QE3. Nanoelectronics

- Single-electron Transistors

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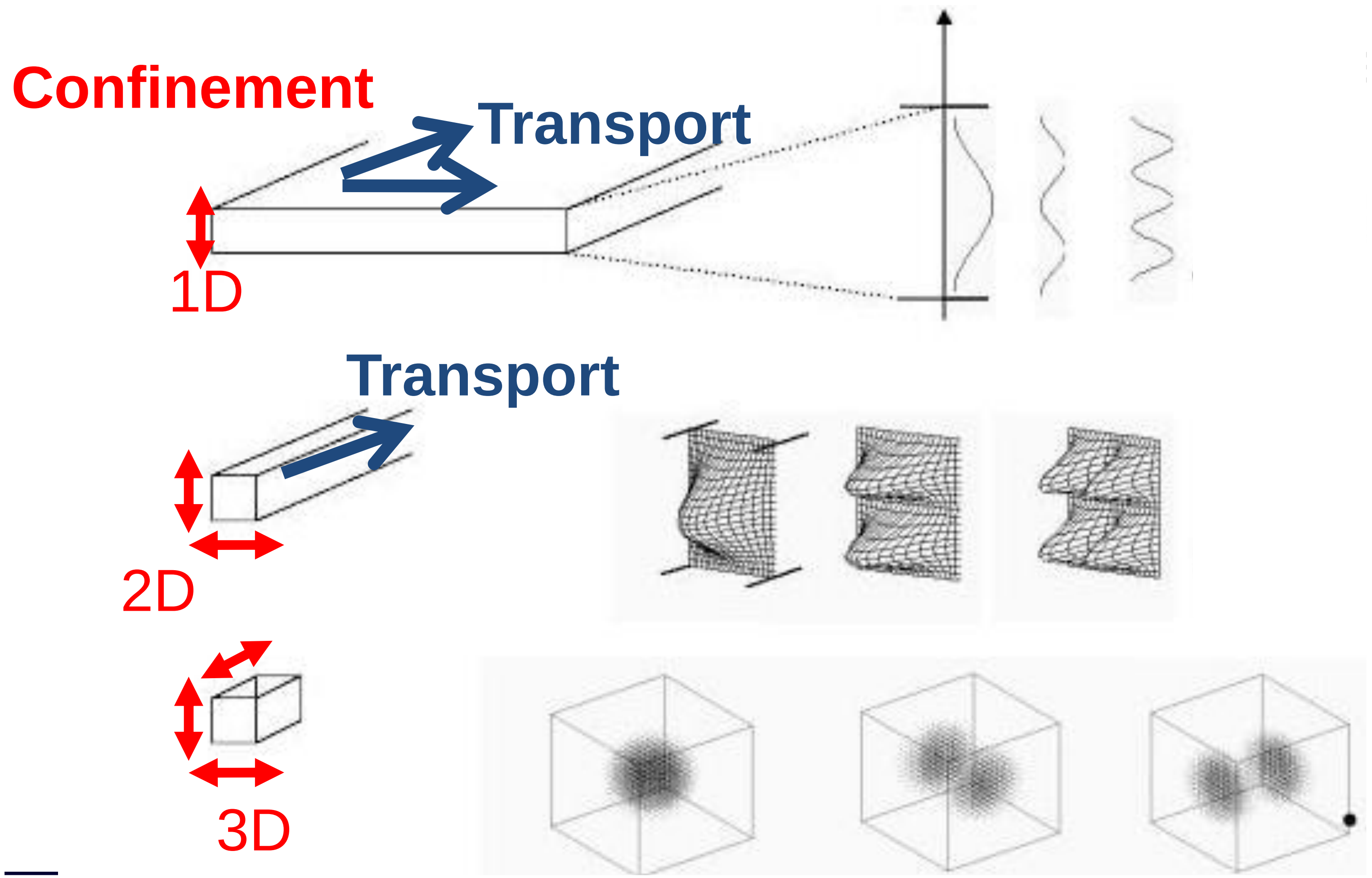
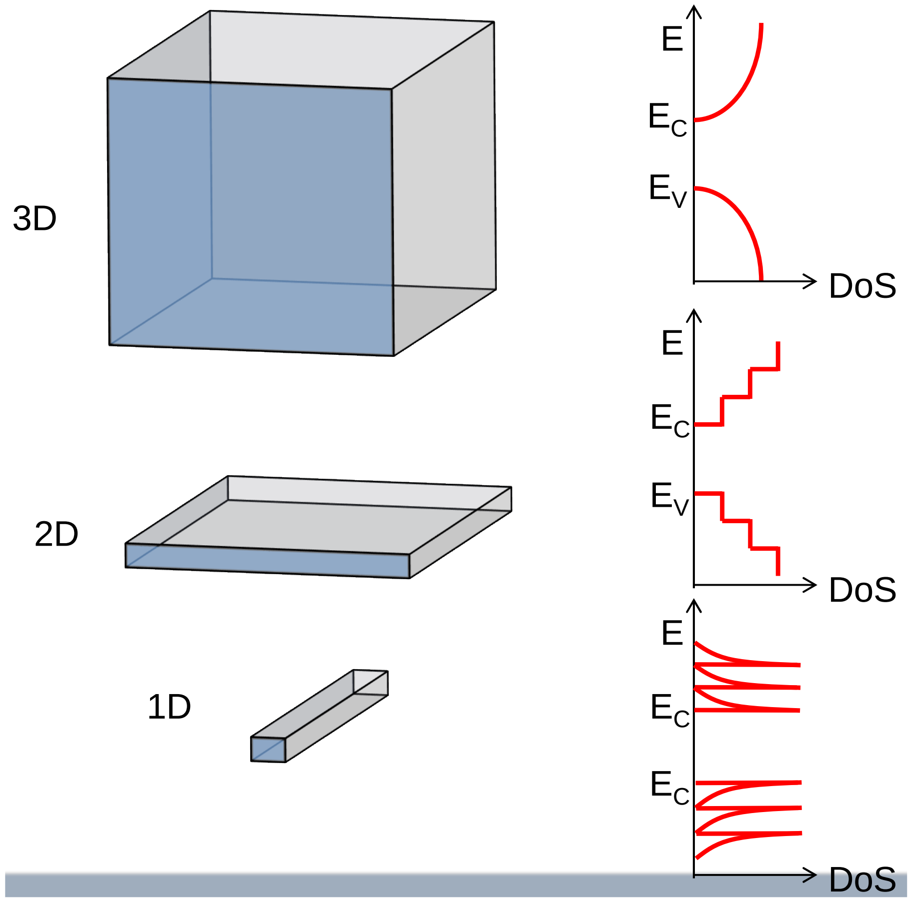
Confinement effects: wavefunctions



