QY2. Quantum Devices

Quantum Tunnelling:

- **Review: Barrier Reflection**
- **Barrier Penetration (Tunneling)**

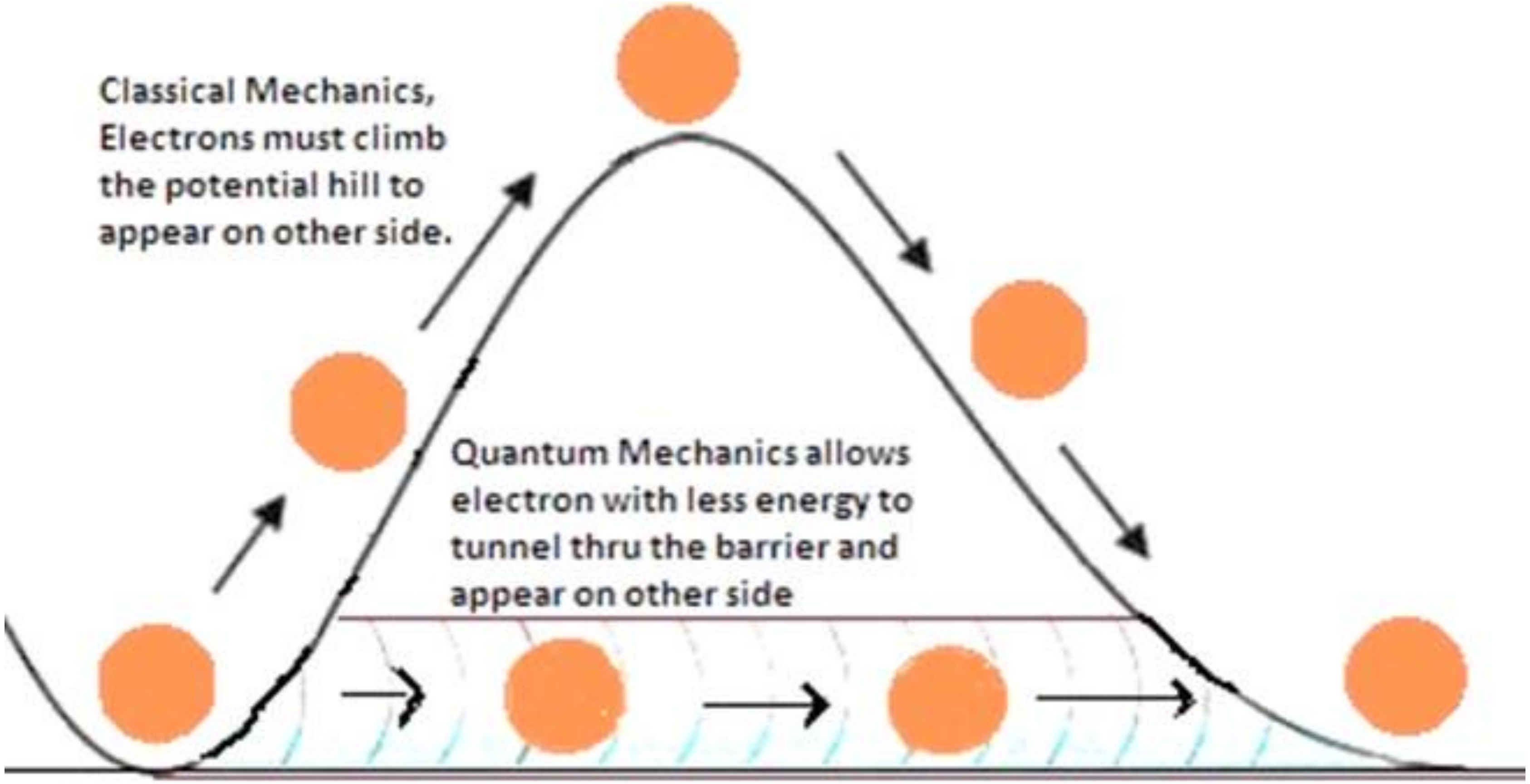
Dr Panagiotis Dimitrakis

DEMOCRITUS UNIVERSITY OF THRACE | DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING

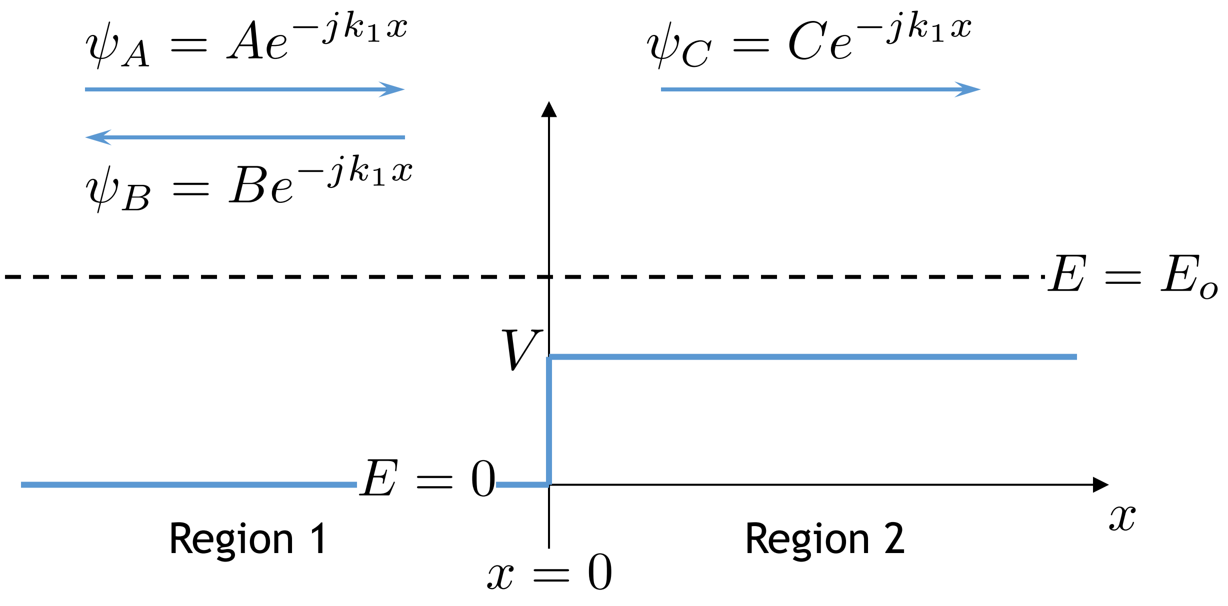
MSc in QUANTUM COMPUTING AND QUANTUM TECNNOLOGIES

