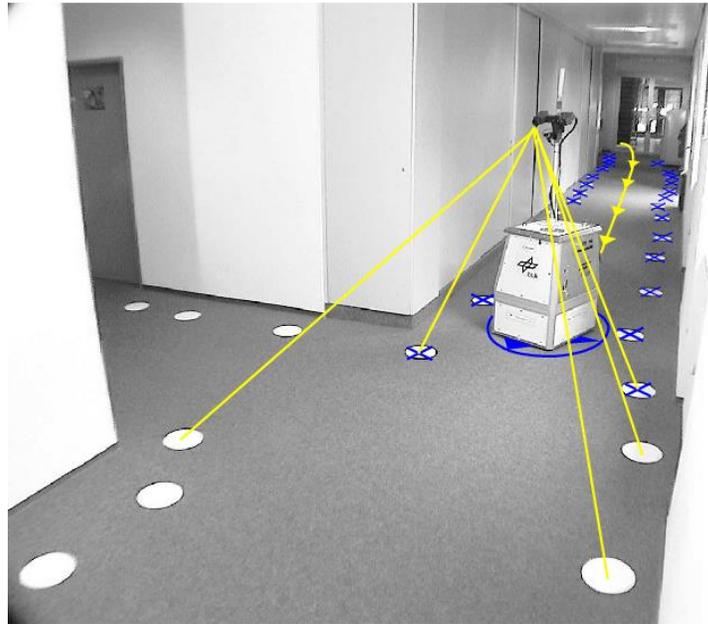


Robust Mechatronics

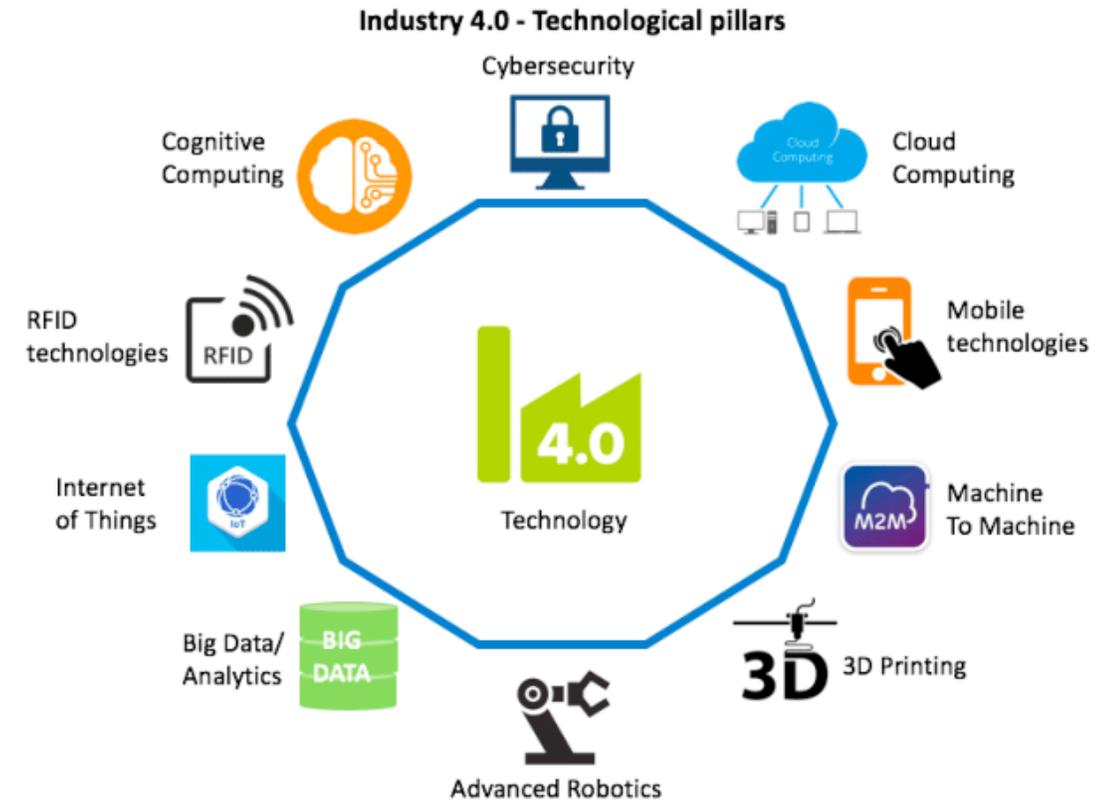
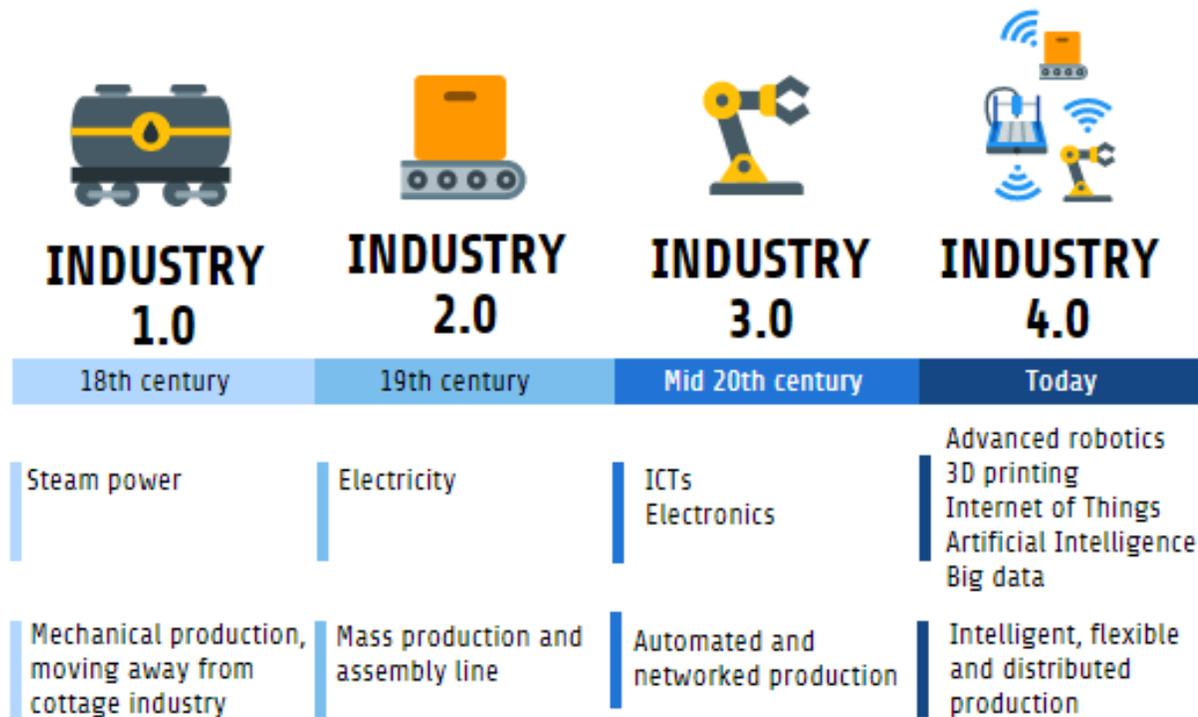
Localization and Mapping for Autonomous Mobile Systems



Dr Loukas Bampis, Assistant Professor
Mechatronics & Systems Automation Lab

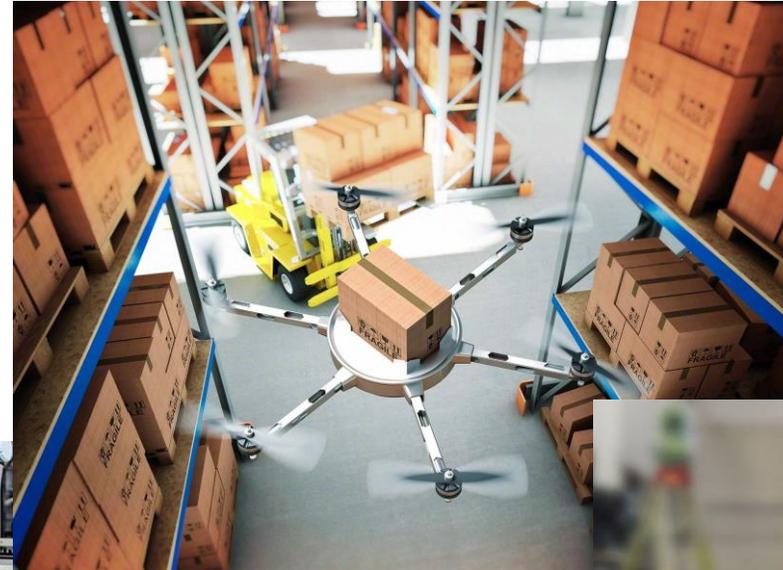
Localization and Mapping for Autonomous Mobile Systems

Industry 4.0 and the Age of Mobile Robotics



Localization and Mapping for Autonomous Mobile Systems

Industry 4.0 and the Age of Mobile Robotics



Localization and Mapping for Autonomous Mobile Systems

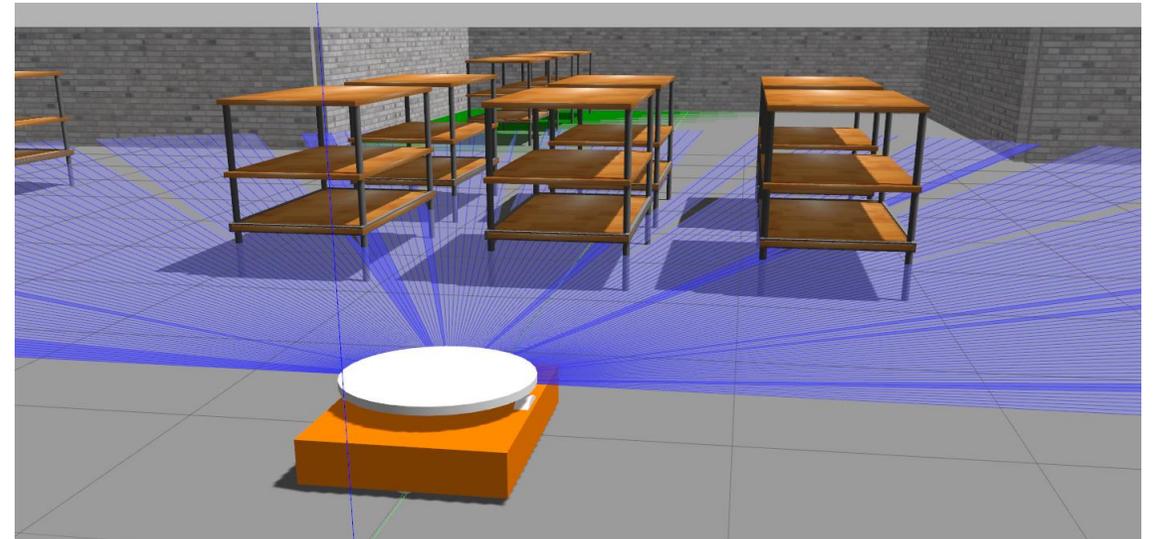
Simultaneous Localization And Mapping

Provide the means for a mobile robot to

- map an unknown environment and
- identify its location within it

Today's lecture in simple terms

- Evolution of Localization and Mapping approaches
- Basic theory and methods
- Re-evaluation and correction of the output



Localization and Mapping for Autonomous Mobile Systems

Now, lets define some terms for this lecture

What is a mobile robot?