Confinement effects: DoS



Free electrons and Confined electrons

The fact that light is made up of photons lead Louis de Broglie in 1923 to make the radical suggestion that all "particles" having energy E and momentum p should have wavelike properties, too.

For photons (waves)

$E=h.f$

$c = \lambda/T = \lambda f$

$E = h.c / \lambda$

$E = p.c$

$p = h / \lambda = h.k/(2\pi)$

Free electron condition

$\lambda_e << L_x, L_y, L_z$

de Broglie wavelength

$p=mv= (2mE)^{1/2}$

$\lambda_e = h / (2m_eE)^{1/2}$

$E = 1$ eV
$\lambda_e = 1,23$ nm

The de Broglie wavelength at the Fermi energy is called the Fermi wavelength, and is denoted by the symbol λ_F . Therefore, for a space to be sufficiently "large" so that the energy levels of the electron form an approximately continuous set, we usually require

$\lambda_F = h / (2m_eE_F)^{1/2}$

Confinement condition

$\lambda_F > L_x$ 1D confinement

$\lambda_F >> L_x, L_y$ 2D confinement

$\lambda_F >> L_x, L_y, L_z$ 3D confinement

Electron movement will be confined in all three directions (i.e., electrons will "feel" the boundaries in the x-, y-, and z-directions), exhibiting energy quantization in three dimensions, and will not be free in any direction. This makes for an effectively zero-dimensional system called a quantum dot.

Fermi energy of materials

Element	Fermi Energy eV	Fermi Temperature x 10^4 K	Fermi Velocity x 10^6 m/s
Li	4.74	5.51	1.29
Na	3.24	3.77	1.07
K	2.12	2.46	0.86
Rb	1.85	2.15	0.81
Cs	1.59	1.84	0.75
Cu	7.00	8.16	1.57
Ag	5.49	6.38	1.39
Au	5.53	6.42	1.40
Be	14.3	16.6	2.25
Mg	7.08	8.23	1.58
Ca	4.69	5.44	1.28
Sr	3.93	4.57	1.18
Ba	3.64	4.23	1.13
Nb	5.32	6.18	1.37
Fe	11.1	13.0	1.98
Mn	10.9	12.7	1.96
Zn	9.47	11.0	1.83
Cd	7.47	8.68	1.62
Hg	7.13	8.29	1.58
Al	11.7	13.6	2.03
Ga	10.4	12.1	1.92
In	8.63	10.0	1.74
Tl	8.15	9.46	1.69
Sn	10.2	11.8	1.90
Pb	9.47	11.0	1.83
Bi	9.90	11.5	1.87
Sb	10.9	12.7	1.96

$$E_F = \left[\frac{(hc)^2}{8mc^2}\right]\left[\frac{3}{\pi}\right]^{2/3} n^{2/3}$$

n free electron density (m^{-3})

Ashcroft, N. W. and Mermin, N. D., Solid State Physics, Saunders, 1976

Energy Levels in QDs

$$E = \frac{\hbar^2 \pi^2}{2m_e}\left(\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2\right)$$

$$= E_{n_x, n_y, n_z}$$

Zero-point energy (zpe)

