

# An Investigation Study Considering the Effect of Magnet Type, Slot Type and Pole-Arc to Pole-Pitch Ratio Variation on PM Brushless DC Motor Design

Yannis L. Karnavas  
Electrical Machines Laboratory  
Dept. of Electrical & Computer Engineering  
Democritus University of Thrace  
Xanthi, Hellas, Greece (GR)  
karnavas@ee.duth.gr

Ioannis D. Chasiotis  
Electrical Machines Laboratory  
Dept. of Electrical & Computer Engineering  
Democritus University of Thrace  
Xanthi, Hellas, Greece (GR)  
ichasiot@ee.duth.gr

Alexandros D. Gkiokas  
Electrical Machines Laboratory  
Dept. of Electrical & Computer Engineering  
Democritus University of Thrace  
Xanthi, Hellas, Greece (GR)  
alexgiok@ee.duth.gr

**Abstract**—This paper deals with a sensitivity study of an outer rotor permanent magnet (PM) brushless DC (BLDC) motor design, concerning the effect of several parameters and factors on its corresponding performance. For the purposes of this study, several design features were considered regarding the type and size of the permanent magnets and the slot geometry. Moreover, the pole-arc to pole-pitch ratio variation was investigated. The primary aim is to draw useful conclusions for the motor overall performance according to preset requirements, such as the rated speed, motor efficiency, output power, cogging torque, overall motor weight etc. The obtained results were validated through simulations using finite element method (FEM) analysis with the aid of commercial software. The concluded remarks reveal important considerations which have to be taken into account by the designer prior and through the design phase of a BLDC motor.

**Keywords**—brushless dc motor; permanent magnets; motor design; finite element analysis; industrial applications

## I. INTRODUCTION

While brushed DC motors have been in commercial use for almost 150 years, brushless DC (BLDC) motors have only been commercially possible since the last few decades. Moreover, current trends show that BLDC motors are now utilized in many domestic and industrial application areas, unlike traditional permanent magnet dc commutator (PMDC) motors [1]-[3]. These trends can be easily justified for many reasons since there are some important advantages of BLDC motors over the brushed counterparts, such as, the higher efficiency, the higher torque per volume ratio, the inherent capability of higher driving speeds and the use of an electronic commutation system which can be programmed for desired performance and operational behavior [3]-[6]. The list is impressive and can't be extensively reported here. Typical examples are a) in medical equipment e.g. dialysis pumps, automatic agitators for medical and chemical use, dental use pumps, ventilator systems pumps in anesthesia etc, b) in industrial equipment e.g. high-speed spraying machines, laser scanning tools, industrial laser barcode readers etc, c) in computers and hobbies e.g. hard disk drives, cooled airflow cooling fans, remote control (R/C) vehicles, drone motors etc. Also, it is obvious that the motors used in each case have very different requirements in terms of power, speed, torque, volume, form and size [4], [5]. On the other hand, there are, of course, some disadvantages. There is a need of sensors to know the position of the rotor at any time in order for the electronic commutation mechanism to drive the motor. This also increases the complexity of

controlling the drive circuitry. In addition, the cost is somewhat higher due to the high cost of permanent magnets. The maximum speed of the BLDC can also be limited due to the magnet "hold-action" against the centrifugal forces. In small motors with high speeds, magnets are glued onto the solid core and consequently, only outer rotor motors can withstand the possible magnet detachment. In addition there is always a risk of demagnetization of the permanent magnets (especially of the rare earth ones) due to possible external field presence, very high temperature rise or heavy mechanical stress [6].

It becomes apparent though, that problems associated with the optimization, analysis and performance of BLDC motors have gained the focus of researchers over the last decades. The overall topology (inner or outer rotor motors), the winding configuration combinations, the number of poles and the number and type of slots selection, the application of different soft and hard magnetic materials, the optimum pole-arc to pole-pitch ratio and the efficiency maximization, are some of the operational key elements and many of them have been studied and also their key role has been underlined with respect to the problem significance [7]-[11].