NATIONAL CENTRE FOR SCIENTIFIC RESEARCH "DEMOKRITOS"

Mission imposible in Classical Mechanics



A Simple Potential Step



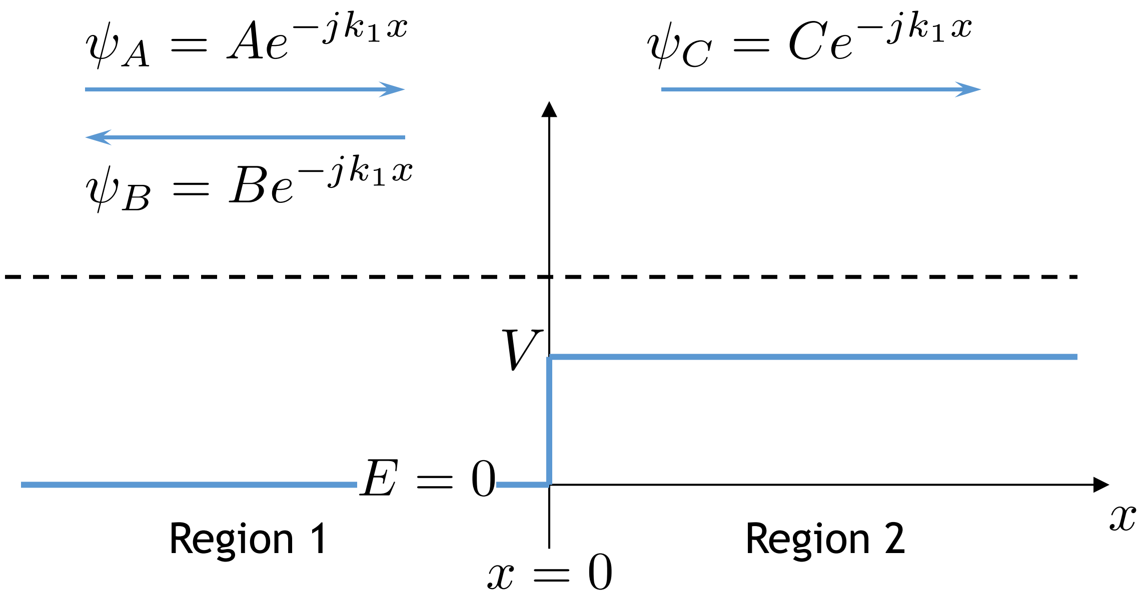
CASE I : $E_o > V$

In Region 1: $$E_o \psi = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} \Rightarrow k_1^2 = \frac{2mE_o}{\hbar^2}$$

In Region 2: $$\left(E_o - V\right)\psi = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} \Rightarrow k_2^2 = \frac{2m\left(E_o - V\right)}{\hbar^2}$$

A Simple Potential Step

CASE I : $E_o > V$



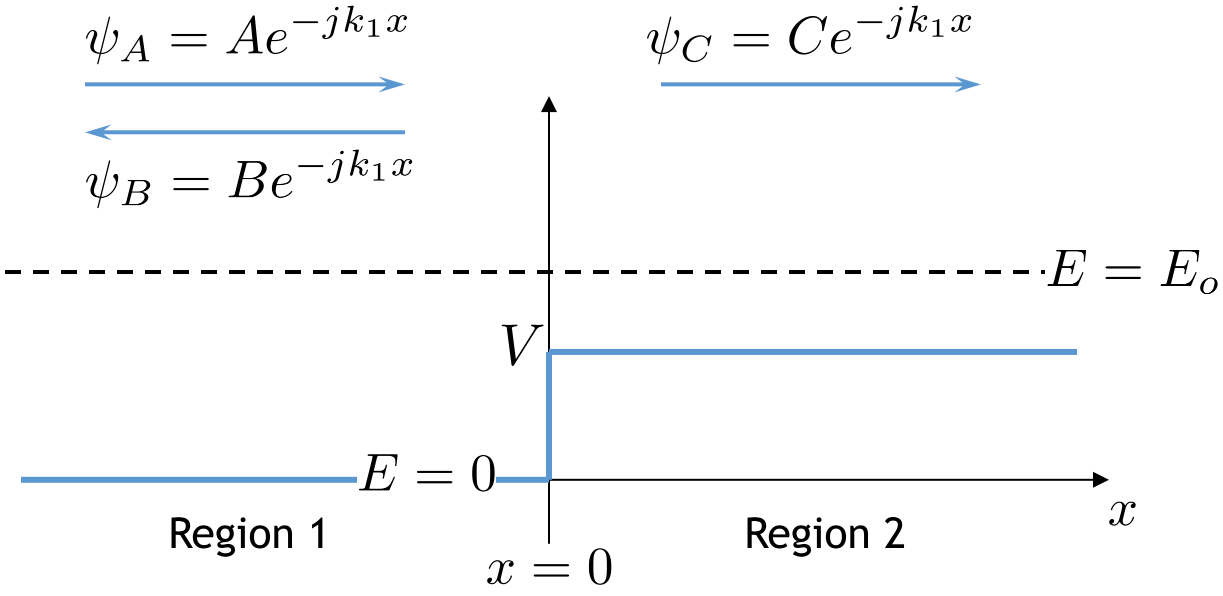
$$\psi_1 = Ae^{-jk_1x} + Be^{jk_1x} \qquad \psi_2 = Ce^{-jk_2x}$$