$L_x = L_y = L_z = l$

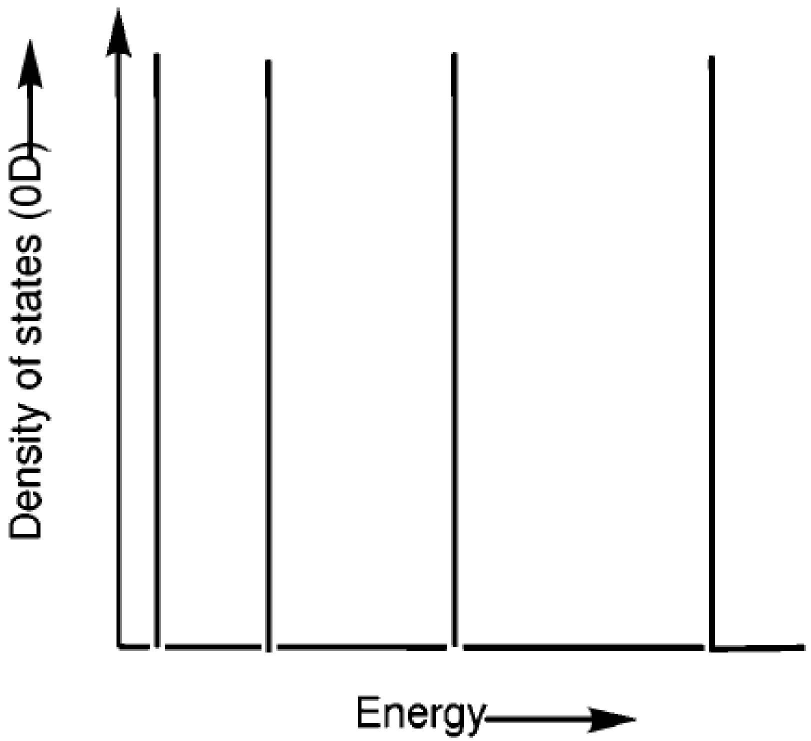
$n_x = n_y = n_z = 1$

Box

$$E_{zpe} = \frac{3h^2}{8m_e^* l^2}$$

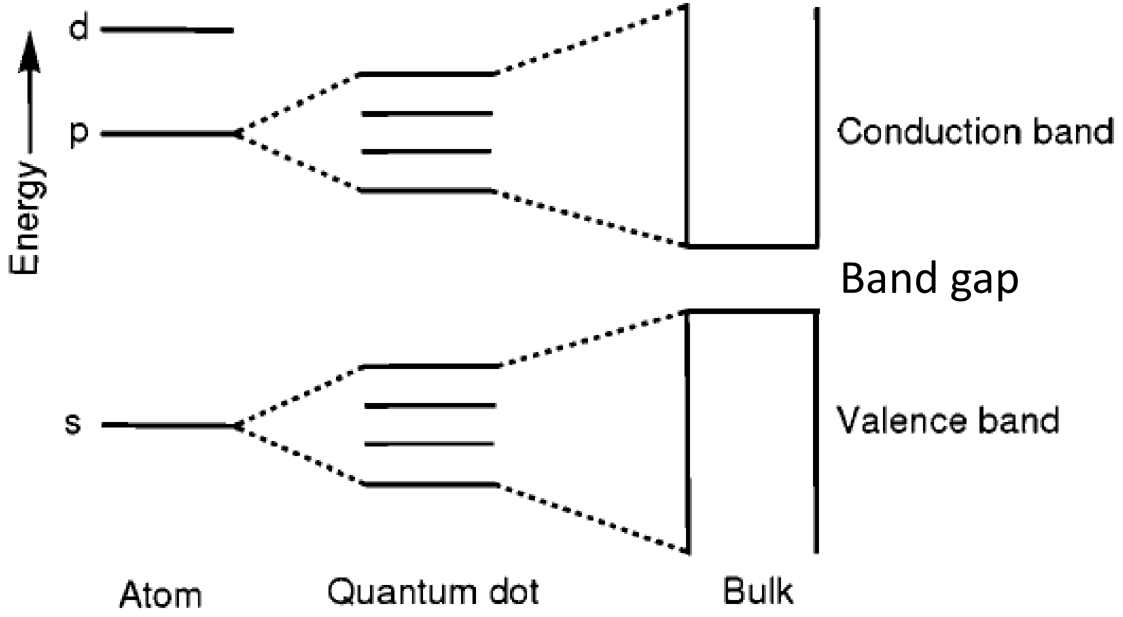
Sphere

$$E_{zpe} = \frac{h^2}{2m_e^* d^2}$$



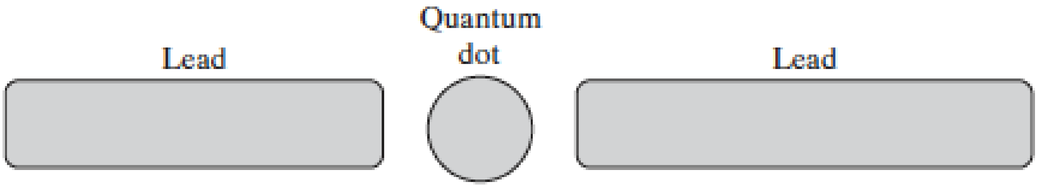
QD Energy Bandgap

QD Energy band diagram

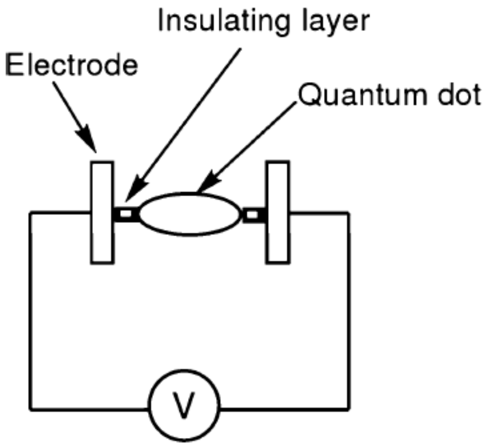


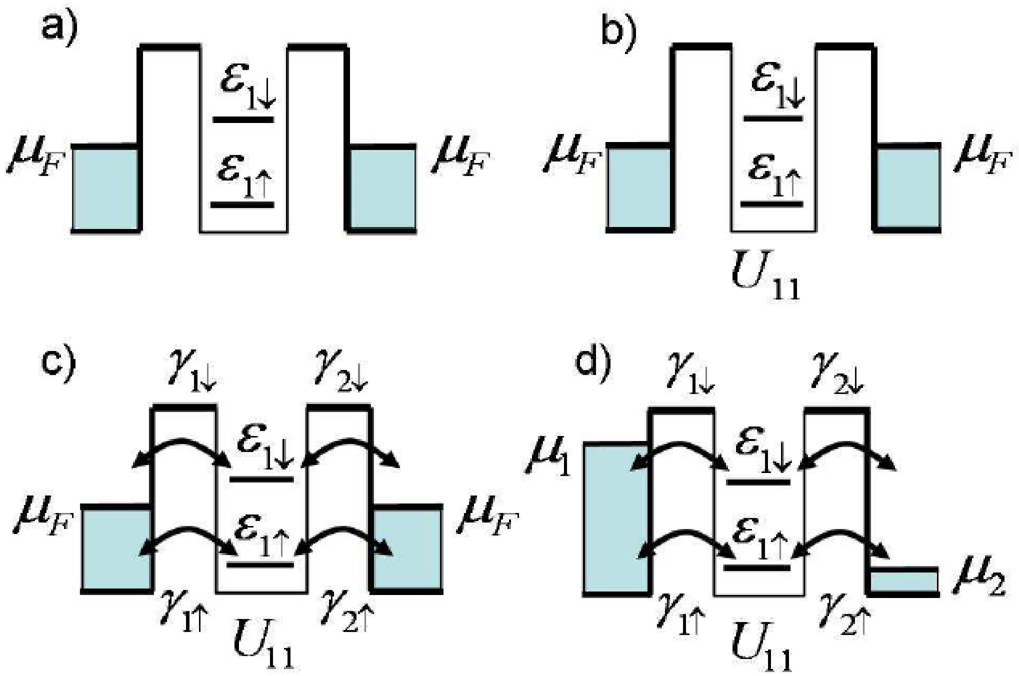
$$E_{g(dot)} = E_{g(bulk)} + E_{conf.} + E_{Coulomb} = E_{g(bulk)} + \frac{h^2}{2m^* l^2} - \frac{1.8e^2}{2\pi\varepsilon\varepsilon_o l}$$

QD measurements



$$Q = CV, \qquad E = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$