Given this context, and the constant need for improving electrical machines in general, the present work focuses on the design and analysis of an outer rotor BLDC motor. Starting from an initial design topology, many variants were developed and analyzed thoroughly by utilizing commercial finite element method (FEM) analysis software. The motor's specifications and other relevant data are taken into account for achieving results on certain quantities. A brief theory reference along with the relevant design considerations is presented in Section II, while the different kinds of magnets as well as the slot types under consideration are described in Section III. Section IV shows the examined motor characteristics, the investigated case studies and the algorithmic procedure followed. Section V reports and comments the corresponding derived results, where Section VI concludes the work.

## II. BRIEF BLDC MOTOR THEORY & DESIGN CONSIDERATIONS

### A. Main Operational Characteristics

A BLDC motor is structurally similar to a DC motor as it exhibits a linear relationship between current and torque, and also voltage and speed. The switching of the current is done electronically and not mechanically as in the conventional

DC machine. This fact essentially separates these two types of motors, and has a central role in anything related to the operation of the BLDC motor from the description and modeling to the design of the motor's power supply system, any control systems applied, the construction and its function, its advantages and disadvantages in relation to other types of DC motors in which the switching is done mechanically. In the outer BLDC motor the rotating part of the machine (rotor) has permanent magnets, while the electromagnets (windings) are fixed. In this way, the problem of moving the current to a moving winding (i.e. like in the simple DC motor case where this function is obtained through the collector-brush system), is addressed. However, in order to correctly switch the current to the stator to create the rotating field and then to develop the torque in the rotor, an electronic control circuit is used, which performs the proper distribution of power to the stator windings. Thus, this system is the electronic analog of the mechanical collector-brush system [12].

### B. Design Considerations

Fig. 1 shows a typical structure of a BLDC motor with outer rotor topology. The four essential parts are the rotor steel ring, the permanent magnets mounted on its inner surface, the stator (formed by thin steel laminations) and the copper windings. At any operational condition, the apparent power of the motor is

$$S = 3E_a I_{ph} \quad (1)$$

where  $E_a$  is the developed back-electromagnetic force (BEMF) and  $I_{ph}$  the phase flowing current. Considering 3 phases ( $N_{ph}=3$ ) then,

$$I_{ph} = \frac{I_s}{N_{ph} n_s} \quad (2)$$

where  $n_s$  is the number of turns per slot, and  $I_s$  the slot total current. Also, the BEMF can be obtained by,

$$E_a = 4.44 k_w f_r \Phi n \quad (1)$$

where  $k_w$  is the winding factor and  $n$  is the rotor speed i.e.,

$$n = 120 f_r / p \quad (3)$$

for a given (by the driving inverter) frequency  $f_r$ . At the same time, the output (or dimensioning) equation of the motor is given by,

$$S = 1.11 k_w \pi^2 B_g A_c D_g^2 L n \quad (4)$$

where  $B_g$  the average magnetic flux density in the airgap,  $A_c$  is the electric loading,  $D_g$  is the airgap diameter and  $L$  is the active axial motor length. Considering a  $p$ -pole motor which develops a coupled magnetic flux  $\Phi$  per pole, then the magnetic loading can be found using,

$$B_g = \frac{p\Phi}{\pi D_r L} \quad (5)$$

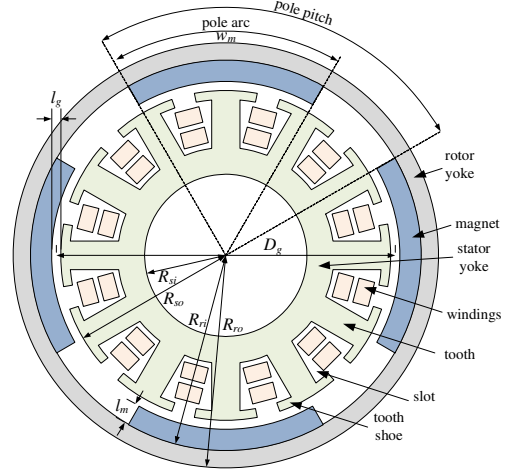


Fig. 1. Typical outer rotor BLDC motor structure.

The maximum limit is usually set by the phenomenon of saturation of the ferromagnetic material in the stator teeth. High magnetic loading means increased ability of the motor to generate torque and power. However, this may result in increased core losses, especially when the stator teeth are highly saturated [13].