- An autonomous machine equipped with a motion system and a set of sensors

What is an unknown environment?

- Our knowledge of the world's structure is zero or the environment is constantly changing

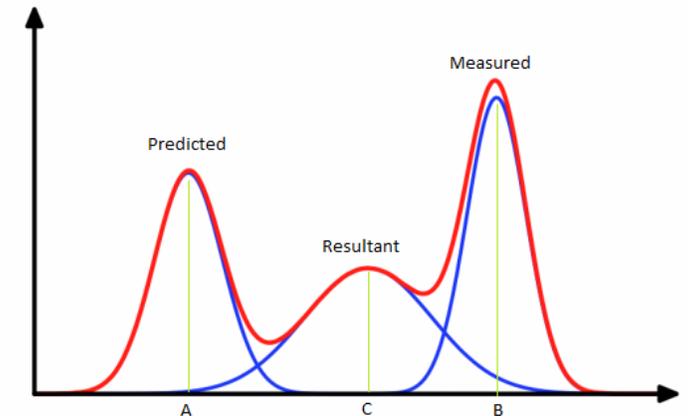
What is localization and mapping for a robot?

- Localization: Identify the robot's pose within a specified environment
- Mapping: Measure the environment's structure given the robot's pose
- Chicken-or-egg problem: Simultaneous Localization and Mapping (SLAM)

Localization and Mapping for Autonomous Mobile Systems

SLAM – Simultaneous Localization And Mapping

- Iterative process
 - Given your best knowledge about the environment, measure your location
 - Given your best localization estimate, update your knowledge about the environment
- Means to perceive the world
 - Noisy sensory inputs
- In its core: Estimation theory
 - Probabilistic Models
 - Most recently: Deep Learning



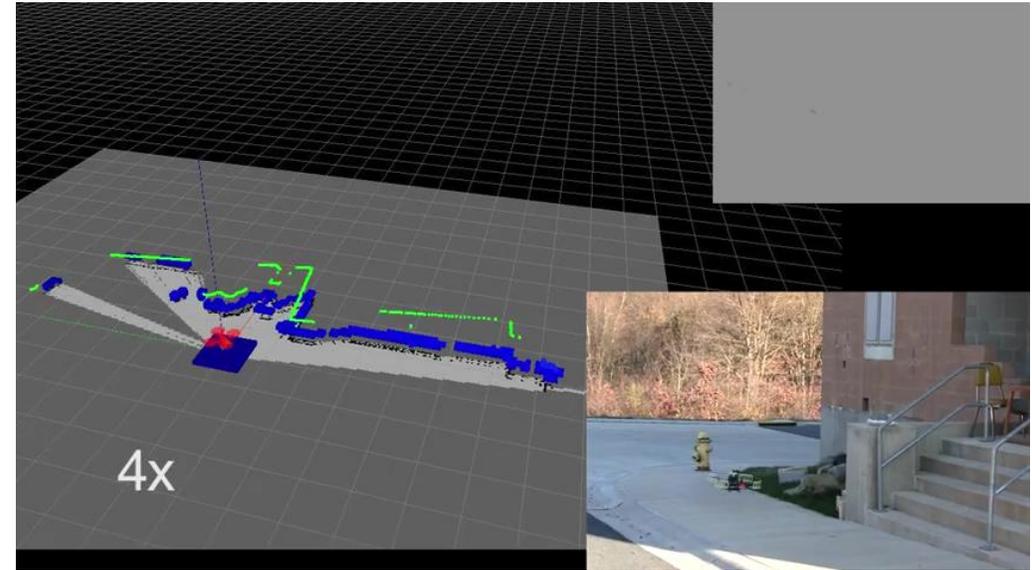
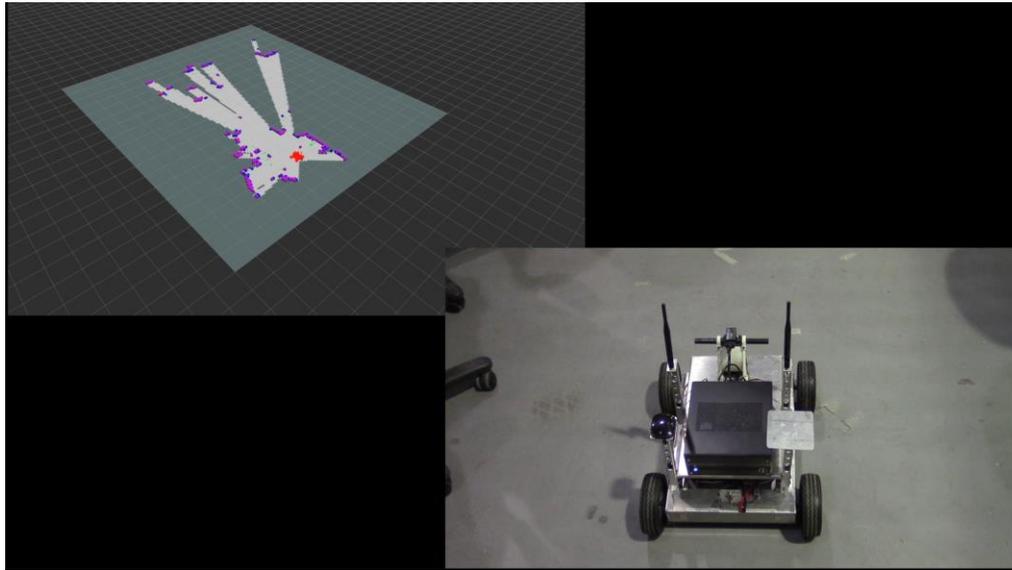
Localization and Mapping for Autonomous Mobile Systems

SLAM VS Deep Learning

- Typical SLAM focuses on geometric problems
- Deep Learning is the master of perception (recognition) problems
- If you want a robot to go towards your refrigerator without hitting a wall, use SLAM
- If you want the robot to identify the items inside your fridge, use Deep Learning
- Most institutes split their graduate level curriculums into:
 - Learning-based Methods -> Deep Learning
 - Geometry-Based Methods -> SLAM

Localization and Mapping for Autonomous Mobile Systems

A video is worth a thousand words



Localization and Mapping for Autonomous Mobile Systems

Perception systems: The means for obtaining information

What kind of sensors do we need?

- Robot's state
- World's geometry

Mimic the living

- Humans do not have 5 senses
- At least 9 according to neuroscience

Localization and Mapping for Autonomous Mobile Systems

Perception systems: The means for obtaining information

Robot's state

- Position:
 - Absolute: e.g., Global Navigation Satellite System (GNSS)
 - Relative: e.g., Wheel encoders
- Inertia:
 - Acceleration: Accelerometers
 - Orientation and Angular velocity: Gyroscopes
 - Earth frame alignment: Magnetometers

Localization and Mapping for Autonomous Mobile Systems

Perception systems: The means for obtaining information

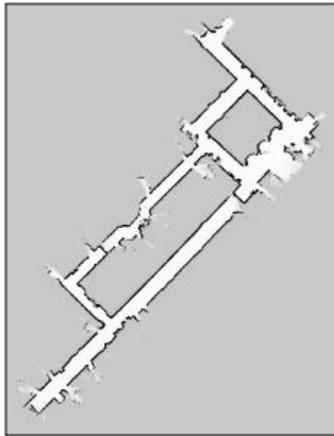
World's geometry

- Range-finders: Proximity
 - Ultrasonic
 - LIDAR
- Cameras: Appearance
 - Monocular
 - Stereo
 - RGBD
 - Infrared
 - Time of flight