ψ is continuous: $\psi_1(0) = \psi_2(0) \Rightarrow A + B = C$

$\dfrac{\partial \psi}{\partial x}$ is continuous: $\dfrac{\partial}{\partial x}\psi(0) = \dfrac{\partial}{\partial x}\psi_2(0) \Rightarrow A - B = \dfrac{k_2}{k_1}C$

A Simple Potential Step

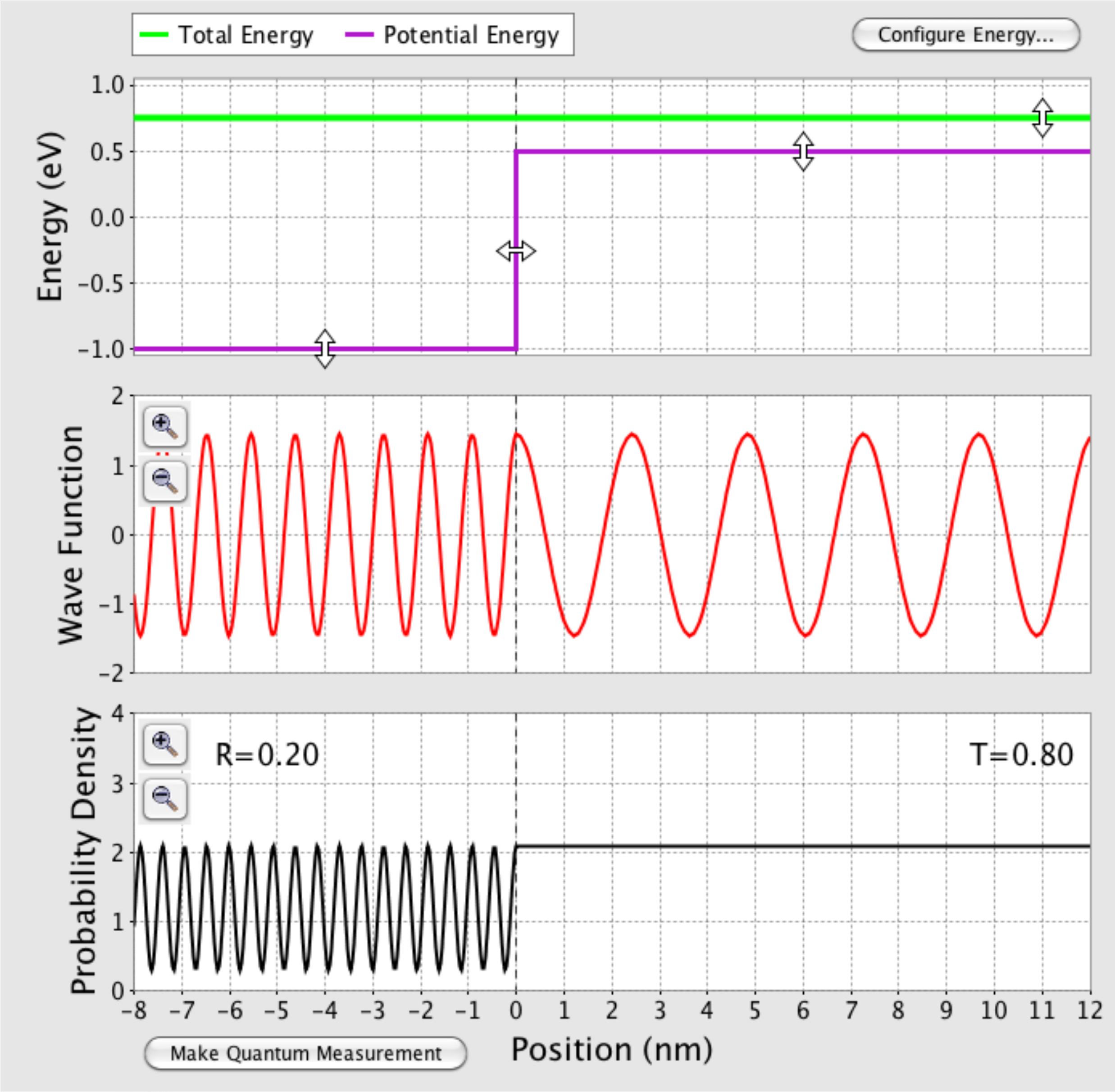
CASE I : $E_o > V$



$$\begin{cases} A + B = C \\ A - B = \dfrac{k_2}{k_1} C \end{cases}$$

$$\frac{B}{A} = \frac{1 - k_2/k_1}{1 + k_2/k_1} = \frac{k_1 - k_2}{k_1 + k_2}$$

$$\frac{C}{A} = \frac{2}{1 + k_2/k_1} = \frac{2k_1}{k_1 + k_2}$$


Total Energy
Potential Energy
Configure Energy...
Energy (eV)
1.0
0.5
0.0
−0.5
−1.0
Wave Function
2
1
0
−1
−2
Probability Density
4
3
2
1
0
R=0.20
T=0.80
−8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8 9 10 11 12
Make Quantum Measurement
Position (nm)