If  $N_s$  is the number of total turns per phase, the specific electric loading can be obtained by

$$A_c = \frac{6 N_s I_{ph}}{\pi D_g} \quad (6)$$

Each phase presents an active resistance so the corresponding copper losses are,

$$P_{Cu} = N_{ph} I_{ph}^2 R_{ph} \quad (7)$$

Consequently, at any motor speed, a torque  $T$  is developed and by calculating the core loss ( $P_{core}$ ) as well as the mechanical losses ( $P_{fw} + P_{stray}$ ), the motor's efficiency can be derived,

$$\eta = \frac{T \omega_m}{T \omega_m + P_{Cu} + P_{core} + P_{fw} + P_{stray}} \cdot 100\% \quad (8)$$

Additionally, the pole pitch  $\tau$ , refers to the maximum circumferential distance corresponding to one pole at diameter  $D_r$  and can be calculated by:

$$\tau = \pi D_r / p \quad (9)$$

In a similar way, the pole arc  $\tau_{act}$ , refers to the actual circumferential distance corresponding to one pole. Thus, w.r.t. to Fig. 1, the pole-arc to pole-pitch ratio is given by:

$$embrace = \frac{\tau_{act}}{\tau} = \frac{w_m p}{2\pi(R_{si} + l_s + l_m)} \quad (10)$$

(where  $l_m$  is the magnet thickness and  $w_m$  the magnet width). This ratio is also called "embrace" and that is how it will be mentioned hereafter.

### III. PERMANENT MAGNETS AND SLOT TYPES USED

In this paragraph the essential description of the permanent magnet types as well as the different slot types which were adopted for investigation in this study is given.

#### A. Demagnetization Curve (B-H Characteristic)

The remanence magnetism  $B_r$  and the electric field intensity  $H_c$  are merged into a parameter known as maximum energy product or maximum material energy  $(BH)_{\max}$ , which gives a magnitude of magnet power and how much it resists demagnetization. The  $(BH)_{\max}$  is calculated in the second quadrant of the material hysteresis loop. This curve is called also demagnetization curve. The more intense the magnetic property, the more the curve tends to become a straight line. On the contrary, the more "weak" the material, a knee point is shown in the second quadrant. Fig. 2 shows typical magnetic flux demagnetization curves at a given temperature for common permanent magnets also used in this study [14].

#### B. AlNiCo Magnets

The AlNiCo magnets were first introduced in 1931 and their major advantages are the long-lasting magnetism, the high temperature resistance and the slight change (degradation) in their characteristic curve due to temperature. Consequently, electric machines with AlNiCo magnets have a large magnetic induction in the gap and allow for high operating temperatures. However, these magnets, as mentioned earlier, have very little resistance to external demagnetization fields. AlNiCo magnets dominated the permanent magnet industry until 1970 when much more economical ceramic magnets (ferrite) began to be preferred to various applications.

#### C. Ceramic magnets (ferrites)

Ceramic magnets first appeared around 1950. These magnets have much higher coercivity field strength compared to AlNiCo magnets but have less residual magnetism value. The maximum permissible operating temperature is high; however, their magnetic properties are highly sensitive to temperature changes. The major advantages of ceramic magnets are the very low cost per unit of energy and the high electrical resistance resulting in the loss of eddy current inside them being almost negligible. The use of ceramic magnets on electric machines leads to more economical designs than AlNiCo magnets and is mainly used in low power machines (up to about 7kW) [15].

#### D. Rare Earth Magnets

Rare earth magnets are the latest generation magnets and have much better magnetic characteristics than the AlNiCo and ceramic counterparts. However, their significant disadvantage is their high cost. In the category of rare earth magnets, there are metal alloys based on samarium and cobaltium (SmCo) and neodymium, ferrum and boron (NdFeB). The SmCo magnets appeared around 1960 while the NdFeB magnets in the early 1970s. The characteristics of these magnets are the large values of the remaining magnetism and the demagnetization field as well as the sufficiently high energy value  $(BH)_{\max}$ . In particular, the demagnetization curve of these magnets is almost straight. Permanent rare earth magnets are used in electric machines from a few Watts to many kW, in a wide range of applications and machine types. They are mainly used where