Localization and Mapping for Autonomous Mobile Systems

The basics of SLAM: Taxonomy

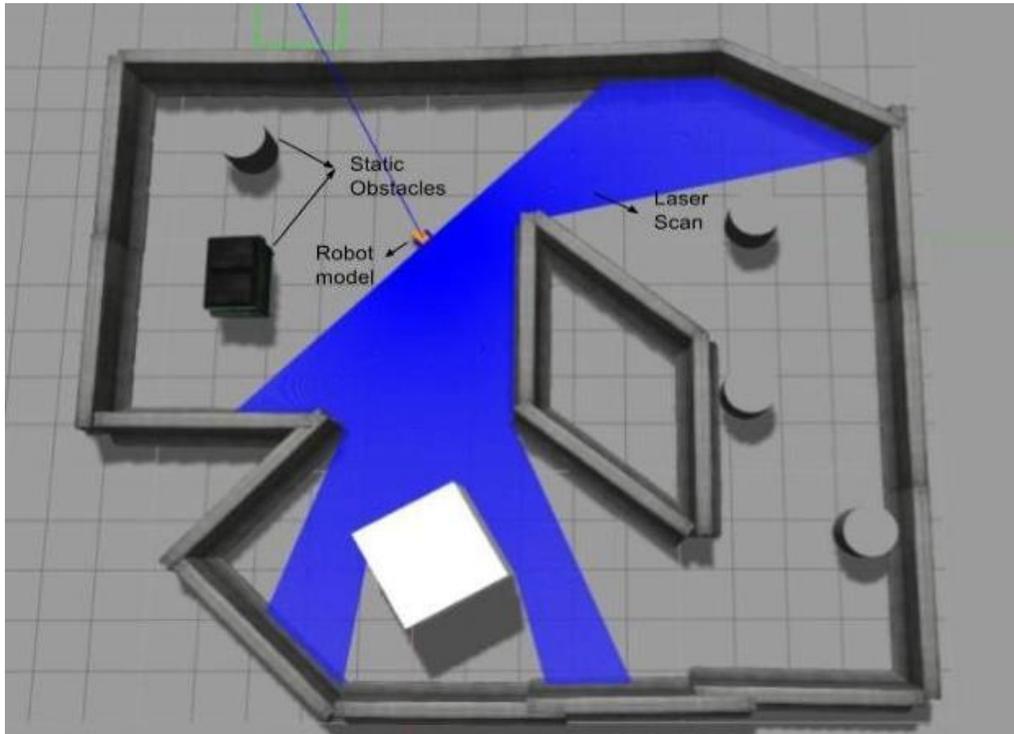
Volumetric VS Feature-Based SLAM



Localization and Mapping for Autonomous Mobile Systems

The basics of SLAM: Taxonomy

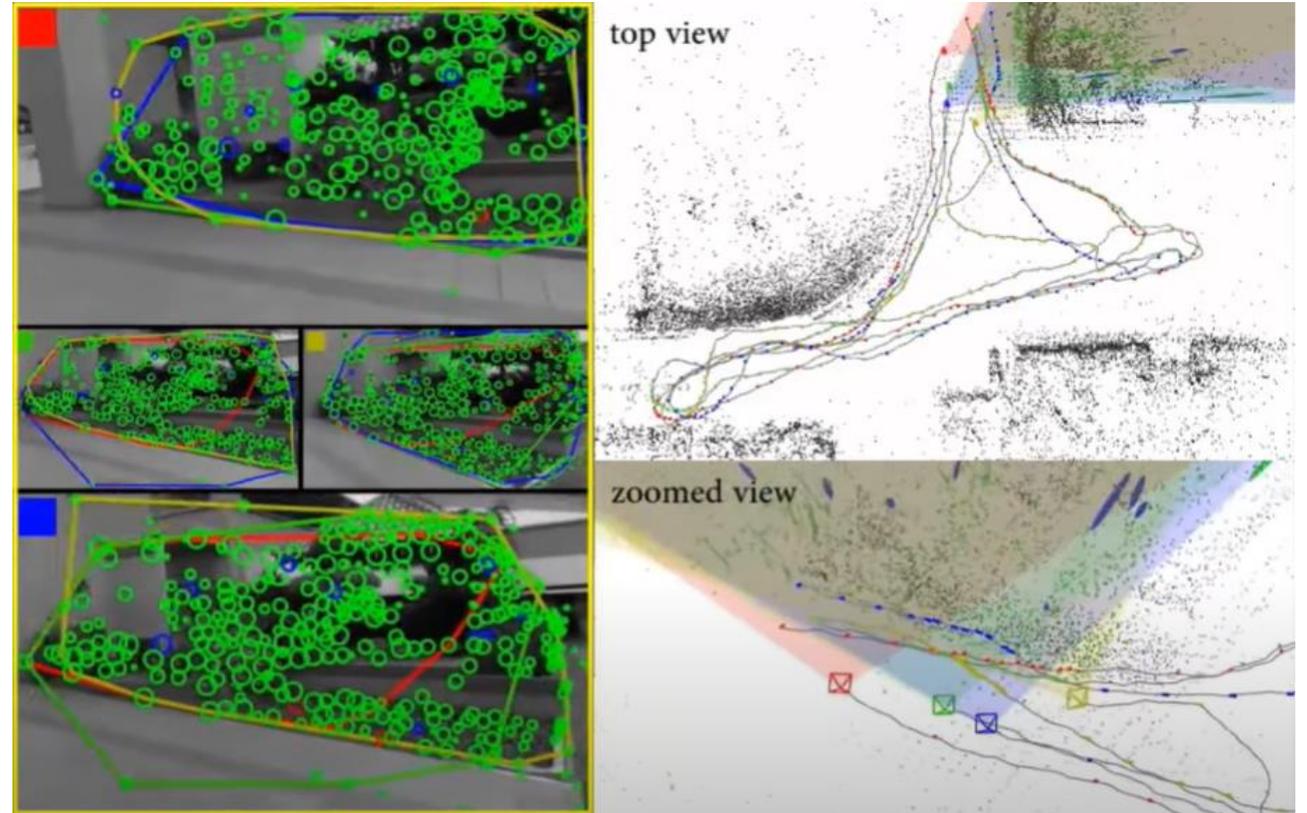
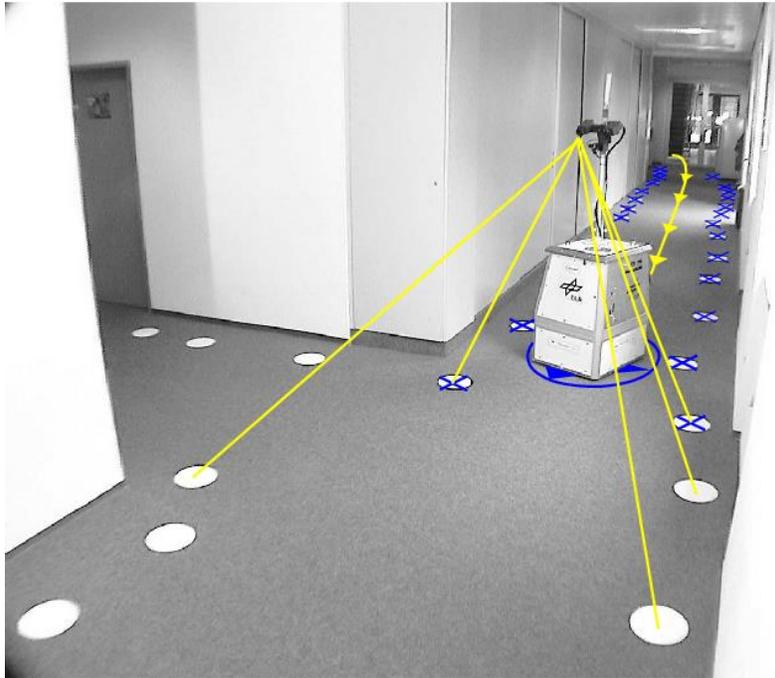
Static VS Dynamic Environments