Quantum Electron Currents

Given an electron of mass m

that is located in space with charge density $\rho = q\left|\psi(x)\right|^2$

and moving with momentum $<p>$ corresponding to $<v> = \hbar k/m$

… then the current density for a *single electron* is given by

$$J = \rho v = q\left|\psi\right|^2 (\hbar k/m)$$

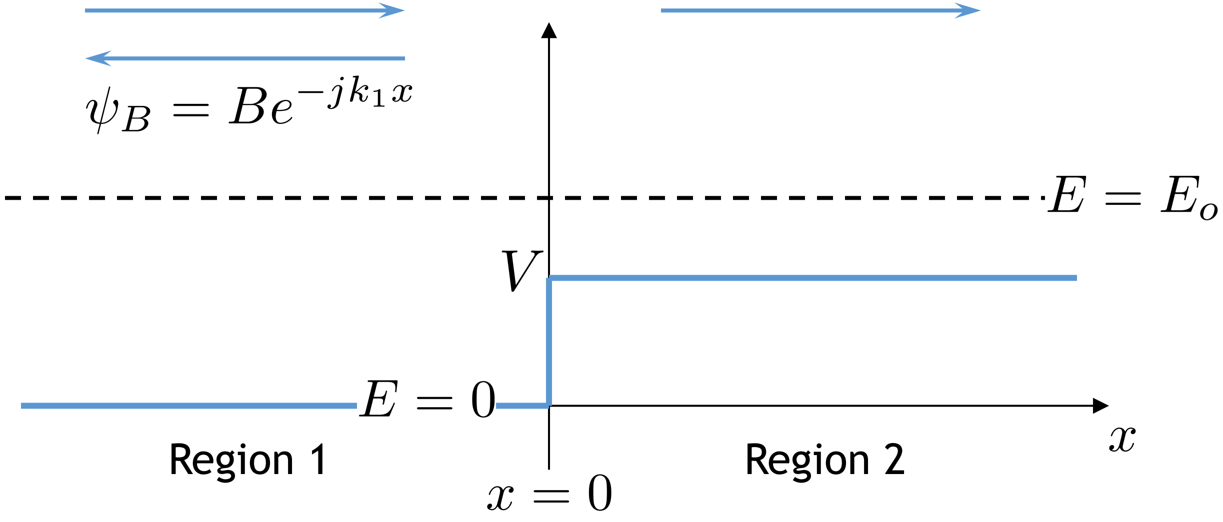
A Simple Potential Step

CASE I : $E_o > V$

$$\psi_A = Ae^{-jk_1x}$$

$$\psi_C = Ce^{-jk_1x}$$

$$\psi_B = Be^{-jk_1x}$$



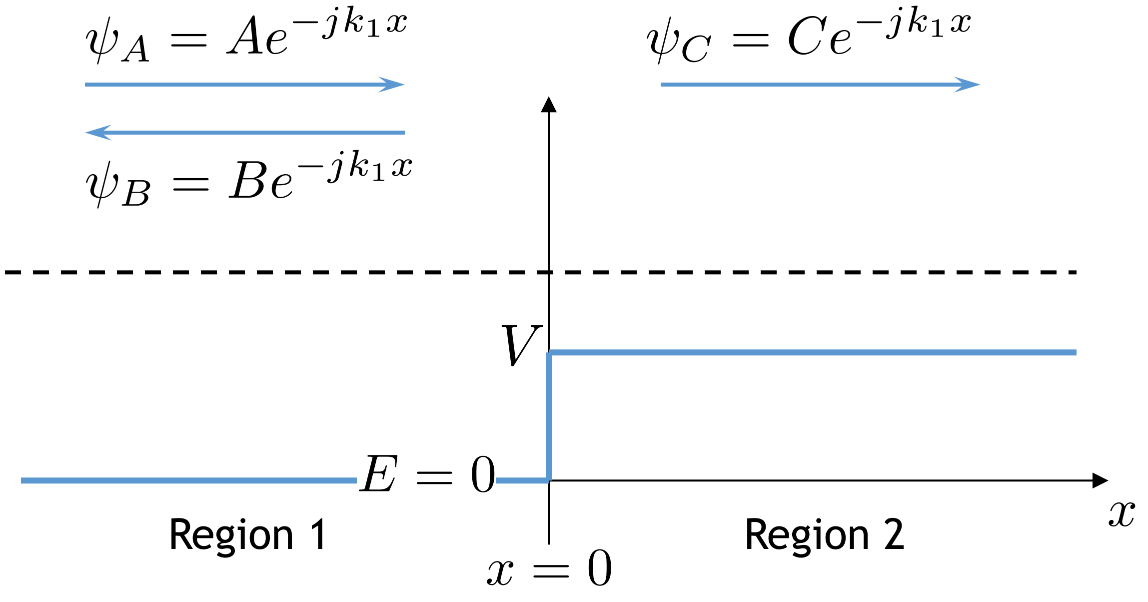
$$\text{Reflection} = R = \frac{J_{reflected}}{J_{incident}} = \frac{J_B}{J_A} = \frac{|\psi_B|^2(\hbar k_1/m)}{|\psi_A|^2(\hbar k_1/m)} = \left|\frac{B}{A}\right|^2$$

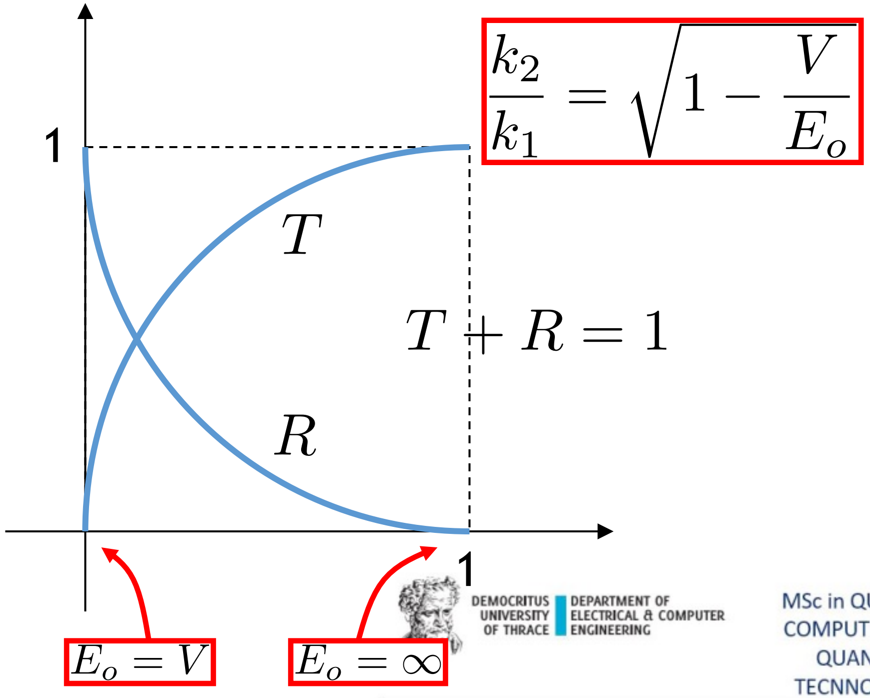
$$\text{Transmission} = T = \frac{J_{transmitted}}{J_{incident}} = \frac{J_C}{J_A} = \frac{|\psi_C|^2(\hbar k_2/m)}{|\psi_A|^2(\hbar k_1/m)} = \left|\frac{C}{A}\right|^2 \frac{k_2}{k_1}$$

$$\frac{B}{A} = \frac{1 - k_2/k_1}{1 + k_2/k_1} \qquad \frac{C}{A} = \frac{2}{1 + k_2/k_1}$$