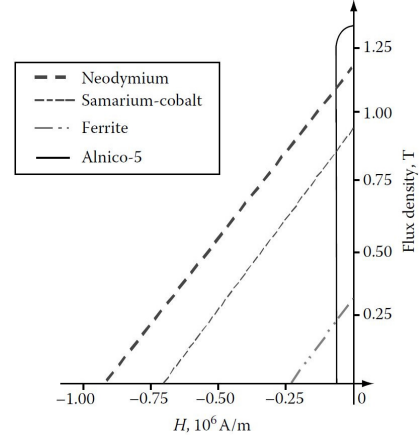


Fig. 2. Indicative B-H curves of four PM types of interest.

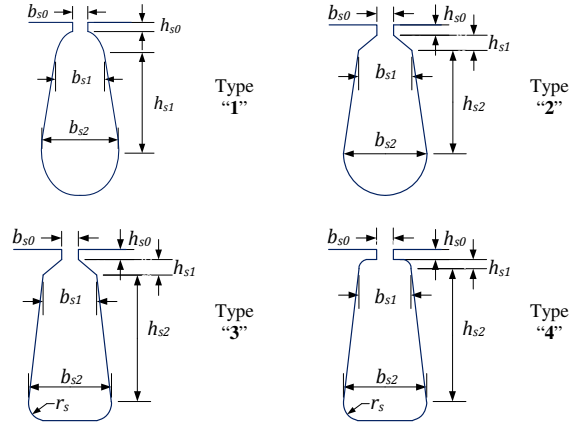


Fig. 3. Slot types examined here.

high performance indexes are required, i.e. power density, efficiency and dynamic behavior [16], [17].

#### E. Slot Types

From a geometrically point of view, in order to any slot shape to be achieved, four types can be used in electrical machines design for the majority of the topologies [18]. The characteristics of each type are depicted in Fig. 3.

### IV. MOTOR CONFIGURATION & CASES UNDER STUDY

With respect to the previous paragraphs, a typical 230V/50Hz, 24 slots, 4 poles, 550W motor is examined here. Table I summarizes the main operational characteristics, whereas Table II shows the relevant geometrical features. Also, Fig. 4 depicts the three phase winding configuration used. Because the magnet types to be tested are 8, and also the slot types are 4, there are 32 different (structural and geometrical) combinations (cases) as shown in Table III. Moreover, for each case, a parametric investigation is performed where: a) the embrace varied from 0.1 to 1.0 in steps of 0.1 and b) the magnet width varied from 0.1 mm to 3.9 mm in steps of 0.1 mm. In this way,  $32 \times 10 \times 39 = 12,480$  variants of the same BLDC motor were designed and analyzed. For each variant, among others, eight important quantities (speed, current, efficiency, output power, cogging torque, torque, magnets weight, and total weight) were

calculated and stored in equally result files. The investigation in step-like algorithmic form is presented next:

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Step 1 : Let the variables under concern be  $V_i$  ( $i=1..N$ )
Step 2 : Formulate initial BLDC motor topology.
Step 3 : Select magnet type.
Step 4 : Select slot type.
Step 5 : Initialize embrace ( $\tau_{act}/\tau$ ).
Step 6 : Initialize magnet thickness ( $l_m$ ).
Step 7 : Adjust BLDC motor topology accordingly.
Step 8 : Perform FEM analysis.
Step 9 : Calculate and save  $V_i$ .
Step 10 : Increase  $l_m$ .
Step 11 : If  $l_m \leq l_m^{\max}$  goto Step 7 else continue.
Step 12 : Increase ( $\tau_{act}/\tau$ ).
Step 13 : if ( $\tau_{act}/\tau$ )  $\leq 1$  goto Step 6 else continue.
Step 14 : Change slot type.
Step 15 : If not all slot types are examined goto
Step 4 else continue.
Step 16 : Change magnet type.
Step 17 : If not all magnet types are examined goto
Step 3 else continue.
Step 18 : End of analysis. Results post-processing
phase.