Localization and Mapping for Autonomous Mobile Systems

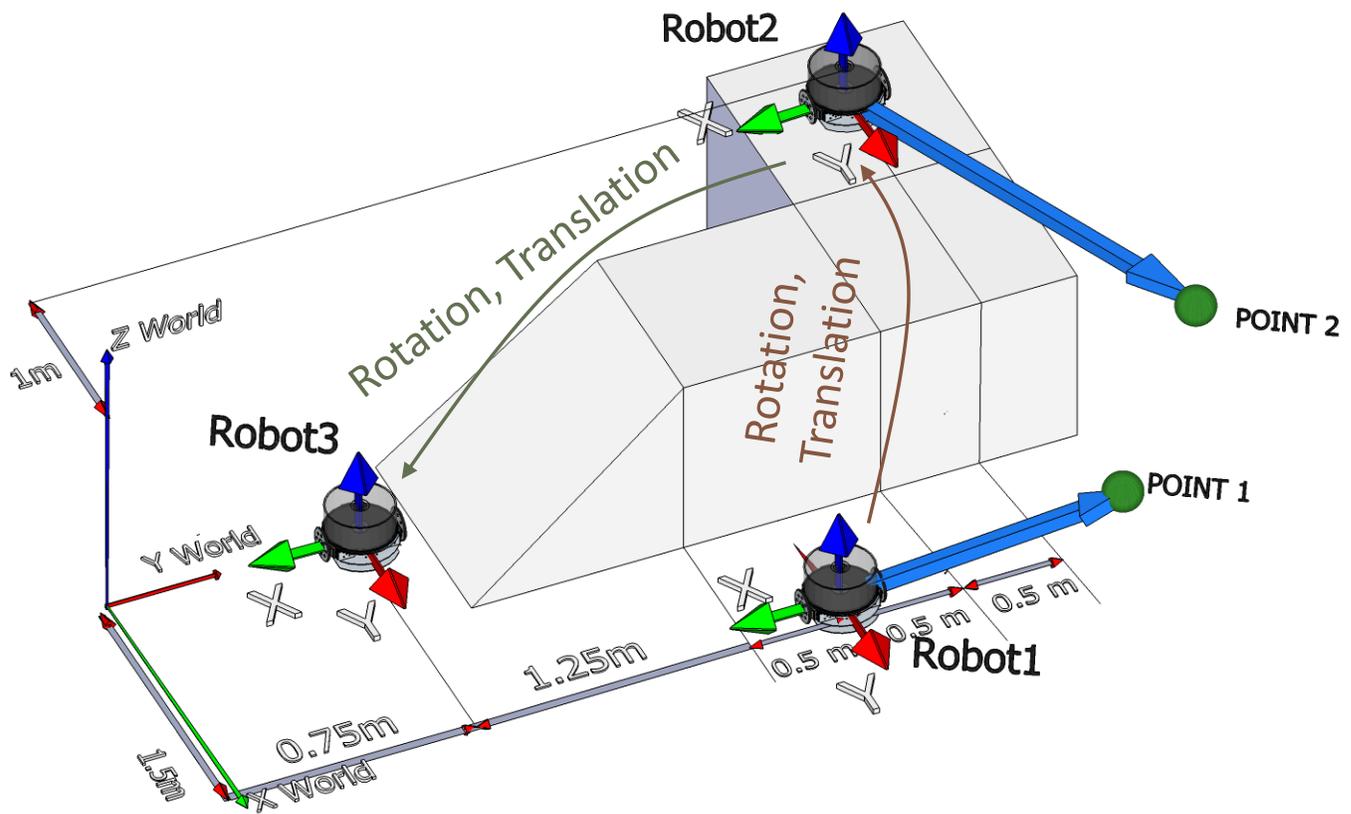
The basics of SLAM: Taxonomy

Single-Robot VS Multi-Robot



Localization and Mapping for Autonomous Mobile Systems

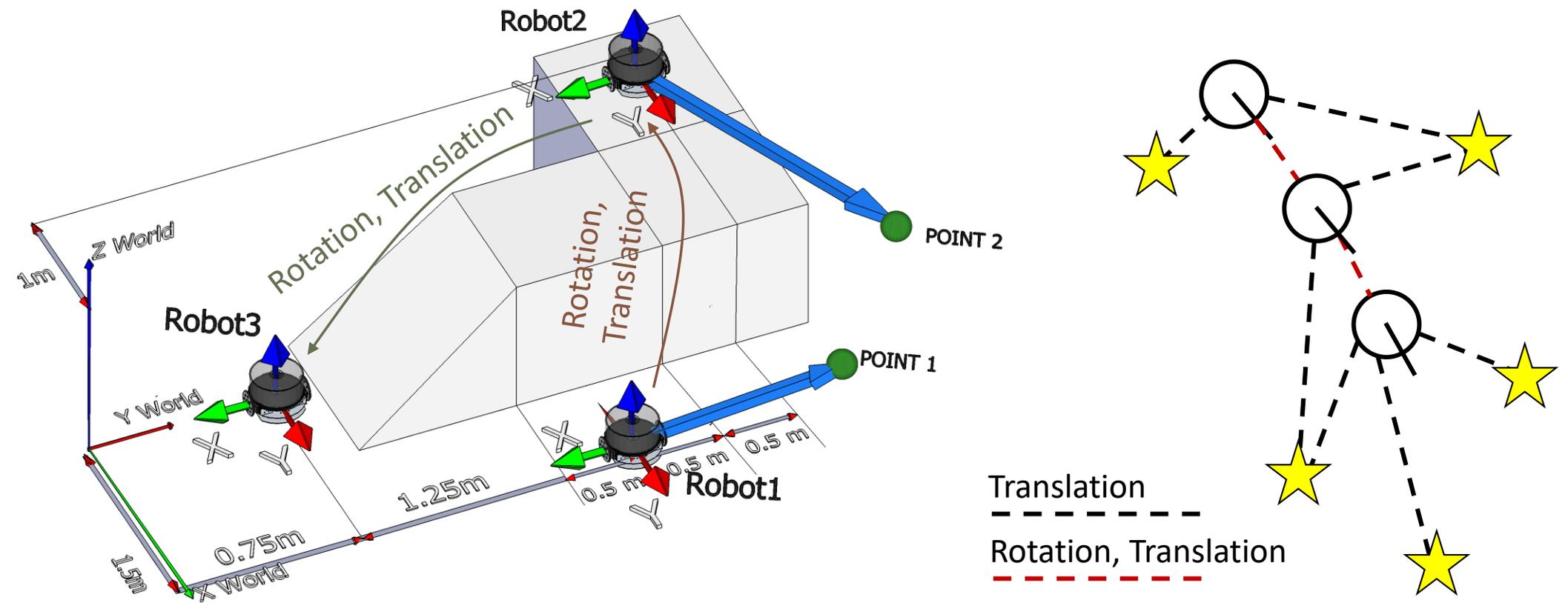
A common representation from SLAM (Graph SLAM)



- Points are represented w.r.t frames of reference placed on each robot pose
- Robot poses are associated through their relative transformations

Localization and Mapping for Autonomous Mobile Systems

A common representation from SLAM (Graph SLAM)

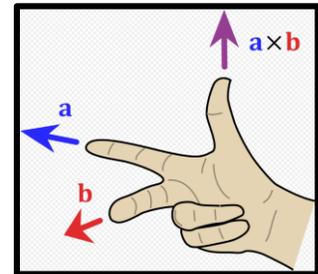
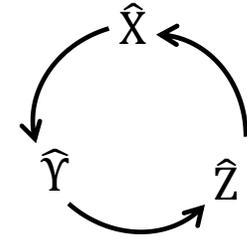
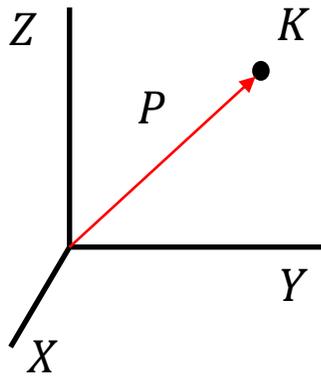


Localization and Mapping for Autonomous Mobile Systems

Frames of reference:: what?

A frame of reference holds:

- Specific position
- Specific orientation
- Specific axis arrangement



$$\hat{X} \times \hat{Y} = \hat{Z}$$

Localization and Mapping for Autonomous Mobile Systems

Frames of reference:: why?

Elements w/o dimensions (points):

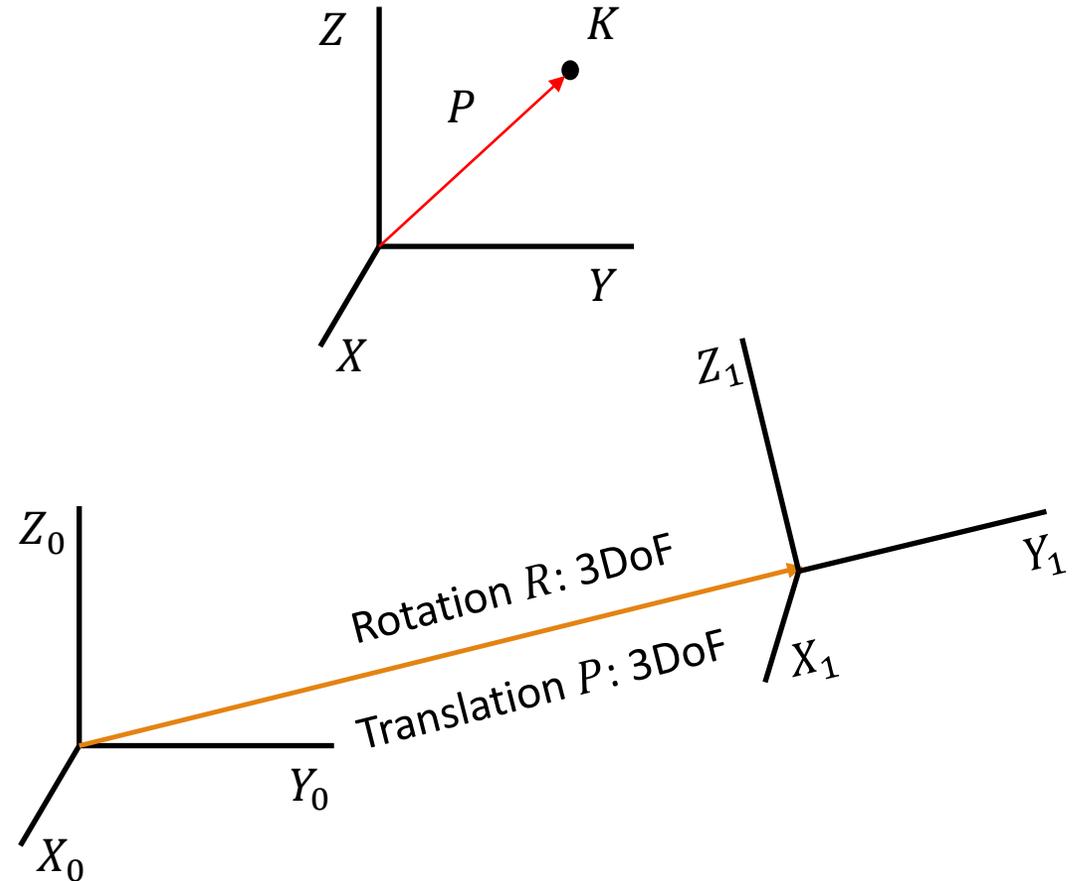
- Definition of location:

$$K = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Elements w/ dimensions (objects):

- Definition of relative pose:

$${}^0_1T = \begin{bmatrix} & {}^0_1R & P \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Localization and Mapping for Autonomous Mobile Systems

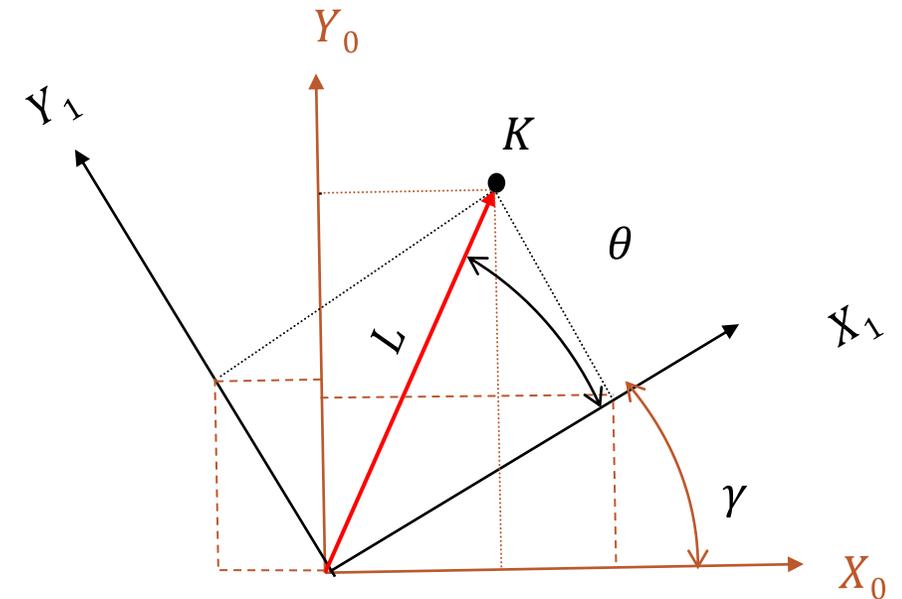
Frames of reference:: How?

Rotation Matrix: 2D

- Knowing the location of K w.r.t. frame 1: $[L \text{ and } \theta]$
- Knowing the rotation of frame 1 w.r.t. frame 0: $[\gamma]$
- Find the location of K w.r.t. frame 0

$$\begin{aligned}x_1 &= L \cos(\theta) \\y_1 &= L \sin(\theta)\end{aligned}$$

$$\begin{aligned}x_0 &= x_1 \cdot \cos(\gamma) - y_1 \cdot \sin(\gamma) \\y_0 &= x_1 \cdot \sin(\gamma) + y_1 \cdot \cos(\gamma)\end{aligned} \Rightarrow \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) \\ \sin(\gamma) & \cos(\gamma) \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = {}^0_1R \cdot \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$



0_1R : 2D Rotation matrix from frame 1 to frame 0

Localization and Mapping for Autonomous Mobile Systems

Frames of reference:: How?