A Simple Potential Step

CASE I : $E_o > V$



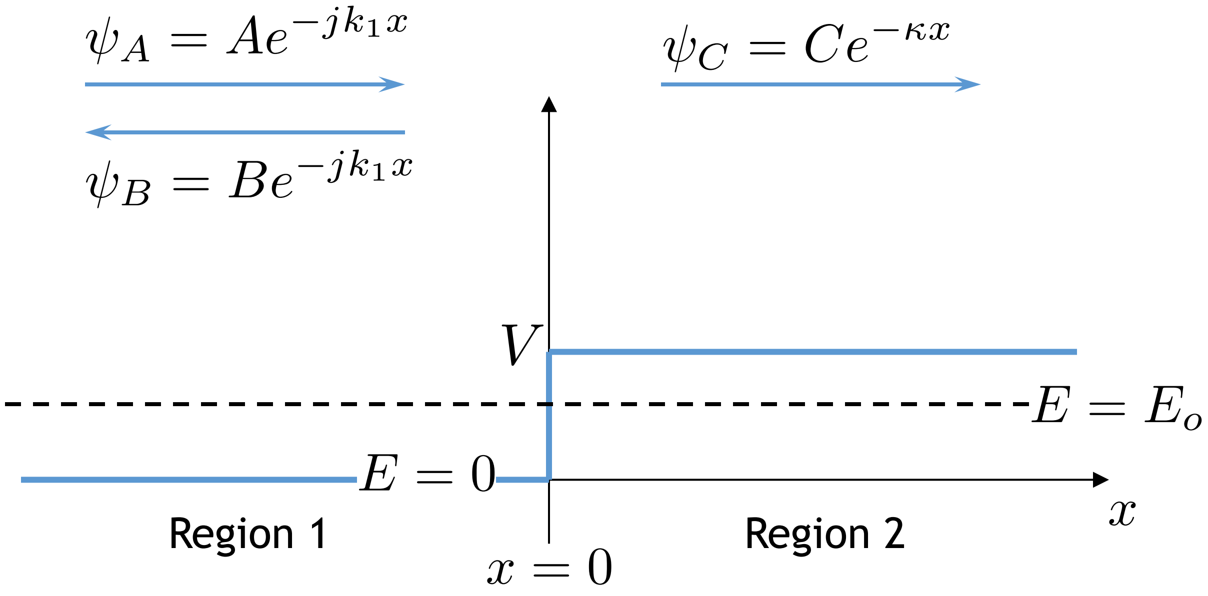


$$\text{Reflection} = R = \left|\frac{B}{A}\right|^2 = \left|\frac{k_1 - k_2}{k_1 + k_2}\right|^2$$

$$\text{Transmission} = T = 1 - R$$

$$= \frac{4k_1 k_2}{|k_1 + k_2|^2}$$

A Simple Potential Step



CASE II : $E_o < V$

In Region 1:

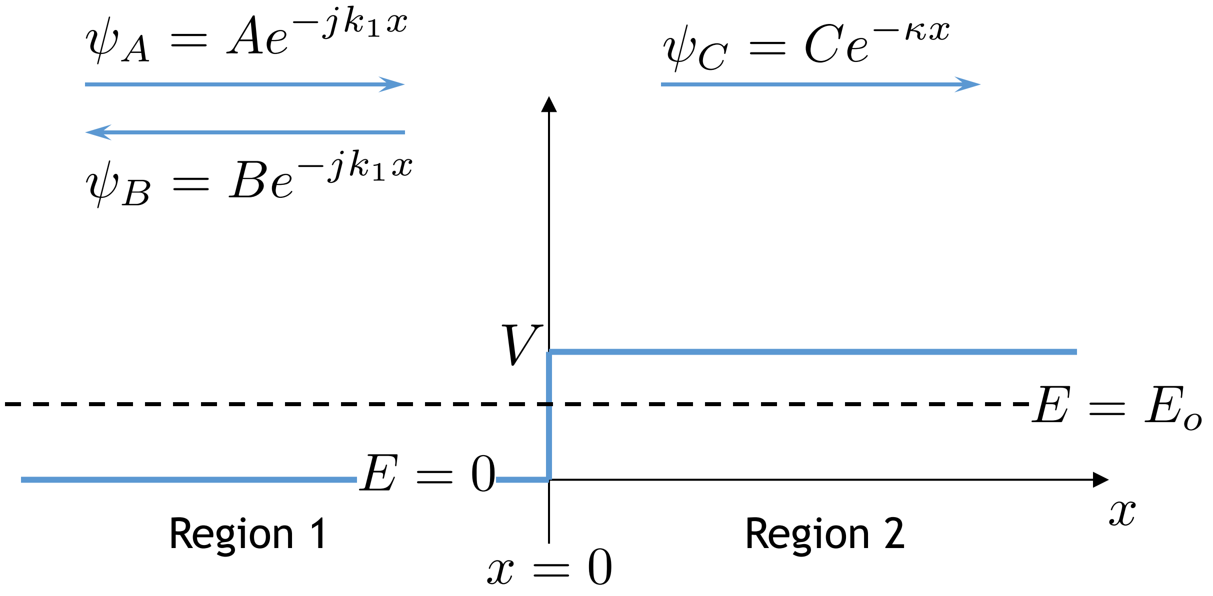
$$E_o\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \quad \Rightarrow \quad k_1^2 = \frac{2mE_o}{\hbar^2}$$