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## V. RESULTS, EVALUATION & DISCUSSION

From this large amount of obtained data, a careful post-processing phase is executed utilizing MATLAB plot capabilities and finally visualization takes place. In Fig. 5 and Fig. 6, 16 out of 256 plots are shown indicatively, for limited space reasons. With respect to Table III, Fig. 5 depicts plot results for case “C\_13”, while Fig. 6 shows the same quantities plot results for case “C\_24”. The following observations for each one of the eight quantities of interest are given below categorized by the magnet type used.

### A. Magnet Type: AlNiCo5

- 1) *Nominal speed*: Presents highest values for small magnet thicknesses ( $l_m$ ).
- 2) *Nominal Current*: It exhibits the same behavior as speed, while for larger values there is no variation.
- 3) *Efficiency*: Gets highest values for large embrace and  $w_m$  values.
- 4) *Output Power*: Tends to be maximum for large embrace and  $l_m$  values. In case of slot type 2, 3 and 4, it also tends to be very high for small  $l_m$ .
- 5) *Cogging Torque*: It is minimum for embrace=0.5.
- 6) *Rated Torque*: Increases as both the embrace and  $l_m$  increase and becomes maximum at their maxima.
- 7) *PM weight*: Increases as embrace and  $l_m$  increase.
- 8) *Total motor weight*: Higher for small  $l_m$  and large embrace.

### B. Magnet Type: AlNiCo9

- 1) *Nominal speed*: Is getting highest values for small  $l_m$  as well as for embrace values of 0.7-0.9 and  $w_m$  of 3 mm-3.8 mm. When embrace and  $l_m$  are maximum, speed drops.
- 2) *Nominal Current*: For the minimum values of embrace and  $l_m$ , it becomes maximum.
- 3) *Efficiency*: presents maximum values for embrace in the range 0.7-0.9 and  $l_m$  in the range 3mm-3.8 mm. A drop is noticed at their maximum values.
- 4) *Output Power*: Gets maximum and kept constant for embrace greater than 0.7 and  $l_m$  values greater than 3 mm.

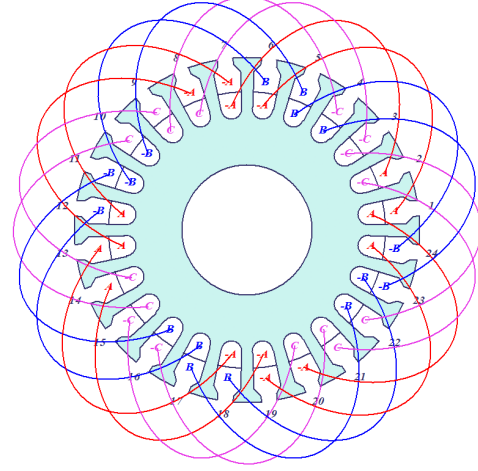


Fig. 4. Three phase winding (double layer) connection.

TABLE I. OPERATIONAL CHARACTERISTICS OF BLDC MOTOR

Quantity	Symbol	Value	Unit
Nominal Voltage	$V_N$	230	V
Nominal Input Power	$P_N$	550	W
Nominal (inverter) Frequency	$f_r$	50	Hz
Nominal Speed	$n$	1500	rpm
Desired Output Power		$\geq 400$	W
Friction/windage Losses		12	W
Number of phases		3	-
Winding connection (Y/ $\Delta$ )		Y	-

TABLE II. STRUCTURAL CHARACTERISTICS OF BLDC MOTOR

Quantity	Symbol	Value	Unit
Stator Outer Diameter	$D_{so}$	87	mm
Stator Inner Diameter	$D_{si}$	26	mm
Rotor Outer Diameter	$D_{ro}$	120	mm
Rotor Inner Diameter	$D_{ri}$	88	mm
Shaft Diameter	$D_{sh}$	26	mm
Axial Length	$L$	65	mm
Airgap length	$l_g$	1	mm
Stacking factor	$sf$	0.95	-
Rotor skew		no	-
Stator/Rotor Steel Type		M19	-
Mass density		7650	kg/m <sup>3</sup>
Number of poles	$p$	4	-
Number of stator Slots	$Q_s$	24	-
Winding layers		2	-
Coil pitch		5	-