Rotation Matrix: 2D

- Rotation around Z

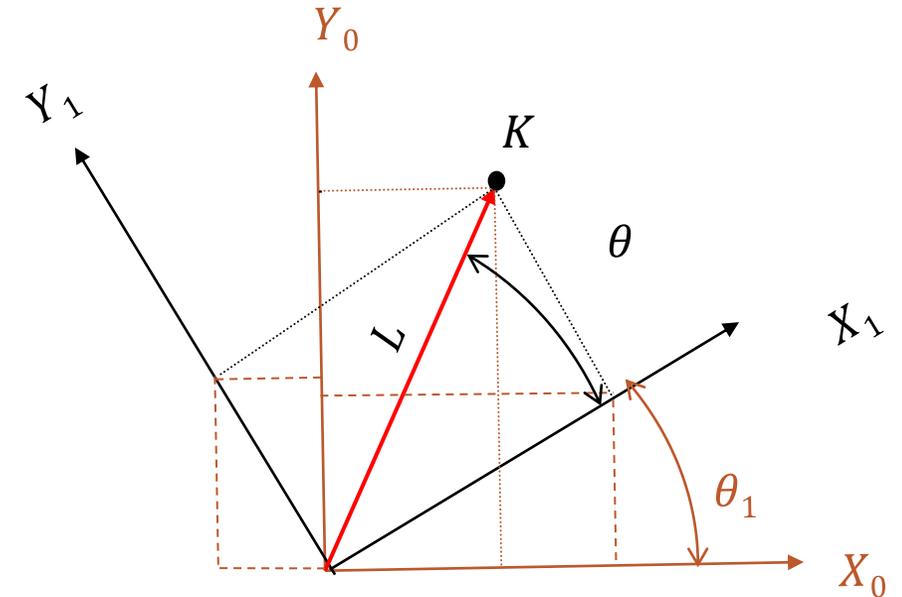
$${}^0_1R = \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) \\ \sin(\gamma) & \cos(\gamma) \end{bmatrix}$$

Rotation Matrix: 3D

- Rotation around Z

$${}^0_1R = \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Z_1 = Z_0 \rightarrow \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} = \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$$



Localization and Mapping for Autonomous Mobile Systems

Frames of reference:: How?

Similarly for the rest of the axes' rotation.

- Rotation around Z

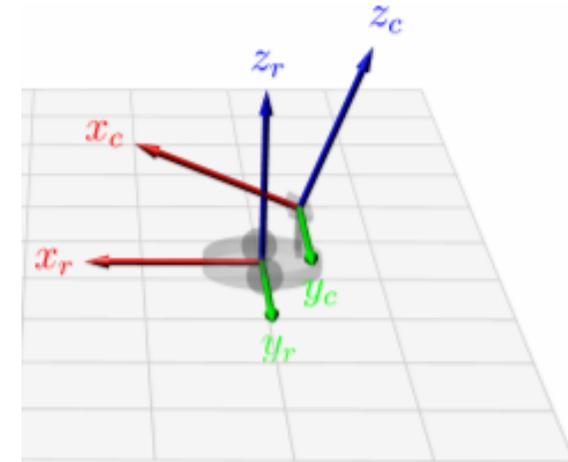
$${}^B_A R_Z = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Rotation around X

$${}^B_A R_X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\gamma) & -\sin(\gamma) \\ 0 & \sin(\gamma) & \cos(\gamma) \end{bmatrix}$$

- Rotation around Y

$${}^B_A R_Y = \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix}$$



- Rotation around all 3 axes

$${}^B_A R = R_Z(\alpha)R_Y(\beta)R_X(\gamma)$$

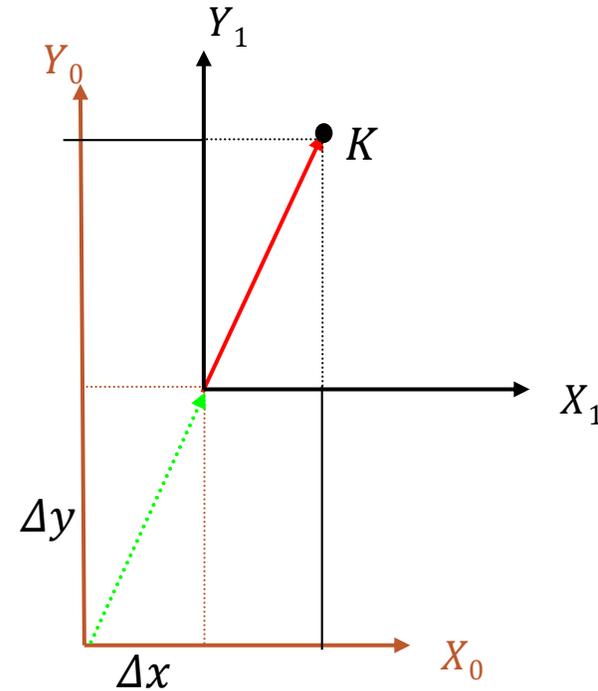
Localization and Mapping for Autonomous Mobile Systems

Frames of reference:: How?

Translation Matrix P : 2D

$$\begin{aligned}x_0 &= x_1 + \Delta x \\ y_0 &= y_1 + \Delta y\end{aligned} \Rightarrow \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + P$$



Localization and Mapping for Autonomous Mobile Systems

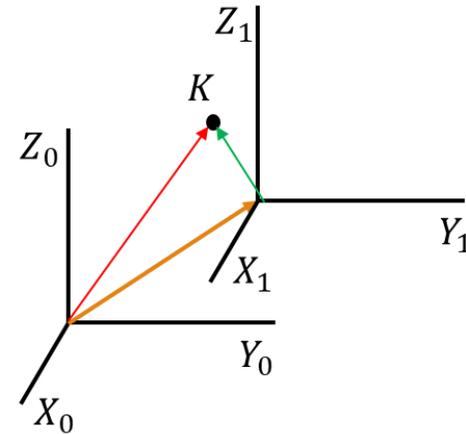
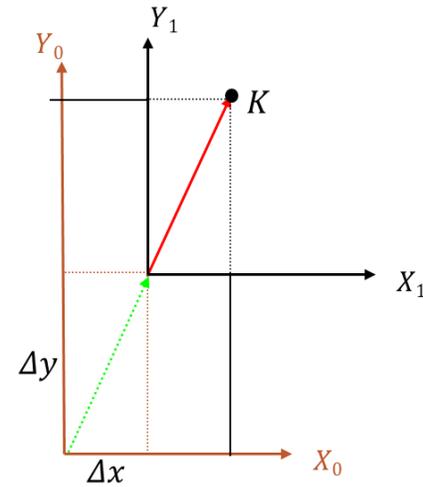
Frames of reference:: How?

Translation Matrix P : 2D

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \Rightarrow \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + P$$

Translation Matrix P : 3D

$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} \Rightarrow \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + P$$



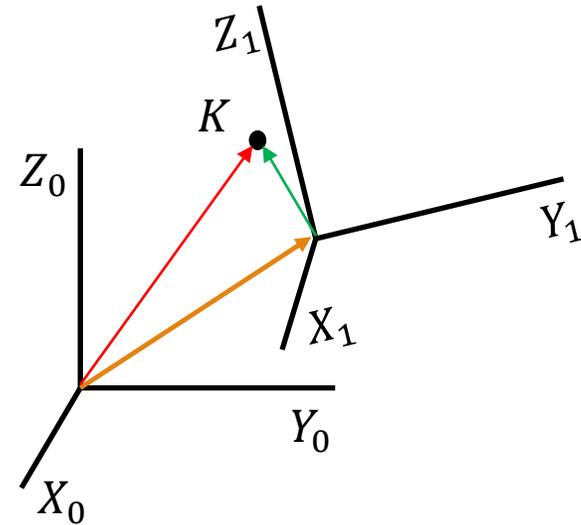
Localization and Mapping for Autonomous Mobile Systems

Frames of reference:: How?

Combining R and $P \rightarrow$ Transformation Matrix T

$$\left. \begin{array}{l} {}^0K = {}^0_1R {}^1K + P \\ {}^0_1T = \left[\begin{array}{ccc|c} {}^0_1R & & & P \\ \hline 0 & 0 & 0 & 1 \end{array} \right] \end{array} \right\} \left[\begin{array}{c} {}^0K \\ 1 \end{array} \right] = \left[\begin{array}{ccc|c} {}^0_1R & & & P \\ \hline 0 & 0 & 0 & 1 \end{array} \right] \left[\begin{array}{c} {}^1K \\ 1 \end{array} \right]$$

