research: process, practice and the development of teaching. *Journal of Mathematics Teacher Education*, 1, 3-31.

Mason, J. (2002). *Researching Your own Practice: The Discipline of Noticing*. London: Routledge Falmer.

Morris, A. (2006). Assessing pre-service teachers' skills for analyzing teaching. *Journal of Mathematics Teacher Education*, 9(5), 471–505.

Scherer, P. & Steinbring, H. (2006). Noticing children's learning processes – teachers jointly reflect on their own classroom interaction for improving mathematics teaching. *Journal of Mathematics Teacher Education*, 9, 157–185.

Spitzer, S. M., Phelps, C. M., Beyers, J. E. R., Johnson, D. Y. & Sieminski, E. M. (in press). Developing prospective elementary teachers' abilities to identify evidence of student mathematical achievement. *Journal of Mathematics Teacher Education*, 9, 157–185.

Strauss, A. & Corbin, J. (1998). *Basics of Qualitative Research*, Sage Publications.