In Region 2:

$$\left(E_o - V\right)\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \quad \Rightarrow \quad \kappa^2 = \frac{2m\left(E_o - V\right)}{\hbar^2}$$

A Simple Potential Step

CASE II : E_o < V



$$\psi_1 = Ae^{-jk_1x} + Be^{jk_1x} \qquad \psi_2 = Ce^{-\kappa x}$$

ψ is continuous: $\psi_1(0) = \psi_2(0)$ ⇒ $A + B = C$

$\dfrac{\partial\psi}{\partial x}$ is continuous: $\dfrac{\partial}{\partial x}\psi(0) = \dfrac{\partial}{\partial x}\psi_2(0)$ ⇒ $A - B = -j\dfrac{\kappa}{k_1}C$

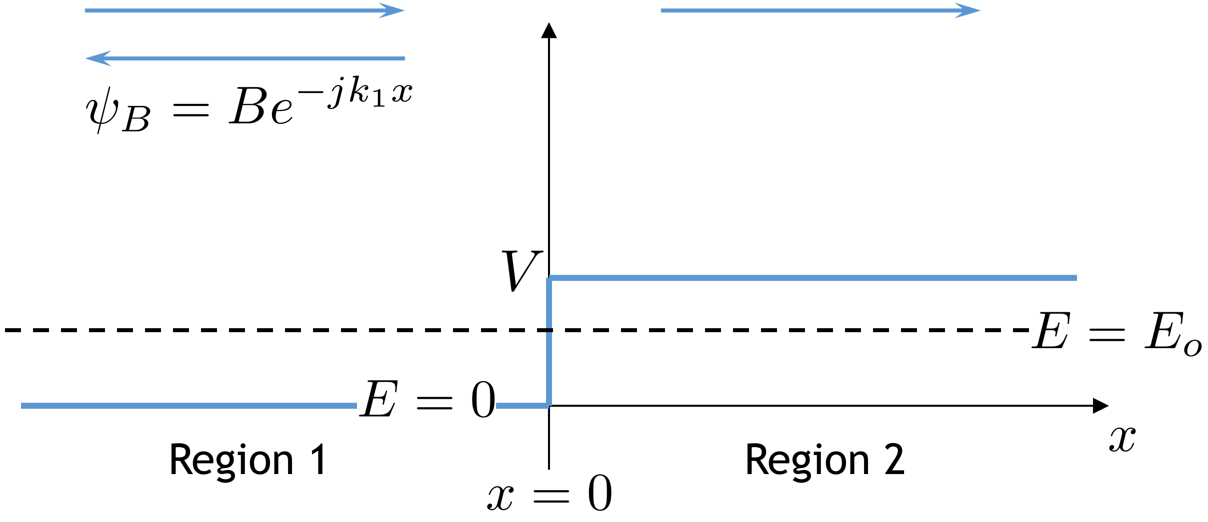
A Simple Potential Step

$$\psi_A = Ae^{-jk_1x}$$

$$\psi_B = Be^{-jk_1x}$$

$$\psi_C = Ce^{-\kappa x}$$

CASE II : $E_o < V$



$$\frac{B}{A} = \frac{1 + j\kappa/k_1}{1 - j\kappa/k_1} \qquad \frac{C}{A} = \frac{2}{1 - j\kappa/k_1}$$

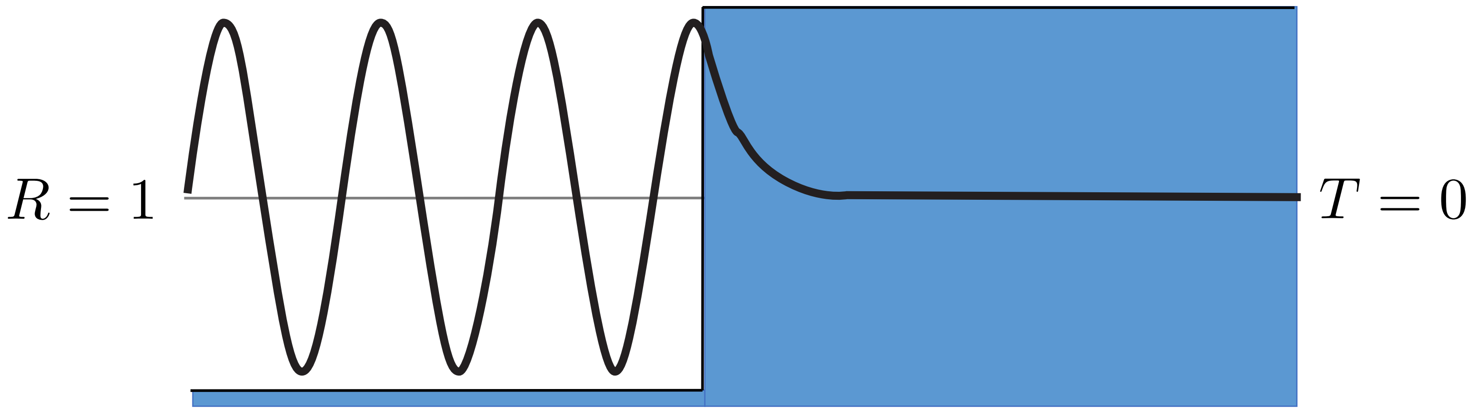
$$\begin{cases} A + B = C \\ A - B = -j\dfrac{\kappa}{k_1}C \end{cases}$$