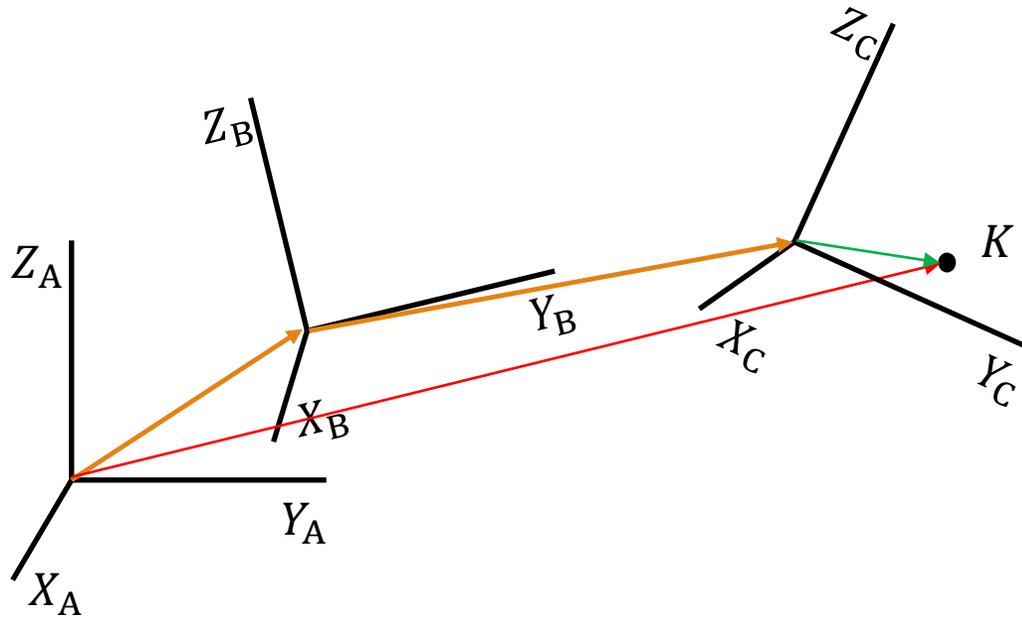
$${}^0K = {}^0_1T {}^1K$$



Localization and Mapping for Autonomous Mobile Systems

Frames of reference:: Kinematic Chain

- Gradual description of a point from one frame of reference to its previous one

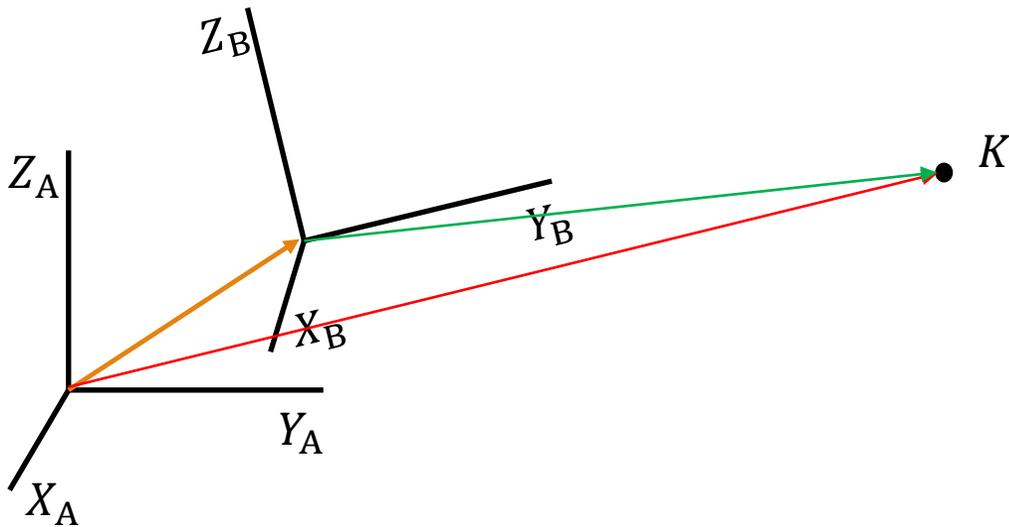


$${}^B K = {}^B T_C {}^C K$$

Localization and Mapping for Autonomous Mobile Systems

Frames of reference:: Kinematic Chain

- Gradual description of a point from one frame of reference to its previous one



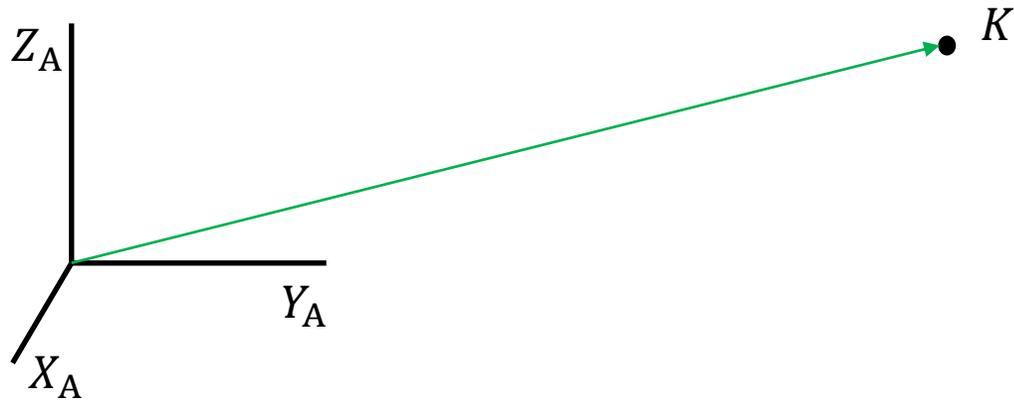
$${}^B K = {}^B T^C K$$

$${}^A K = {}^A T^B K$$

Localization and Mapping for Autonomous Mobile Systems

Frames of reference:: Kinematic Chain

- Gradual description of a point from one frame of reference to its previous one



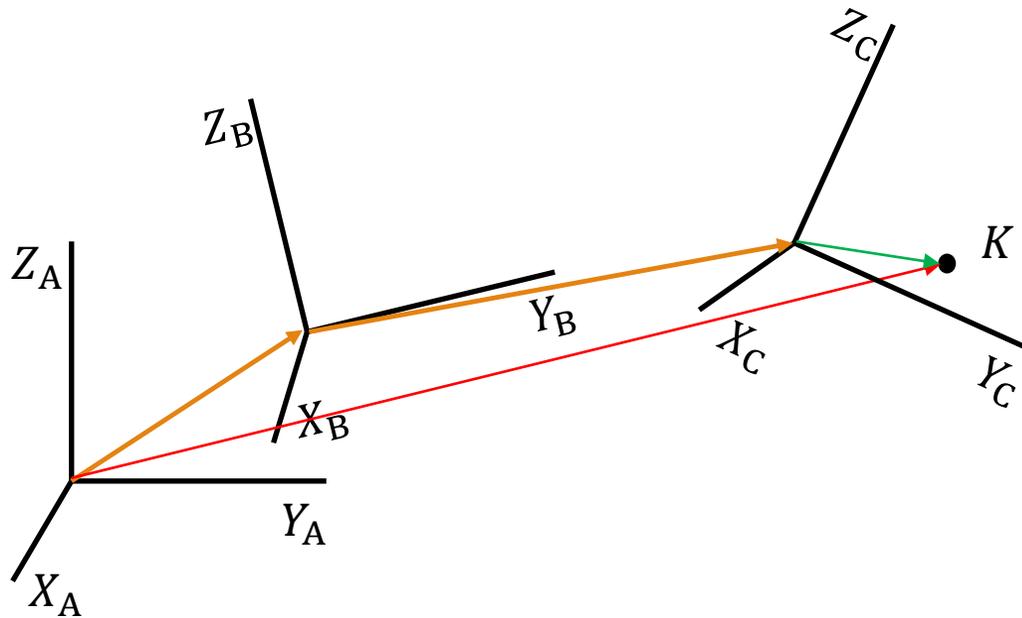
$${}^B K = {}^B T^C {}^C K$$

$${}^A K = {}^A T^B {}^B K$$

Localization and Mapping for Autonomous Mobile Systems

Frames of reference:: Kinematic Chain

- Gradual description of a point from one frame of reference to its previous one



$${}^B K = {}^B T {}^C K$$

$${}^A K = {}^A T {}^B K$$

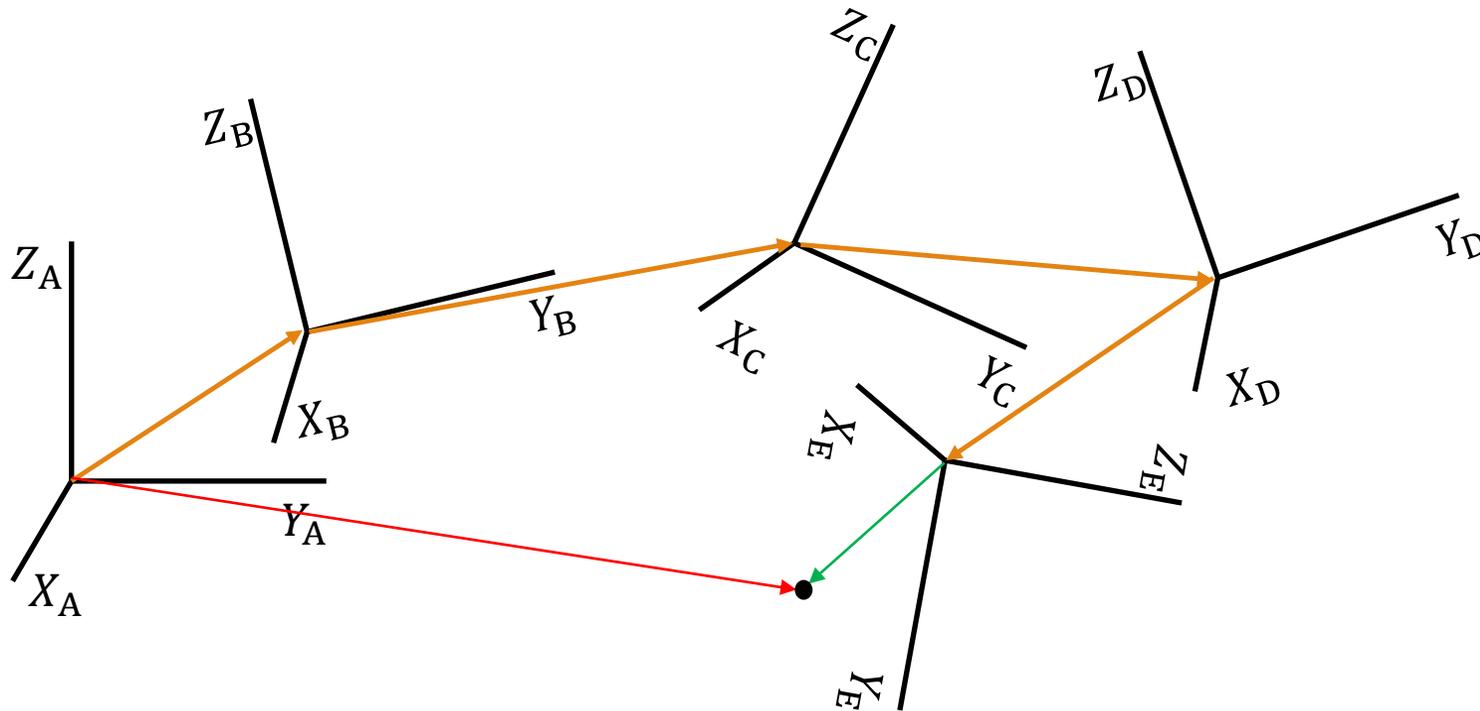
$${}^A K = {}^A T {}^B T {}^C K$$

$${}^C T = {}^B T {}^A T$$

Localization and Mapping for Autonomous Mobile Systems

Frames of reference:: Kinematic Chain

- Gradual description of a point from one frame of reference to its previous one

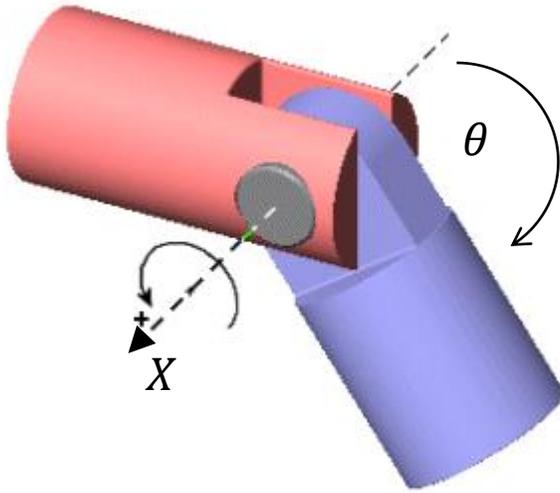


$${}^A_T E = {}^A_T B {}^B_T C {}^C_T D {}^D_T E$$

Localization and Mapping for Autonomous Mobile Systems

Frames of reference:: Applications

Actuators: Every joint can be defined by a Transformation Matrix



Revolute joint

Rotation around X axis with negative rotation direction

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-\theta) & -\sin(-\theta) & 0 \\ 0 & \sin(-\theta) & \cos(-\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\gamma) & -\sin(\gamma) \\ 0 & \sin(\gamma) & \cos(\gamma) \end{bmatrix}$$

$$T = \left[\begin{array}{ccc|c} & R & & P \\ \hline 0 & 0 & 0 & 1 \end{array} \right]$$