Single Electron Capacitor

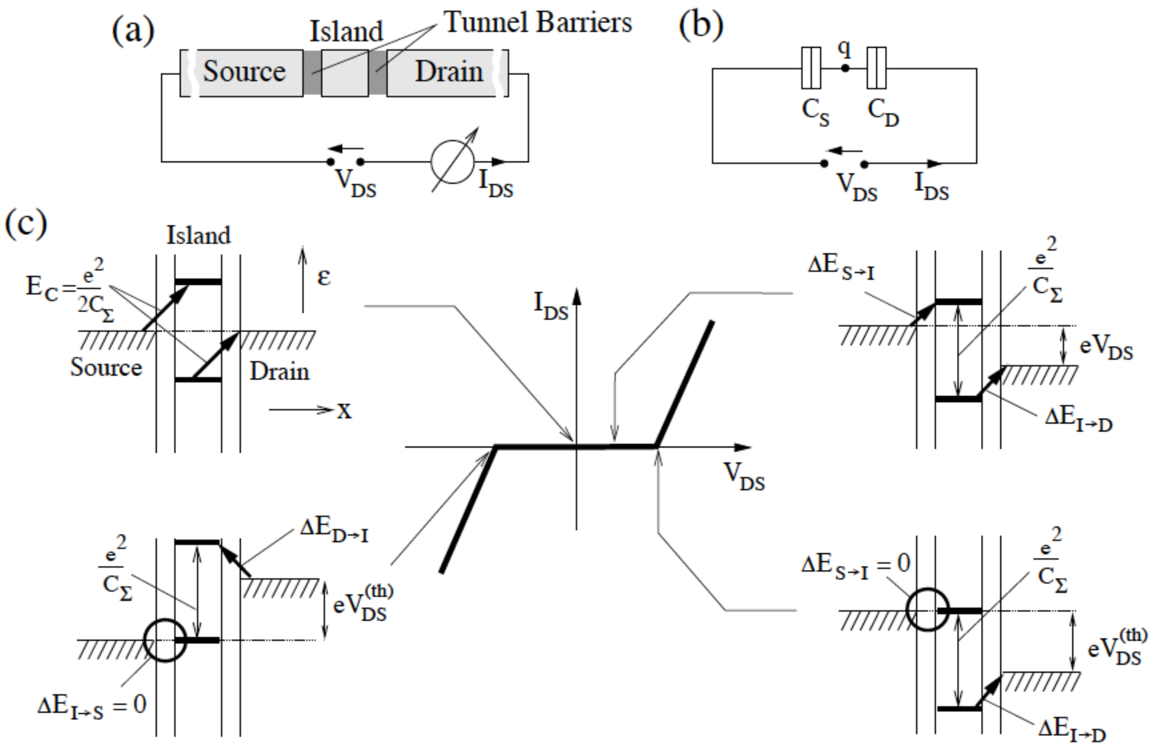


Fig. 1.2: (a) Two-terminal arrangement for discussing the Coulomb blockade effect in electrical transport. (b) The respective capacitance circuit. Note $C_\Sigma = C_\mathrm{S} + C_\mathrm{D}$. (c) Sketch of the expected non-linear $I_\mathrm{DS}(V_\mathrm{DS})$ characteristic with energy schemes for distinct V_DS values reflecting the energetical position of the Fermi levels of the island for charge states $q = -e$ and $q = e$ relatively to the Fermi level of source and drain.