TABLE III. CASE STUDIES EXAMINED

Feature→ Case ↓	Magnet Type	Slot Type	Feature→ Case ↓	Magnet Type	Slot Type
C_01	AlNiCo5	1	C_17	NdFeB30	1
C_02		2	C_18		2
C_03		3	C_19		3
C_04		4	C_20		4
C_05	AlNiCo9	1	C_21	NdFeB35	1
C_06		2	C_22		2
C_07		3	C_23		3
C_08		4	C_24		4
C_09	Ceramic5	1	C_25	SmCo24	1
C_10		2	C_26		2
C_11		3	C_27		3
C_12		4	C_28		4
C_13	Ceramic8D	1	C_29	SmCo28	1
C_14		2	C_30		2
C_15		3	C_31		3
C_16		4	C_32		4

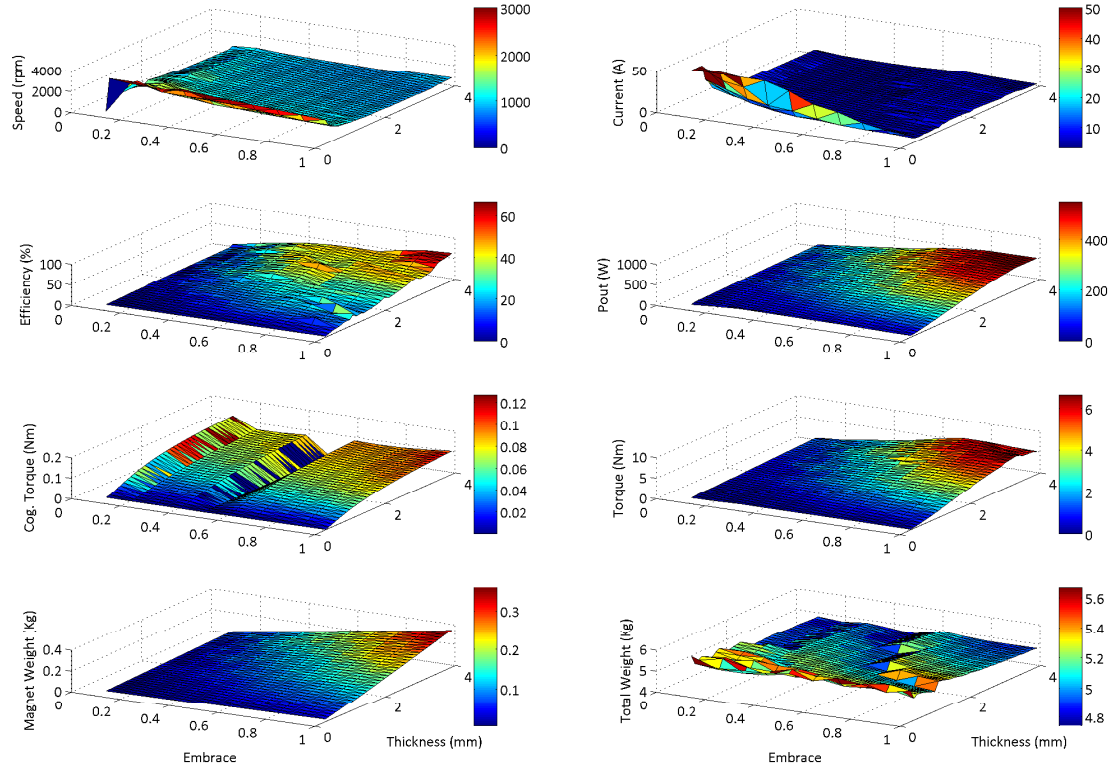


Fig. 5. Results of embrace and magnet thickness variation effect on BLDC motor speed, current, efficiency, output power, cogging torque, torque, magnets weight and total motor weight for Case "C\_13" (Table III).