$$R = \left|\frac{B}{A}\right|^2 = 1 \qquad T = 0$$

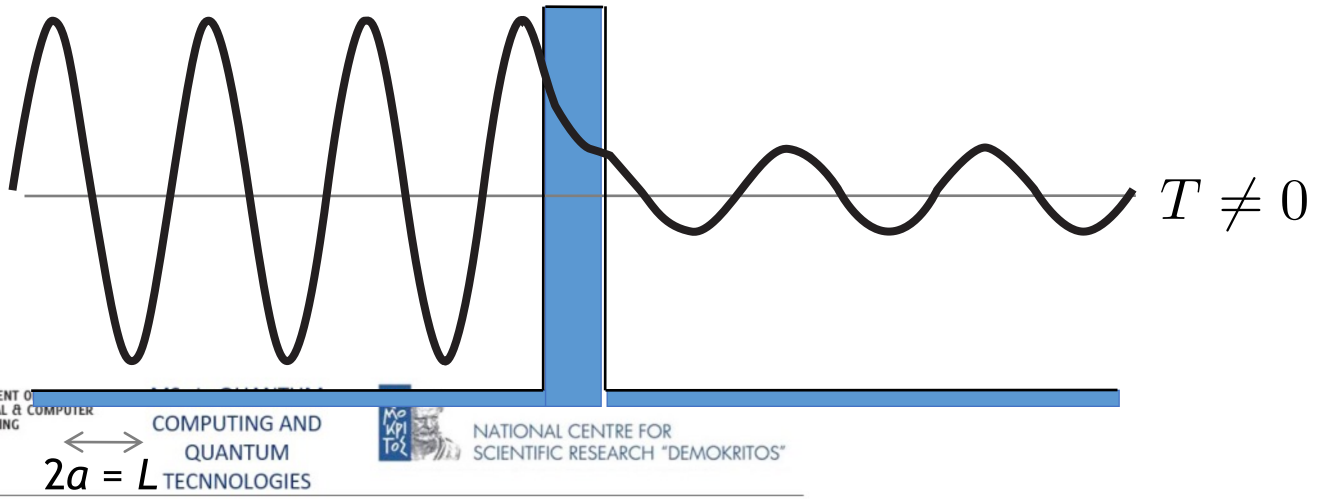
Total reflection → Transmission must be zero

Quantum Tunneling Through a Thin Potential Barrier

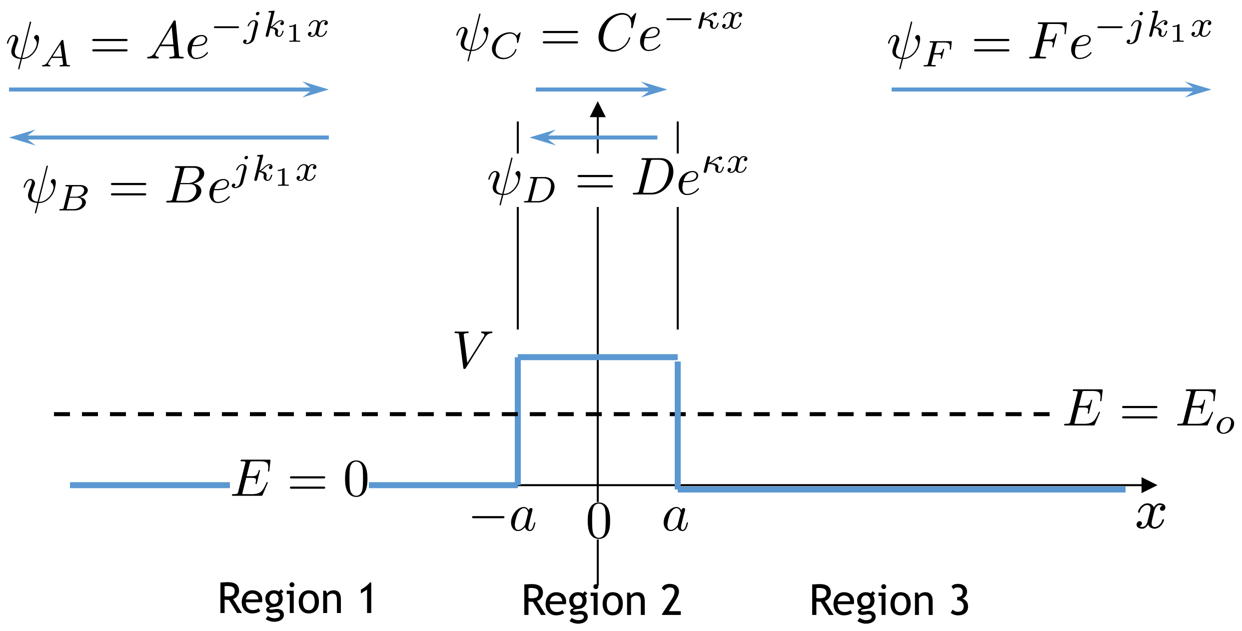
Total Reflection at Boundary



Frustrated Total Reflection (Tunneling)



A Rectangular Potential Step



CASE II : $E_o < V$

In Regions 1 and 3: $$E_o\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \Rightarrow k_1^2 = \frac{2mE_o}{\hbar^2}$$

In Region 2: $$(E_o - V)\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \Rightarrow \kappa^2 = \frac{2m(V - E_o)}{\hbar^2}$$

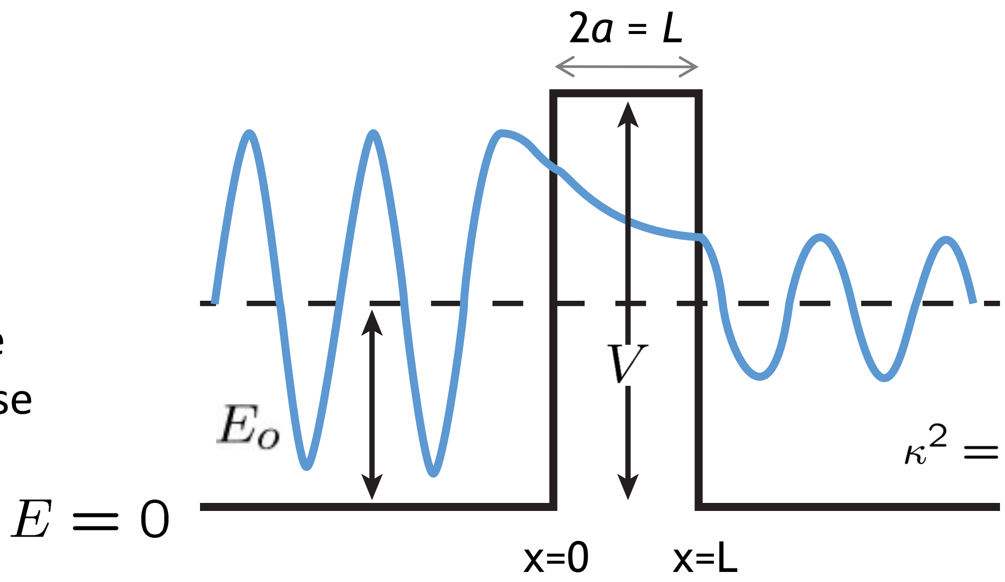
In Region 3?