Electrostatic Barriers, $V_{DS}>0$

$$\Delta E_{\mathrm{S}\to\mathrm{I}} = E_\mathrm{C} - e\,\frac{C_\mathrm{D}}{C_\Sigma}\,V_\mathrm{DS}$$

$$\Delta E_{\mathrm{I}\to\mathrm{D}} = E_\mathrm{C} + e\,\frac{C_\mathrm{D}}{C_\Sigma}\,V_\mathrm{DS} - e\,V_\mathrm{DS}$$

Ecurrent suppression

$$|V_\mathrm{DS}| \geq V_\mathrm{DS}^{(\mathrm{th})} \equiv \min\left(\frac{e}{2\,C_\mathrm{S}};\frac{e}{2\,C_\mathrm{D}}\right)$$

Single Electron Transistor – SET

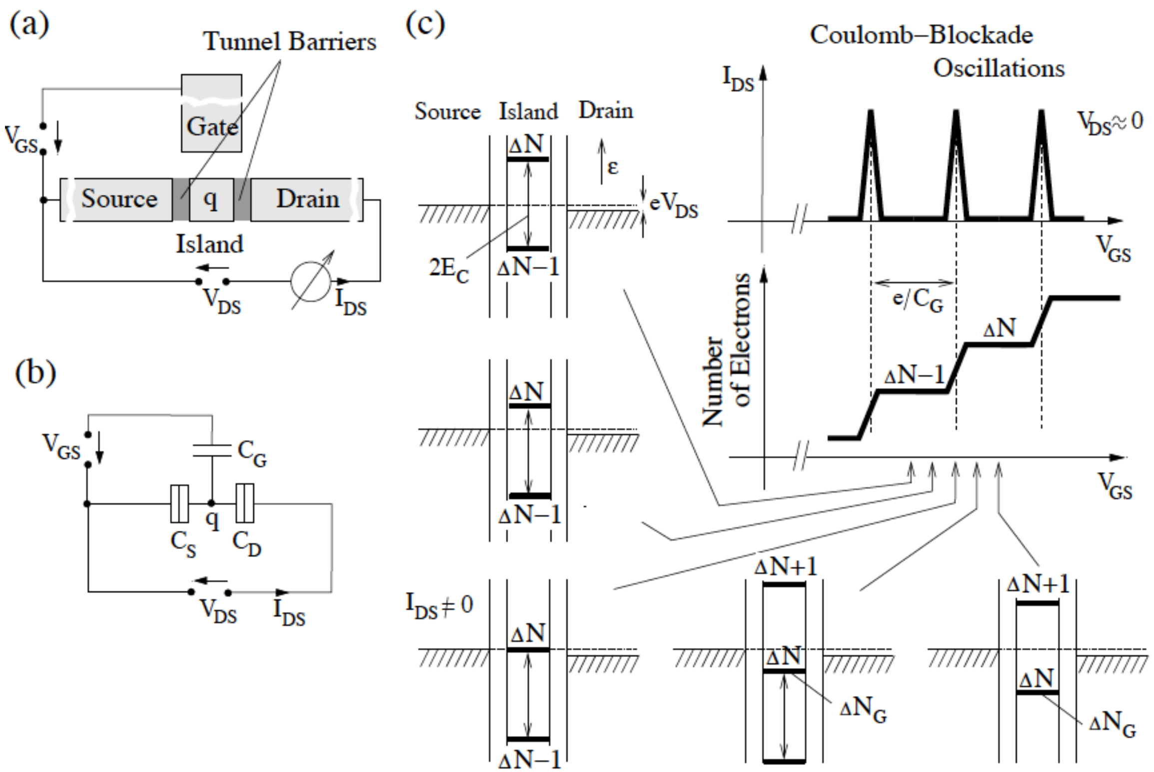


Fig. 1.3: (a) Three-terminal arrangement of a single-electron transistor. (b) The respective capacitance circuit. Note $C_\Sigma = C_S + C_D + C_G$. (c) With increasing gate voltage V_{GS}, electrons are accumulated on the island. Whenever the charge state can energetically fluctuate by e, i.e., the energy for two charge states is degenerate, current I_{DS} flows for small applied V_{DS} through the island, leading to a periodically modulated $I_{DS}(V_{GS})$-characteristic – the Coulomb blockade oscillations. For distinct V_{GS} values, the respective energy schemes are given.

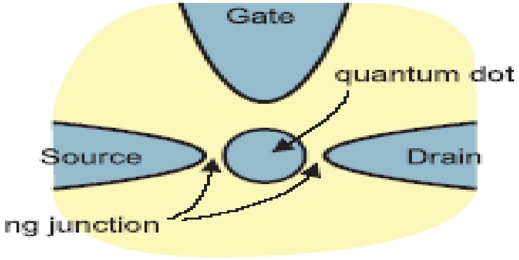
Electrostatic Barrier, Single electron charging

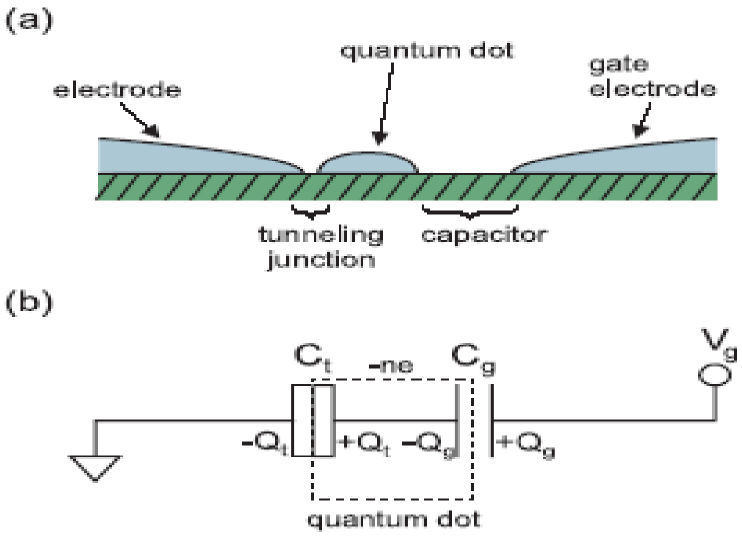
$$\Delta E_{S \to I} = E_C - e\,\frac{C_G}{C_\Sigma}\,V_{GS} \overset{!}{=} 0$$