Localization and Mapping for Autonomous Mobile Systems

Frames of reference:: Applications

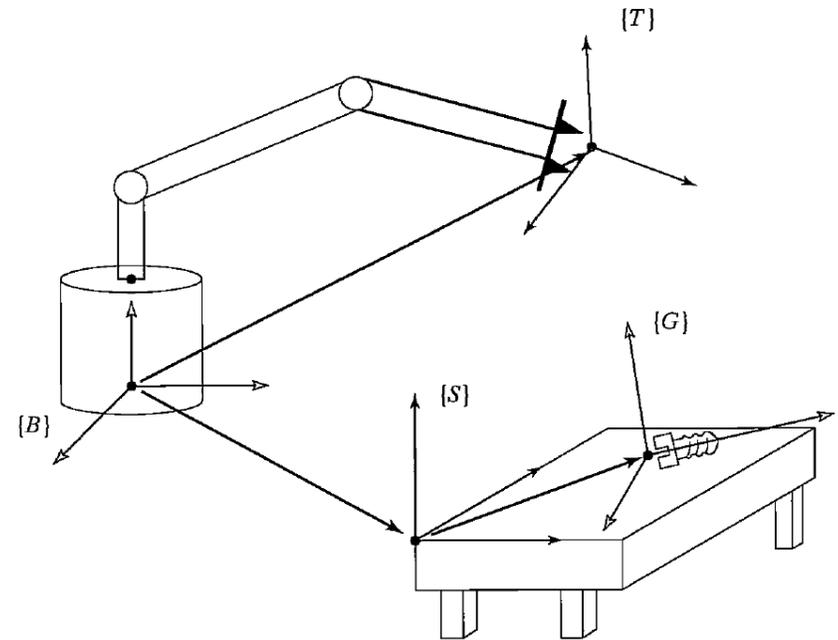
Actuators: Every joint can be defined by a Transformation Matrix

This way, the relative transformation between the end-effector and the target can be computed \Rightarrow Manipulation

Unknowns: ${}^T_G T$

Knowns: ${}^B_T T$, ${}^B_S T$, ${}^S_G T$

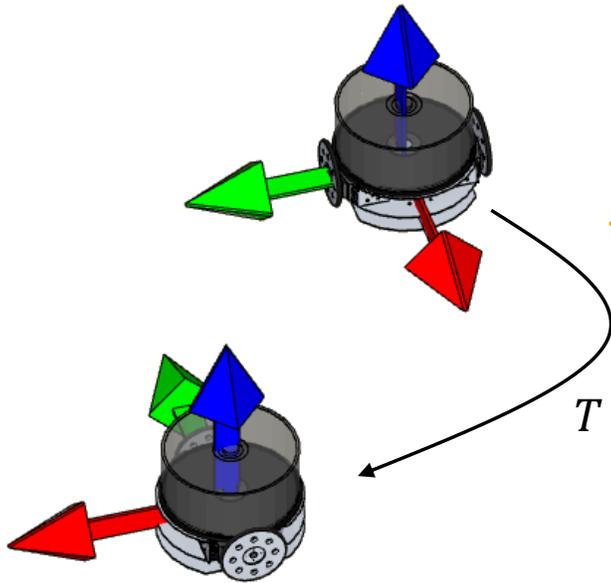
Solution: ${}^T_G T = [{}^B_T T]^{-1} {}^B_S T {}^S_G T$



Localization and Mapping for Autonomous Mobile Systems

Frames of reference:: Applications

AGVs: Every robot movement can be defined by a Transformation Matrix



Rotation around Z axis with negative rotation direction and Translation on X and Y

$$T = \begin{bmatrix} 1 & 0 & 0 & \Delta x \\ 0 & \cos(-\theta) & -\sin(-\theta) & \Delta y \\ 0 & \sin(-\theta) & \cos(-\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\gamma) & -\sin(\gamma) \\ 0 & \sin(\gamma) & \cos(\gamma) \end{bmatrix}$$

$$P = \begin{bmatrix} \Delta x \\ \Delta y \\ 0 \end{bmatrix}$$

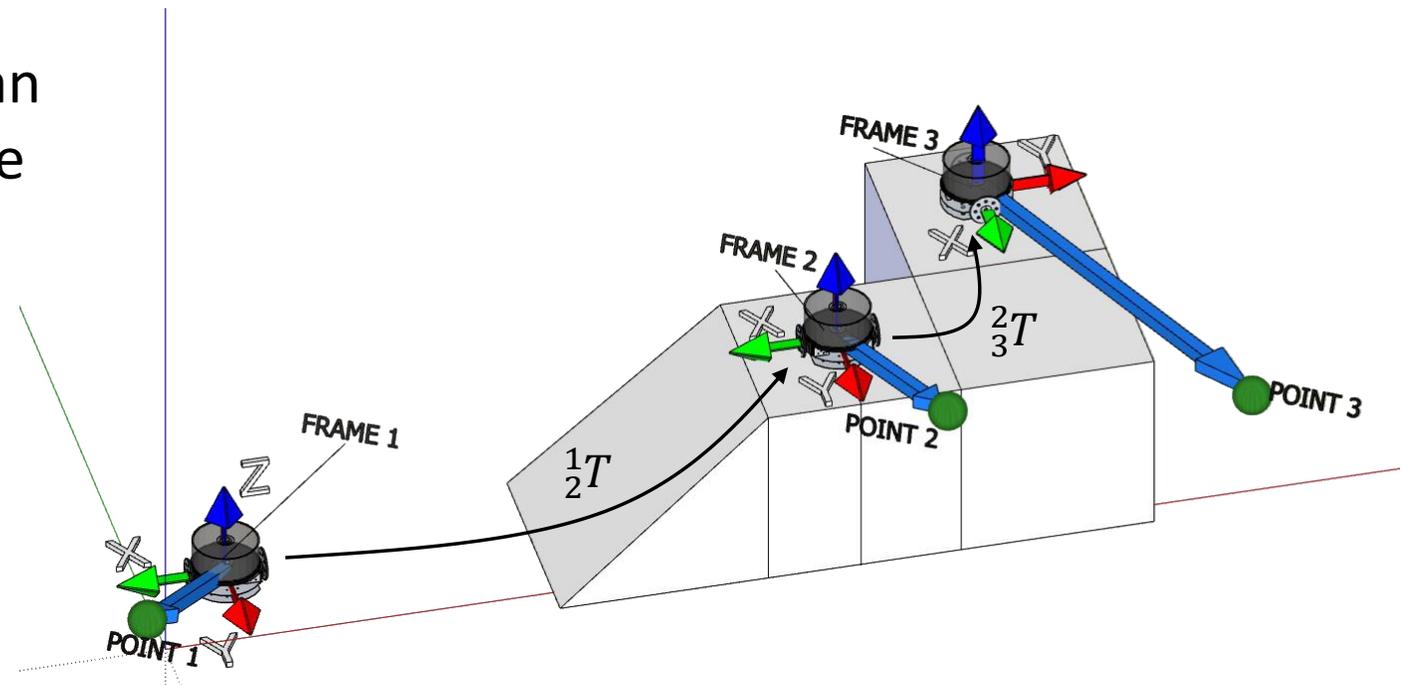
$$T = \left[\begin{array}{ccc|c} R & & & P \\ \hline 0 & 0 & 0 & 1 \end{array} \right]$$

Localization and Mapping for Autonomous Mobile Systems

Frames of reference:: Applications

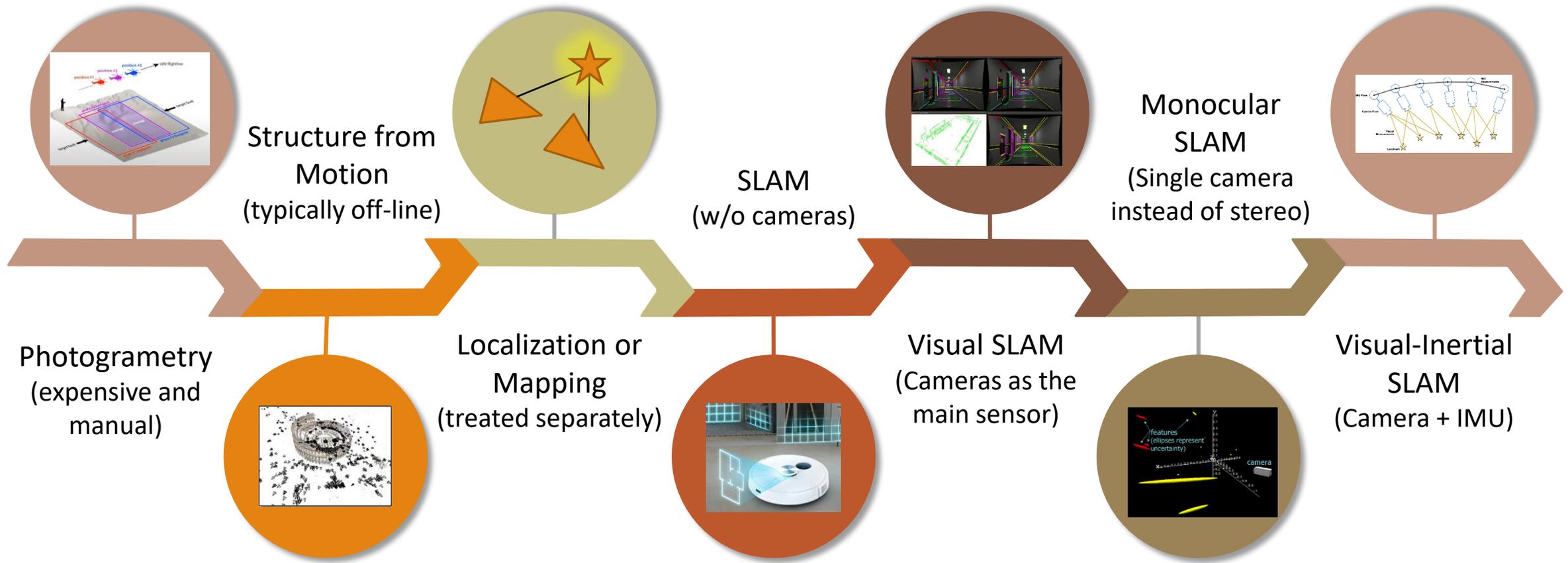
AGVs: Every robot movement can be defined by a Transformation Matrix