for $E_o < V$:

$$T = \left|\frac{F}{A}\right|^2 = \frac{1}{1 + \frac{1}{4}\frac{V^2}{E_o(V-E_o)}\sinh^2(2\kappa a)}$$

A Rectangular Potential Step

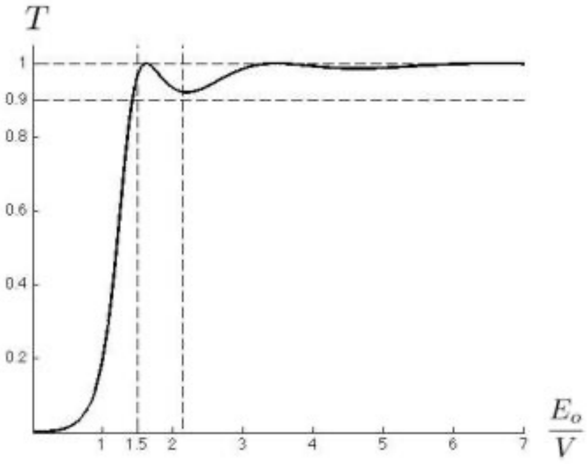
Real part of Ψ for $E_o < V$, shows hyperbolic (exponential) decay in the barrier domain and decrease in amplitude of the transmitted wave.



$$\kappa^2 = \frac{2m\,(V - E_o)}{\hbar^2}$$

for $E_o < V$:

$$T = \left|\frac{F}{A}\right|^2 = \frac{1}{1 + \frac{1}{4}\frac{V^2}{E_o(V - E_o)}\sinh^2(2\kappa a)}$$



Transmission Coefficient versus E_o/V for barrier with $2m(2a)^2V/\hbar = 16$

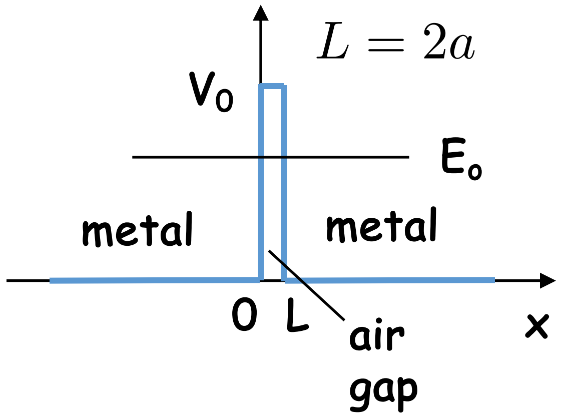
$$\sinh^2(2\kappa a) = \left[e^{2\kappa a} - e^{-2\kappa a}\right]^2 \approx e^{-4\kappa a}$$

$$T = \left|\frac{F}{A}\right|^2 \approx \frac{1}{1 + \frac{1}{4}\frac{V^2}{E_o(V - E_o)}}e^{-4\kappa a}$$

Example: Barrier Tunneling

- Let's consider a tunneling problem:

An electron with a total energy of E_o= 6 eV approaches a potential barrier with a height of V_0 = 12 eV. If the width of the barrier is L = 0.18 nm, what is the probability that the electron will tunnel through the barrier?



$$T = \left|\frac{F}{A}\right|^2 \approx \frac{16E_o(V - E_o)}{V^2}e^{-2\kappa L}$$

$$\kappa = \sqrt{\frac{2m_e}{\hbar^2}(V - E_o)} = 2\pi\sqrt{\frac{2m_e}{h^2}(V - E_o)} = 2\pi\sqrt{\frac{6\text{eV}}{1.505\text{eV-nm}^2}} \approx 12.6 \text{ nm}^{-1}$$

$$T = 4e^{-2(12.6 \text{ nm}^{-1})(0.18 \text{ nm})} = 4(0.011) = \boxed{4.4\%}$$

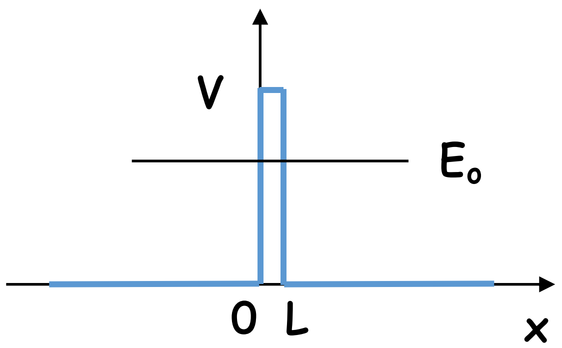
Question: What will T be if we double the width of the gap?

Multiple Choice Questions

Consider a particle tunneling through a barrier:

1. Which of the following will increase the likelihood of tunneling?

a. decrease the height of the barrier b. decrease the width of the barrier c. decrease the mass of the particle



2. What is the energy of the particles that have successfully "escaped"?

   a. < initial energy

   b. = initial energy

   c. > initial energy

Although the *amplitude* of the wave is smaller after the barrier, no energy is lost in the tunneling process

…to be continued

Thank you!

E-class Support

Lesson	Kassap	Fu
1	3.2	1.1, 1.3, 1.9, B4, B5
2	3.5	B8