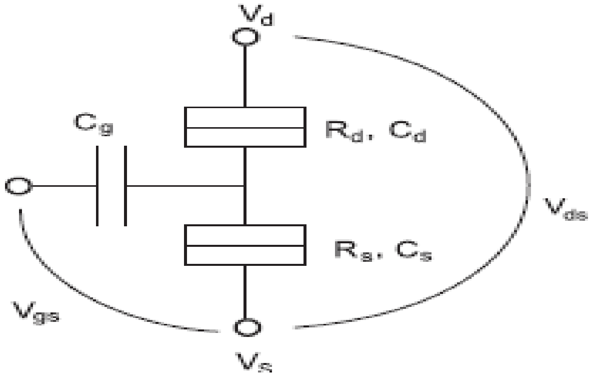
$$V_{GS}^{(th)} = \frac{E_C}{e\,C_G/C_\Sigma} = \frac{e}{2\,C_G}$$

Single Electron Transistor – SET

- Conditions for observing single electron tunneling phenomena
    - $E_c > k_b T$
        - $E_c = e^2/2C_\Sigma$
    - $R_t > R_k$
        - $R_k = h/e^2$ (25.8 KOhms)







Example

Calculate the size of a sphere shaped quantum dot of Si that would produce observable single electron effect at room temperature.

Solution. The energy change on charging of the quantum dot capacitor should be much larger than kT in order to observe the single electron effects.

At 300 K, $kT = 1.38 \times 10^{-23}$ J K^{-1} x 300 K = 414 x 10^{-23} J

Taking 1 eV = 1.602 x 10^{-19} J, kT = 258.43 x10^{-4} eV = 25.84 meV.

The energy change on charging of the quantum dot by a single electron

$$= q^2/2C = e^2/2C$$

The capacitance of the *sphere* shaped capacitor, $C = 4\pi\varepsilon\varepsilon_0 r = 4\pi \times 11.5 \times 8.85 \times 10^{-12} \times r$

(taking the dielectric constant of silicon to be 11.5 and the permittivity of vacuum , $\varepsilon_0 = 8.85 \times 10^{-12}$ F.m^{-1}.)

$C = 1278.294 \times r \times 10^{-12}$ F = $1.278 \times r \times 10^{-18}$ F, when r is taken in nm.

Energy change on charging by a single electron

$$= e^2/2C = (1.6 \times 10^{-19})^2/2 \times 1.278 \times r\ 10^{-18} \text{ J} = = 0.0626/r \text{ eV}$$

This energy should be much larger than kT for the single electron effect to be observable

i.e. $0.0626/r$ eV >> 0.02584 eV at 300K

or $0.0626/r \approx 0.5 \times 0.02584 = 0.1292$ (say) or $r \approx 0.5$ nm

Hence the quantum dot should have a radius of the order of 0.5 nm for this effect to be observable at room temperature.

Single Electron Transistor – SET

- Conditions for observing single electron tunneling phenomena
  - $E_c > k_b T$
    - $E_c = e^2/2C_\Sigma$
  - $R_t > R_k$
    - $R_k = h/e^2$ (25.8 KOhms)

Another requirement for observing the single electron effects is that the fluctuations in the number of electrons in the quantum dot should be negligible. The time constant for an R-C circuit is RC. The time taken by an electron to move in or out of a junction should be of this order.

According to the Heisenberg uncertainty principle, the product of the energy change accompanying this transfer and the time taken should be larger than h, the Planck's constant

$$\Delta E.\Delta t > h$$

$$\text{or } (e^2/2C).RC > h$$

$$\text{or } R > 2\,h/e^2 = 51.6\ \text{k}\Omega.$$

Single electron charging (II)