This way, the observed points can be projected to a common frame of reference \Rightarrow Map



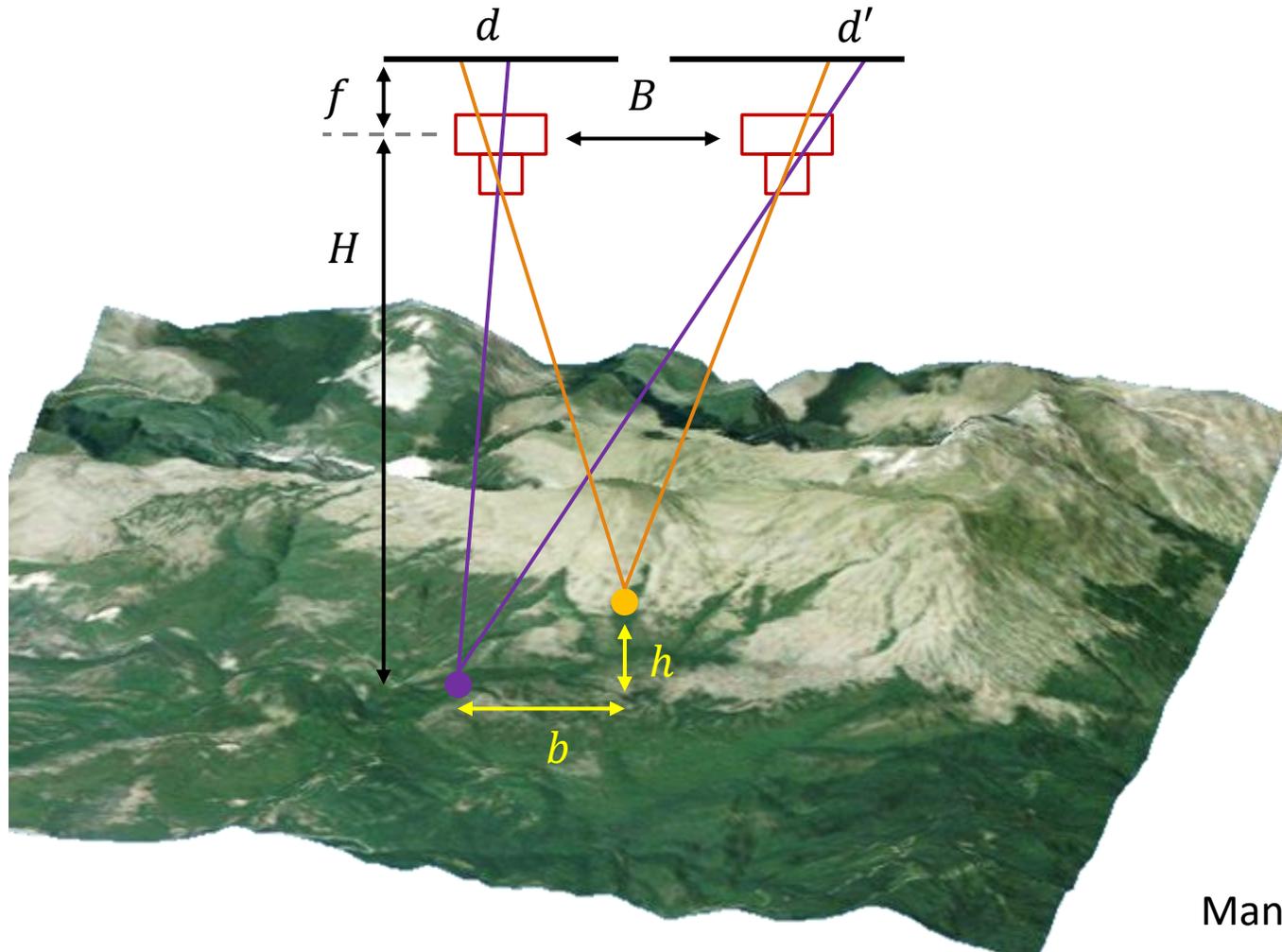
Localization and Mapping for Autonomous Mobile Systems

Evolution of SLAM



Localization and Mapping for Autonomous Mobile Systems

It all started with stereo-photogrammetry



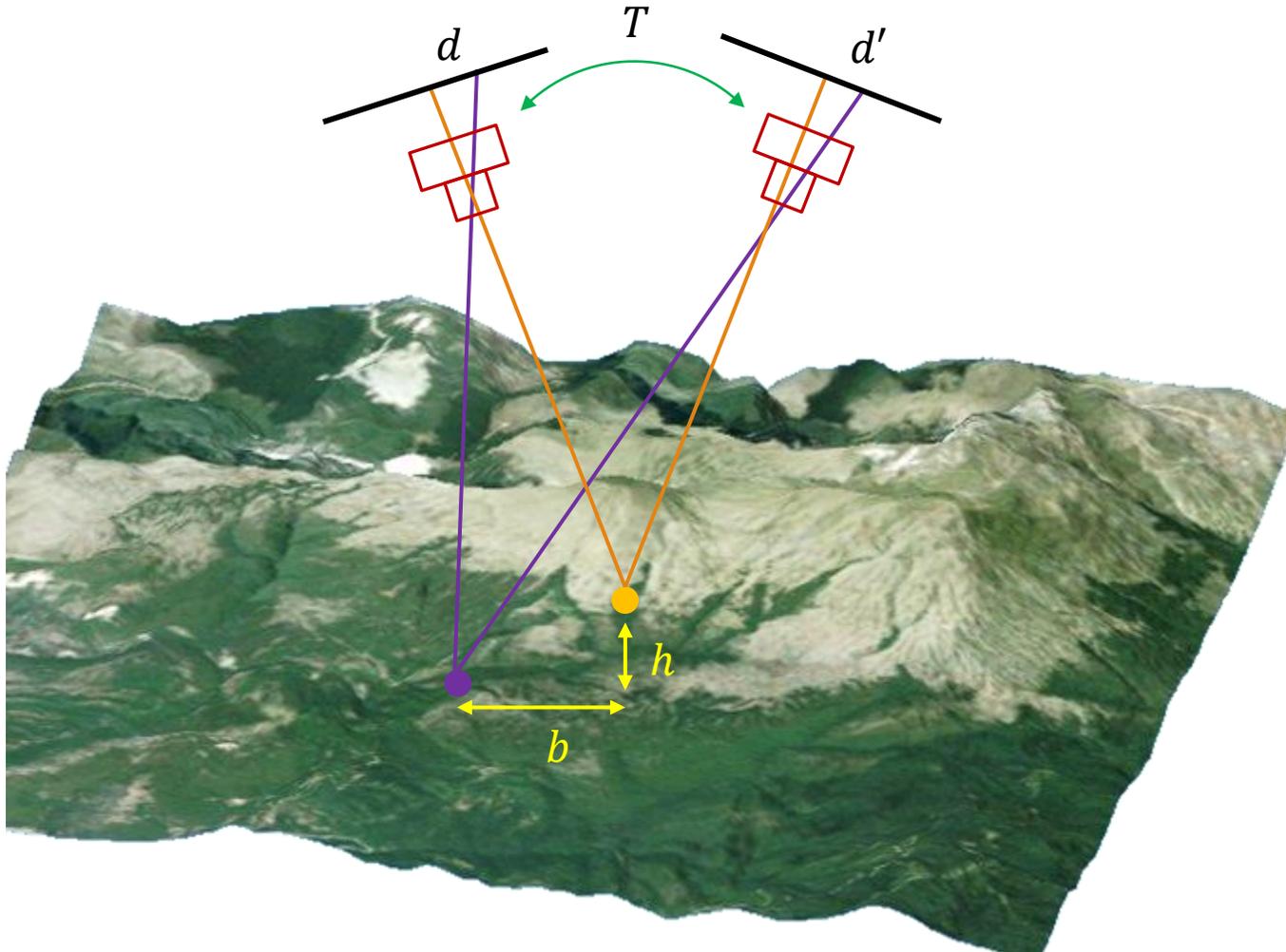
- Required: H, f, B
- Measured: d, d'
- Computed: b, h



Manual measurements: Mirror Stereoscope

Localization and Mapping for Autonomous Mobile Systems

Structure from Motion – SfM



- Required: Multiple points associations among different frames (automated)
- Measured: d, d' (automated)
- Computed: T, b, h

Localization and Mapping for Autonomous Mobile Systems

Structure from Motion – SfM

Falls into the “Shape from X ” problem

Methods:

- Stereo
- Shading
- Photometric Stereo
- Texture
- Contours
- Silhouettes
- Motion

Localization and Mapping for Autonomous Mobile Systems

SLAM and SfM

SLAM is using tools from SfM

- SfM is traditionally performed offline
 - Recovery of 3D shape from 2D images
 - Depending on the scale, reconstruction can take hours or days
 - It is typically performed on high-performance computers
 - Google Maps and Google Street View were built using SfM
- SLAM mostly refers to online applications
 - Mapping and localization on-the-fly
 - Real-time
 - Low-power sensors (e.g., a single RGB camera and an IMU)
 - It is typically performed on-board using low-power processing units

Localization and Mapping for Autonomous Mobile Systems

Structure from Motion – SfM

Falls into the “Shape from X” problem

Methods:

- Stereo
- Shading
- Photometric Stereo
- Texture
- Contours
- Silhouettes
- Motion

Localization and Mapping for Autonomous Mobile Systems

Structure from Motion – SfM

Shape from X - Shape from Motion

- Humans are able to recover 3D from motion