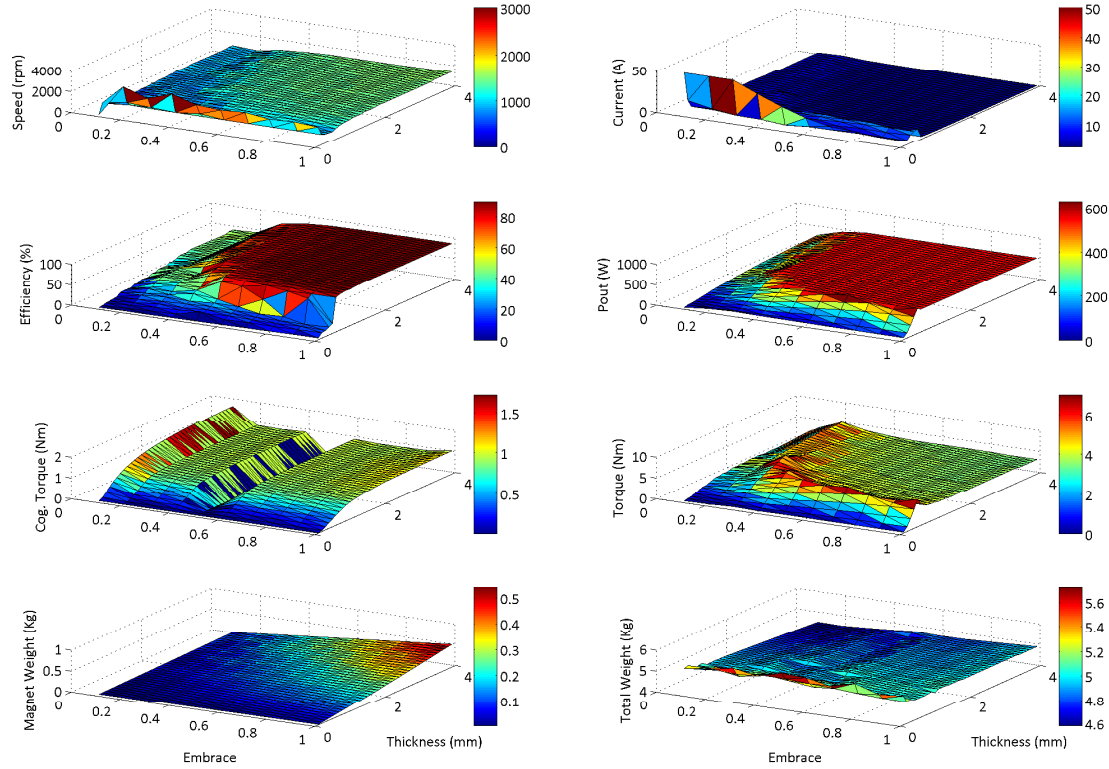


Fig. 6. Results of embrace and magnet thickness variation effect on BLDC motor speed, current, efficiency, output power, cogging torque, torque, magnets weight and total motor weight for Case "C\_24" (Table III).

- 5) *Cogging Torque*: Minimum for embrace=0.5.
- 6) *Rated Torque*: It becomes maximum for a magnet thickness of 3 mm and an embrace greater than 0.7.
- 7) *PM weight*: Increases as the embrace and  $w_m$  increases.
- 8) *Total motor weight*: Higher for small  $l_m$  and large embrace.

#### C. Magnet Type: Ceramic5 & Ceramic8D

- 1) *Nominal speed*: Exhibits high values for a small  $l_m$ .
- 2) *Nominal Current*: For  $l_m$  and embrace values greater than 0.3 mm and 0.2 respectively presents low and constant values.
- 3) *Efficiency*: Gets its highest values for large values of embrace and  $l_m$ .
- 4) *Output Power*: Increases as the values of the embrace and  $l_m$  increase and becomes maximum for their maxima.
- 5) *Cogging Torque*: Minimum for embrace=0.5 and maximum for embrace=0.1.
- 6) *Rated Torque*: Increases as embrace and  $l_m$  increase and becomes maximum for their maxima values.
- 7) *PM weight*: Increases as the embrace and  $l_m$  increases.
- 8) *Total motor weight*: tends to get large values when for a small  $l_m$  value, non-feasible configurations start to appear.