Electrostatic Energy, ΔN electron charging

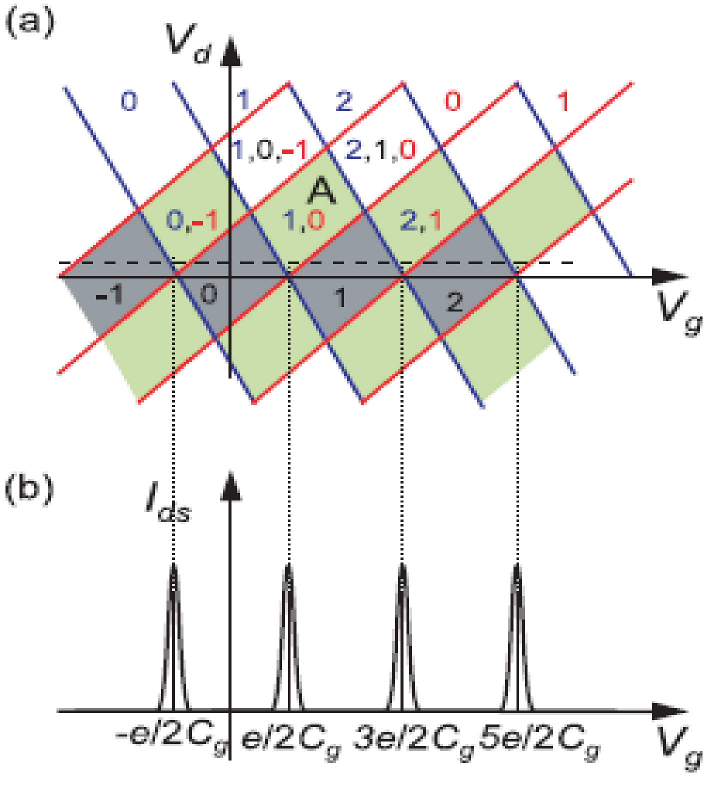
$$E_{\mathrm{elst}}(\Delta N; V_{\mathrm{GS}}, V_{\mathrm{DS}}) = -\Delta N\, e \left(\frac{C_{\mathrm{G}}}{C_{\Sigma}} V_{\mathrm{GS}} + \frac{C_{\mathrm{D}}}{C_{\Sigma}} V_{\mathrm{DS}} \right) + \frac{(\Delta N\, e)^2}{2\, C_{\Sigma}}$$

Electrostatic Barriers, ΔN + 1 electron charging

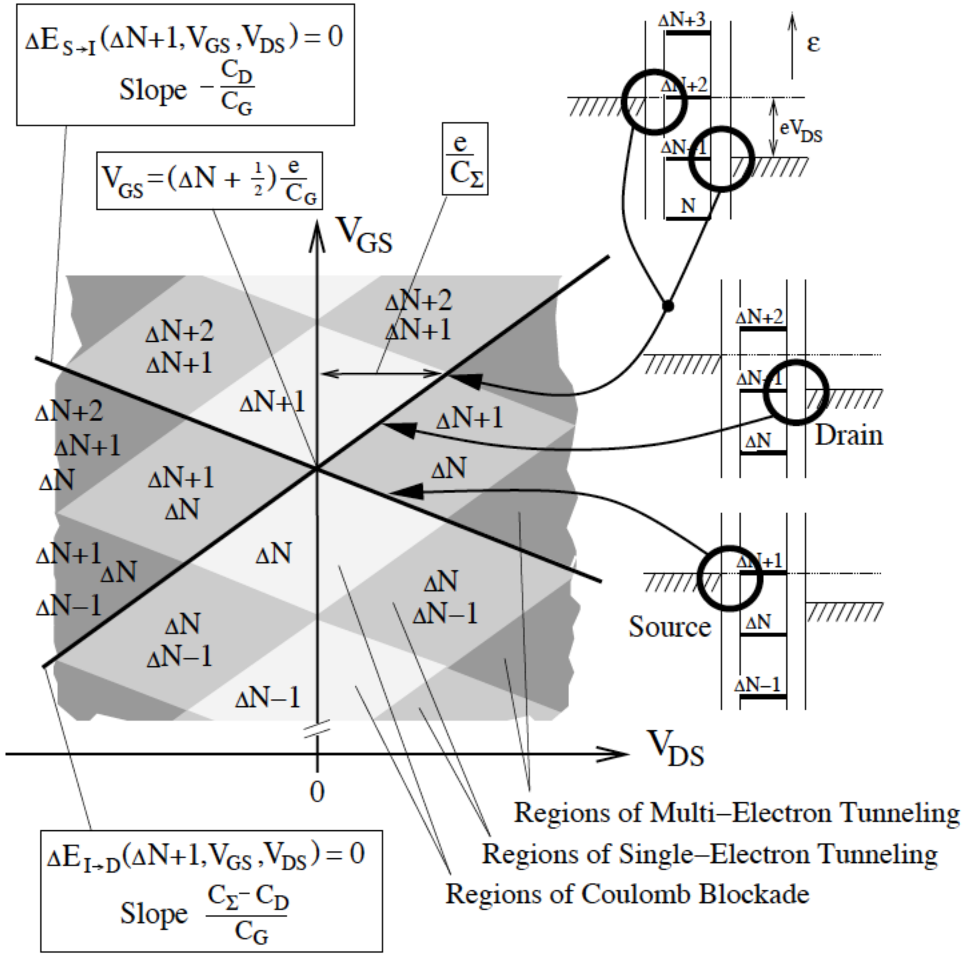
$$\begin{aligned} \Delta E_{\mathrm{S}\to\mathrm{I}}(\Delta N + 1; V_{\mathrm{GS}}, V_{\mathrm{DS}}) &= E_{\mathrm{elst}}(\Delta N + 1; V_{\mathrm{GS}}, V_{\mathrm{DS}}) - E_{\mathrm{elst}}(\Delta N; V_{\mathrm{GS}}, V_{\mathrm{DS}}) \\ &= (\Delta N + \tfrac{1}{2}) \frac{e^2}{C_{\Sigma}} - e \frac{C_{\mathrm{G}}}{C_{\Sigma}} V_{\mathrm{GS}} - e \frac{C_{\mathrm{D}}}{C_{\Sigma}} V_{\mathrm{DS}} \,. \end{aligned}$$

$$\begin{aligned} \Delta E_{\mathrm{I}\to\mathrm{D}}(\Delta N; V_{\mathrm{GS}}, V_{\mathrm{DS}}) &= E_{\mathrm{elst}}(\Delta N - 1; V_{\mathrm{GS}}, V_{\mathrm{DS}}) - e\, V_{\mathrm{DS}} - E_{\mathrm{elst}}(\Delta N; V_{\mathrm{GS}}, V_{\mathrm{DS}}) \\ &= -(\Delta N - \tfrac{1}{2}) \frac{e^2}{C_{\Sigma}} + e \frac{C_{\mathrm{G}}}{C_{\Sigma}} V_{\mathrm{GS}} - e \left(1 - \frac{C_{\mathrm{D}}}{C_{\Sigma}}\right) V_{\mathrm{DS}} \,. \end{aligned}$$