#### D. Magnet Type: NdFeB30

- 1) *Nominal speed*: For minimum values of  $l_m$  it gets maximum values.
- 2) *Nominal Current*: Same behavior as speed.
- 3) *Efficiency*: It becomes maximum and kept constant for a  $l_m$  greater than 1 mm and a embrace greater than 0.4.
- 4) *Output Power*: It becomes maximum and kept constant for a  $l_m$  greater than 0.6 mm and an embrace greater than 0.4.
- 5) *Cogging Torque*: Minimum for embrace=0.5 and maximum for embrace=0.1.
- 6) *Rated Torque*: Presents maximum values for and embrace between 0.2-0.4 or for  $l_m=0.1$  mm.
- 7) *PM weight*: Increases as the embrace and  $l_m$  increases.
- 8) *Total motor weight*: Same as Ceramic magnets.

#### E. Magnet Type: NdFeB35

- 1) *Nominal speed*: For low  $l_m$  values it is getting high, while for embrace values greater than 0.4 and  $l_m$  greater than 1 mm, it obtains a constant value. For slot type 3, this is more intense.
- 2) *Nominal Current*: For low  $l_m$  values it takes high values. For slot type 2 and 3, the highest values are shown for large embrace values, while for slot type 1 and 4 for small embrace values.
- 3) *Efficiency*: Becomes maximum and kept constant for  $l_m$  greater than 0.7 mm and an embrace greater than 0.4.
- 4) *Output Power*: Becomes maximum and kept constant for  $l_m$  greater than 0.6 mm and an embrace value greater than 0.4.
- 5) *Cogging Torque*: Minimum for embrace=0.5 and maximum for embrace=0.1.

- 6) *Rated Torque*: Presents maximum values for embrace between 0.2-0.4 or for  $l_m=0.1$  mm.
- 7) *PM weight*: increases as the embrace and  $l_m$  increase.
- 8) *Total motor weight*: Same as Ceramic magnets.

#### F. Magnet Type: SmCo24 & SmCo28

- 1) *Nominal speed*: for minimum values of variable  $l_m$  it gets maximum values.
- 2) *Nominal Current*: For low  $l_m$  values it takes high values. For slot type 1, 2 and 3, the largest values are shown for low embrace values, while for slot type 4 it shows higher values for an embrace between 0.5-0.8.
- 3) *Efficiency*: Gets maximum and constant values for  $l_m$  greater than 0.7 mm and an embrace value greater than 0.4.
- 4) *Output Power*: it becomes maximum and kept constant for  $l_m$  greater than 0.5 mm and an embrace value greater than 0.5.
- 5) *Cogging Torque*: Minimum for embrace=0.5 and maximum for embrace=0.1.
- 6) *Rated Torque*: Shows the highest values for embrace between 0.2-0.4 or for  $l_m=0.1$  mm.
- 7) *PM weight*: Increases as the embrace and  $l_m$  increases.
- 8) *Total motor weight*: Same as Ceramic magnets.

#### G. Overall Discussion

As more general conclusions from the obtained results the following are valid:

- For small magnet thickness values, speed and current vary slightly regardless of the magnet type. Additionally, for NdFeB and SmCo magnets, after a certain point, they obtain maximum and constant values.
- For AlNiCo type and Ceramic type magnets, efficiency and rated power increase as the embrace and magnet thickness increase. For NdFeB and SmCo magnets, after a certain point, they obtain maximum and constant values.
- Minimization of cogging torque is observed in all case for an embrace value of 0.5.
- For AlNiCo type and Ceramic type magnets, the nominal torque increases as the embrace and magnet thickness increases. For NdFeB and SmCo magnets the torque obtains maximum values for small embrace and/or small magnet thicknesses, while their further increase reduces torque.
- Higher efficiency (greater than 80%) and higher output power (greater than 500W) is noted for NdFeB and SmCo magnets.

## VI. CONCLUSIONS

Many factors play a vital role in the design of brushless DC motors. Since this kind of electrical machines are used widely in a large number of applications nowadays, the careful considerations of these factors can lead to more effective designs. This study considered the permanent magnet type and its dimensions, the slot type and the pole-arc to pole-pitch ratio variations and their effect on many parameters of interest such as the speed, the current, the



efficiency, the power, the torque and the weight of the overall BLDC structure, in a quantitative and qualitative manner. FEM analysis validation was performed and several conclusions were extracted through the commenting of the obtained results.

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