SET Measurements

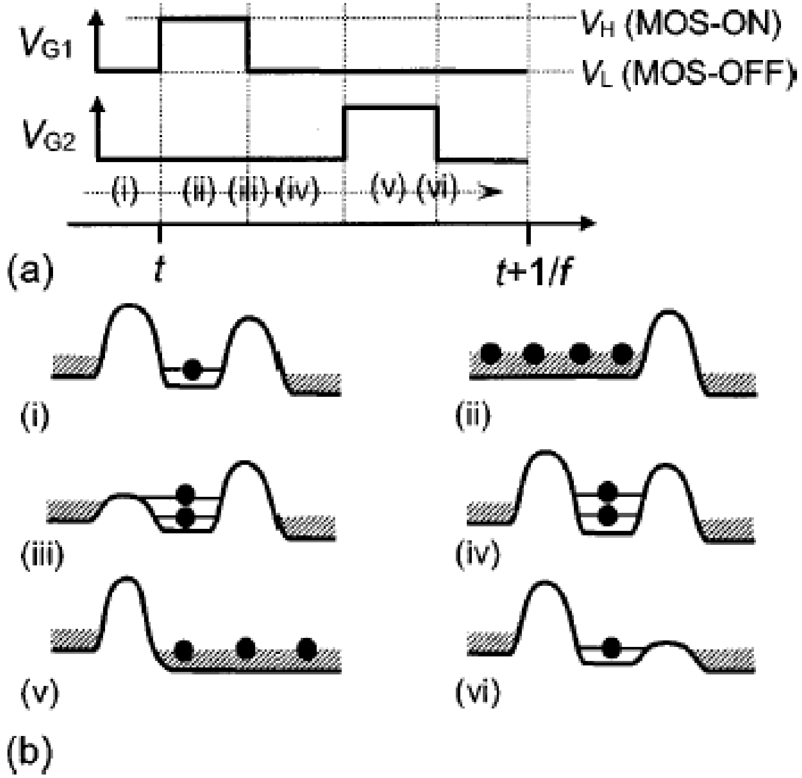


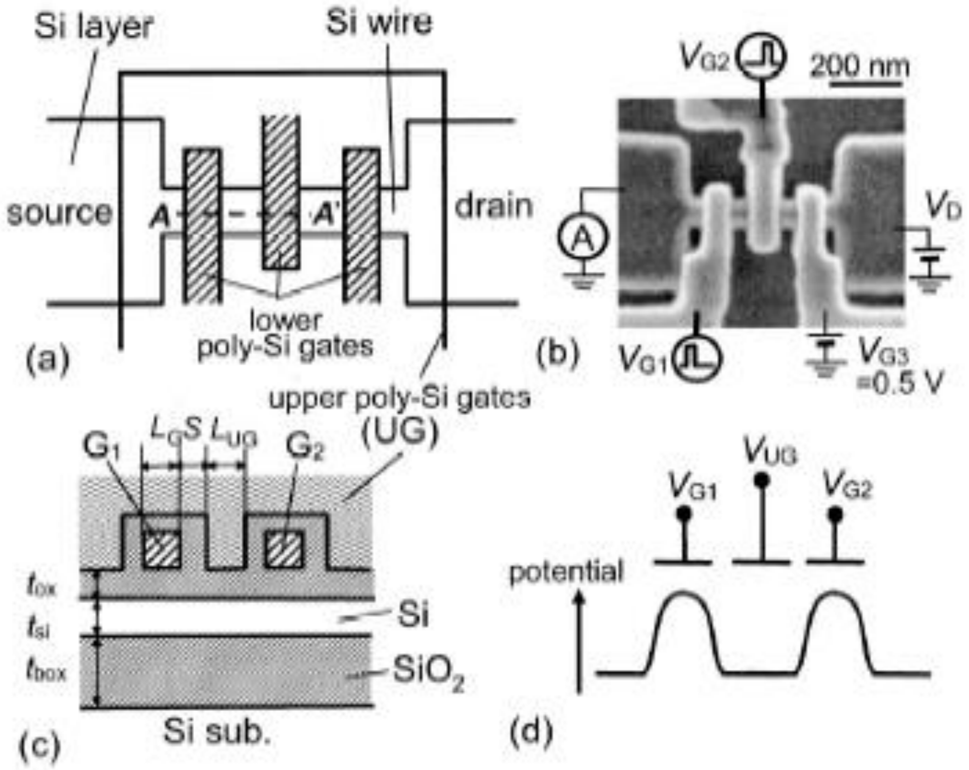
Current flows when $V_g = ne/2C_g$



Example

30 nm wide Si-wire channel and poly-Si gates defined by E-beam lithography





Current quantization due to single electron transfer in Si-wire charge coupled device, Applied Physics Letters, Vol 84 (8), 23 Feb 2004, 1323

The End

THANK YOU

E-class Support

Lesson	Kassap	Hanson
1		Chap.4, 9.1, 9.2, 9.3.1
2		Chap. 7
3		